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Technological advances in self-insurance and self-protection

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Abstract

This study investigates how technological advances in self-insurance (or self-protection) affect the optimal level of self-insurance (or self-protection) and that of insurance, if insurance is also taken into account. Conditions are derived for determining the signs of changes in the optimal levels of decision variables due to improved technology. Two cross-derivatives are found to be the key factors. Classification of technological advances is suggested based on the two cross-derivatives. The results show that when analyzed pairwise, “neutral” technological advances, according to the classification, decrease the optimal level of self-insurance and that of insurance, but increase the optimal level of self-protection and that of insurance.

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1. Introduction

Self-insurance, by definition, mitigates the severity of loss, whereas self-protection reduces the likelihood of loss. In what follows, these two risk reduction measures are collectively termed "self-efforts." This study is motivated by a proposition in the appendix of a seminal paper by Ehrlich and Becker (1972). The proposition, which has seemingly received little attention due to the lack of details, argues that the improvement of self-protection technology leads to an increase in the optimal level of self-protection and that of insurance under certain assumptions. Since their paper was published, there have been numerous extensions of it being worked out from various aspects. Nevertheless, it appears that none of the extensions has been found relevant to technological advances in self-efforts.

This study aims to cover a greater range of topics and provide insights into the fundamental causes in its own context. In addition to revisiting their proposition that analyzes self-protection along with insurance in different settings, this study also explores the effects of improved self-insurance or self-protection technology in the event that either self-insurance or self-protection is the only decision variable, and that self-insurance and insurance are considered jointly. Moreover, this study takes a closer look at the factors, especially two cross-derivatives, which determine whether the optimal levels of self-efforts and insurance increase, decrease, or remain the same with improved technologies. An approach is suggested to categorize technological advances by the two cross-derivatives.

Two main characteristics of modeling distinguish this study from Ehrlich and Becker (1972): (i) *Two-period framework.* Menegatti (2009) gives several convincing examples and points out that a suitable choice between a one-period model and a two-period model depends on whether the effort and the occurrence of loss are "contemporaneous." In this sense, two-period framework, other than their one-period models, may be natural to model intertemporal decisions for a scenario like purchasing fire extinguishers at present to reduce the loss potentially occurring sometime in the future. Furthermore, it seems inappropriate or even infeasible to analyze some decision variables, *e.g.* saving, financial or physical investments, in a one-period framework. A merit of two-period framework is allowing future studies to incorporate this type of decision variables into existing two-period models and make comparisons. (ii) *Nonlinear costs.* This study generalizes their linear costs of self-efforts and insurance to nonlinear ones so that the effects of nonlinearity can be investigated.

2. Technological Advances

2.1 Definitions of technological advances

We begin with the definitions of technological advances since they are not explicitly defined in Ehrlich and Becker (1972). As a concrete example, either more total capacity of the extinguishing agent contained in the extinguishers (self-insurance effort) or more effective extinguishing agent with improved chemical composition (self-insurance technology) mitigates the loss if it occurs. A parameter τ measuring the level of self-insurance technology is introduced to study its effects. It is called technological advance in self-insurance if the size of potential loss l is reduced at every given level of self-insurance effort η, which can be formulated as

$$l = l(\eta;\tau);\ \frac{\partial l}{\partial \tau} < 0. \tag{1}$$

Likewise, suppose that θ measures the level of self-protection technology. It is called technological advance in self-protection if loss probability p is lowered at every given level of self-protection effort e:

$$p = p(e;\theta);\ \frac{\partial p}{\partial \theta} < 0. \tag{2}$$

Note that efforts η and e are decision variables, whereas technologies τ and θ, parameters for the modeling in Section 3. Geometrically, l decreases with η along the curve $l = l(\eta;\tau)$, whereas an increase in τ causes a downward-shift of the curve. The same interpretation also applies to the self-protection case.

2.2 The cross-derivatives and technological advances classification

As will be shown later, it is to be highlighted that there are two cross-derivatives, $\partial l'/\partial\tau$ and $\partial p'/\partial\theta$, that play crucial roles in determining the signs of changes in the optimal levels of self-efforts as technologies are improved, where $l' = dl(\eta;\tau)/d\eta < 0$ and $p' = dp(e;\theta)/de < 0$ are the "efficiencies of self-efforts" in contrast to the "efficiencies of technologies," $\partial l/\partial\tau < 0$ and $\partial p/\partial\theta < 0$. The intuition behind the cross-derivatives is elaborated as follows: improved extinguishing agent *per se* mitigates the potential loss in terms of technology ($\partial l/\partial\tau < 0$). On the other hand, for example, suppose its density or viscosity may extend or shorten the range to which it can be discharged and thus may enhance ($\partial l'/\partial\tau < 0$) or undermine ($\partial l'/\partial\tau > 0$) the efficiency ($l' < 0$) of extinguishing agent in terms of effort.

With these two cross-derivatives, we are allowed to classify technological advances into three categories by whether technological advances constructively or destructively interfere the efficiencies of self-efforts. For convenience, the three categories are tentatively termed as follows:

(i) *Undermining technological advances* $(\partial l'/\partial\tau > 0,\ \partial p'/\partial\theta > 0)$;

(ii) *Neutral technological advances* $(\partial l'/\partial\tau = 0,\ \partial p'/\partial\theta = 0)$;

(iii) *Enhancing technological advances* $(\partial l'/\partial\tau < 0,\ \partial p'/\partial\theta < 0)$.

To picture the ideas mentioned above, we summarize with Fig.1, 2, and 3 for the self-insurance case. The figures for the self-protection case can be easily obtained by replacing (l, η, τ) with (p, e, θ). Note that the slope of a curve in the figures is the efficiency of self-effort l'. Thus, the steeper the slope, the better the efficiency.

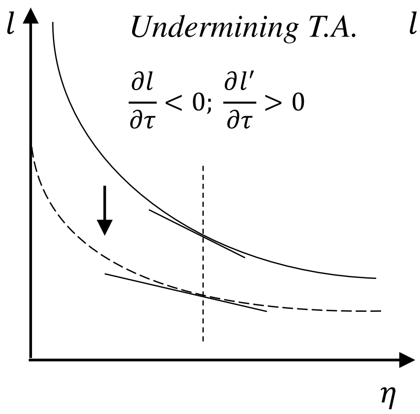

Fig. 1. The slope becomes flatter at every level of efforts after the shift due to undermining T.A.

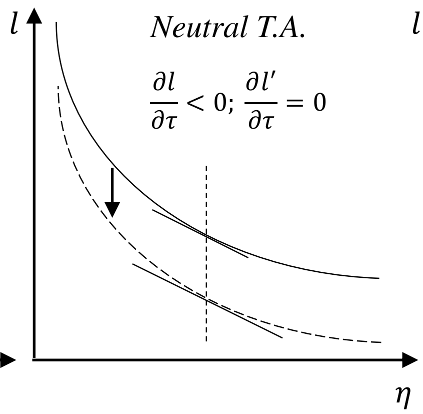

Fig. 2. The slope remains the same at every level of efforts after the shift due to neutral T.A.

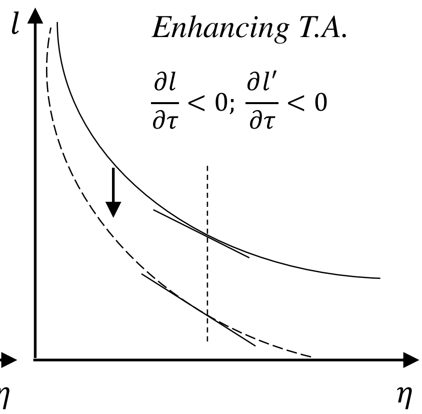

Fig. 3. The slope becomes steeper at every level of efforts after the shift due to enhancing T.A.

3. The Models

3.1 Self-Insurance

This subsection considers the case that the only means of risk mitigation under consideration is self-insurance, *e.g.*, pesticide used to reduce the agricultural damage caused by pests. Either a larger amount of pesticide (effort) or improved composition of pesticide (technology) mitigates the damage. Advanced composition kills pests more effectively but may shorten (undermining T.A.) or lengthen (enhancing T.A.) the duration of pesticide that remains in the fields (efficiency of effort). Risk averters are to optimally allocate their initial wealth w_i over two periods: the present and the future, denoted by subscript 1 and 2 respectively. They face a potential loss with probability $p \in (0,1)$ and size $l(\eta;\tau)$, which can be mitigated by self-insurance effort η $(l' < 0;\ l'' \geq 0)$ as well as the level of self-insurance technology τ $(\partial l/\partial\tau < 0)$. Suppose that subscript *L* stands for the loss state, whereas *N*, the no-loss state. Their utilities u_i are assumed to be separable and additive over two periods and discounted by $\beta \in (0,1]$, which sum up to the objective function U. They maximize U by choosing the optimal level of η with cost function $c(\eta)$ $(c' > 0;\ c'' \geq 0)$, as shown in the following model:

$$\max_{\eta} U = u_1 + \beta[pu_{2L} + (1-p)u_{2N}], \quad (3)$$

where $u_1 = u\big(w_1 - c(\eta)\big)$; $u_{2L} = u\big(w_2 - l(\eta;\tau)\big)$; $u_{2N} = u(w_2)$.

Let the asterisk symbol (*) denote dependent variables evaluated at a critical point. Suppose that there exists η^* so that the first-order condition holds:

$$U_\eta^* = -c'^* u_1'^* - \beta p u_{2L}'^* l'^* = 0. \quad (4)$$

The second-order condition is satisfied under the assumptions made above:

$$U_{\eta\eta}^* = -c''^* u_1'^* + (c'^*)^2 u_1''^* + \beta p u_{2L}''^* (l'^*)^2 - \beta p u_{2L}'^* l''^* < 0. \quad (5)$$

First of all, one might ask whether an agent would choose to stay with old technology if only "undermining" technological advances were available. Proposition 1 rules out the possibility:

Proposition 1. (See Appendix A for proof.)

All technological advances defined in this study, even undermining technological advances, are always desired by an agent.

After the comparative statics analysis, we have the condition for determining the sign of the optimal level of self-insurance as in (6) (See Appendix B for details):

$$\text{sgn}\left\{\frac{\partial \eta^*}{\partial \tau}\right\} = \text{sgn}\left\{\frac{\frac{\partial l'^*}{\partial \tau}}{\frac{\partial l^*}{\partial \tau}} + l'^* R_{2L}^*\right\}, \quad (6)$$

where "sgn" represents a sign function and $R_{2L}^* = -u_{2L}''^*/u_{2L}'^*$, the coefficient of absolute risk aversion evaluated at the optimum in the loss state during period 2. It may be worth noting the following two points:

(i) With $\partial l^*/\partial \tau < 0$, it can be observed from (6) that the optimal level of self-effort is inclined to increase, remain the same or decrease with enhancing, neutral, or undermining technological advances, respectively. It is true throughout the paper.

(ii) In fact, all the conditions for determining the signs in this paper take the same form as in (6) (see Appendix B, C, E, and G for the conditions in each case). There are two terms in the braces on the right hand side of the equity: the first term concerns technological improvement, whereas the second term concerns the other factors as a whole, including risk aversion if self-insurance is involved. These two terms simultaneously determine the sign. In some cases, factors other than technological advances are irrelevant and hence the second term vanishes from the braces. Thus, with $\partial l^*/\partial \tau < 0$, the sign depends only on the cross-derivative as shown in (11) and (20).

To be concise, we will not repeatedly go through the above two points in the latter part of this paper. From (6), we readily obtain Proposition 2:

Proposition 2. (See Appendix B for proof.)

The optimal level of self-insurance declines with neutral or undermining technological advances:

$$\frac{\partial l'^*}{\partial \tau} \geq 0 \Rightarrow \frac{\partial \eta^*}{\partial \tau} < 0. \quad (7)$$

3.2 Self-Protection

In this subsection, all the settings remain the same as in Subsection 3.1 except that the only risk management tool considered here is self-protection, *e.g.*, burglar alarms used to reduce the probability of burglary. Either a larger number of alarms (effort) or improved effectiveness of alarms (technology) lowers the likelihood of burglary. However, for example, the component or the design that improves effectiveness may shorten (undermining T.A.) or extend (enhancing T.A.) each alarm's range of detection (efficiency of effort). To model the above scenario, suppose that the probability of loss $p(e;\theta)$ $(p' < 0;\ p'' \geq 0)$ may be lowered by either more self-protection effort e with cost function $k(e)$ $(k' > 0;\ k'' \geq 0)$ or better self-protection technology θ $(\partial p/\partial \theta < 0)$. The model can be expressed as

$$\max_{e} U = u_1 + \beta\left[p(e;\theta)u_{2L} + \big(1 - p(e;\theta)\big)u_{2N}\right], \quad (8)$$

where $u_1 = u\big(w_1 - k(e)\big);\quad u_{2L} = u(w_2 - l);\quad u_{2N} = u(w_2)$.

The first-order condition:

$$U_e^* = -k'^* u_1'^* + \beta p'^*(u_{2L}^* - u_{2N}) = 0. \quad (9)$$

The second-order condition holds:

$$U_{ee}^* = -k''^* u_1'^* + (k'^*)^2 u_1''^* + \beta p''^*(u_{2L}^* - u_{2N}) < 0. \quad (10)$$

The comparative statics analysis yields Proposition 3:

Proposition 3. (See Appendix C for proof.)

As self-protection technology is improved, the sign of change in the optimal level of self-protection is opposite to that of the cross-derivative $\partial p'^*/\partial \theta$:

$$\operatorname{sgn}\left\{\frac{\partial e^*}{\partial \theta}\right\} = -\operatorname{sgn}\left\{\frac{\partial p'^*}{\partial \theta}\right\}. \quad (11)$$

3.3 Self-Insurance and Insurance

In the following model, self-insurance and insurance are analyzed jointly, *e.g.*, allocating wealth among consumption, pesticide, and insurance against damage caused by pests. Insurance premium is determined by a pricing schedule $\pi(q)$ $(\pi' > 0;\ \pi'' \geq 0)$, which is a generalization of linear insurance pricing schedule $\pi(q) = \bar{\pi}q$ with constant price of insurance $\bar{\pi}$, as assumed in Ehrlich and Becker (1972).

$$\max_{\eta,q} U = u_1 + \beta[pu_{2L} + (1-p)u_{2N}], \quad (12)$$

where $u_1 = u\big(w_1 - c(\eta) - \pi(q)\big);\quad u_{2L} = u(w_2 - l(\eta;\tau) + q);\quad u_{2N} = u(w_2)$.
The first-order conditions:

$$U_\eta^* = -c'^* u_1'^* - \beta p u_{2L}'^* l'^* = 0, \quad (13)$$

$$U_q^* = -\pi'^* u_1'^* + \beta p u_{2L}'^* = 0. \quad (14)$$

The second-order conditions are satisfied:

$$U_{\eta\eta}^* = -c''^* u_1'^* + (c'^*)^2 u_1''^* + \beta p u_{2L}''^* (l'^*)^2 - \beta p u_{2L}'^* l''^* < 0, \quad (15)$$

$$U_{qq}^* = -\pi''^* u_1'^* + (\pi'^*)^2 u_1''^* + \beta p u_{2L}''^* < 0, \quad (16)$$

$$U_{\eta q}^* = c'^* \pi'^* u_1''^* - \beta p u_{2L}''^* l'^* < 0, \quad (17)$$

$$H_{\eta,q} = U_{\eta\eta}^* U_{qq}^* - \left(U_{\eta q}^*\right)^2 > 0. \quad \text{(See Appendix D for proof.)} \quad (18)$$

The comparative statics analysis concludes with Proposition 4:

Proposition 4. (See Appendix E for proof.)
Undermining or neutral technological advances decrease the optimal level of self-protection, whereas enhancing or neutral technological advances decrease the optimal level of insurance:

$$\frac{\partial l'^*}{\partial \tau} \geq 0 \Rightarrow \frac{\partial \eta^*}{\partial \tau} < 0;\quad \frac{\partial l'^*}{\partial \tau} \leq 0 \Rightarrow \frac{\partial q^*}{\partial \tau} < 0. \quad (19)$$

In particular, for linear insurance pricing schedule, we have $\pi(q) = \bar{\pi}q$ or $\pi'' = 0$. Thus, the only factor that matters for determining the sign of $\partial\eta^*/\partial\tau$ is the cross-derivative $\partial l'^*/\partial\tau$, whose sign is opposite to $\partial\eta^*/\partial\tau$:

$$\pi(q) = \bar{\pi}q \Rightarrow \text{sgn}\left\{\frac{\partial \eta^*}{\partial \tau}\right\} = -\text{sgn}\left\{\frac{\partial l'^*}{\partial \tau}\right\}. \quad (20)$$

When the above equalities both hold in (19), *i.e.,* in the case of neutral self-insurance technological advances, the optimal level of self-insurance and that of insurance decline.

3.4 Self-Protection and Insurance

Following a similar path in the previous subsection, we study here how the optimal level of self-protection and that of insurance respond to technological advances in self-protection, *e.g.*, allocating wealth among consumption, burglar alarms, and insurance against burglary:

$$\max_{e,q} U = u_1 + \beta\big[p(e;\theta)u_{2L} + \big(1 - p(e;\theta)\big)u_{2N}\big], \quad (21)$$

where $u_1 = u\big(w_1 - k(e) - \pi(q)\big);\quad u_{2L} = u(w_2 - l + q);\quad u_{2N} = u(w_2)$.

The first-order conditions:

$$U_e^* = -k'^* u_1'^* + \beta p'^* (u_{2L}^* - u_{2N}) = 0, \tag{22}$$

$$U_q^* = -\pi'^* u_1'^* + \beta p^* u_{2L}'^* = 0. \tag{23}$$

The second-order conditions:

$$U_{ee}^* = -k''^* u_1'^* + (k'^*)^2 u_1''^* + \beta p''^* (u_{2L}^* - u_{2N}) < 0, \tag{24}$$

$$U_{qq}^* = -\pi''^* u_1'^* + (\pi'^*)^2 u_1''^* + \beta p^* u_{2L}''^* < 0, \tag{25}$$

$$U_{eq}^* = k'^* \pi'^* u_1''^* + \beta p'^* u_{2L}'^* < 0. \tag{26}$$

However, the sign of the Hessian is ambiguous and assumed to be positive as done in Ehrlich and Becker (1972) (See Appendix F for details). By implementing the comparative statics analysis, we have Proposition 5:

Proposition 5. (See Appendix G for proof.)
Both the optimal level of self-protection and that of insurance increase with neutral and enhancing self-protection technological advances:

$$\frac{\partial p'^*}{\partial \theta} \leq 0 \Rightarrow \frac{\partial e^*}{\partial \theta} > 0; \quad \frac{\partial p'^*}{\partial \theta} \leq 0 \Rightarrow \frac{\partial q^*}{\partial \theta} > 0. \tag{27}$$

Proposition 5 can be regarded as a robustness test since the above results are in conformity with the proposition presented in Ehrlich and Becker (1972) in a different context.

4. Conclusion

Undermining, neutral, or enhancing technological advances tend to decrease, maintain, or increase the optimal levels of self-efforts, respectively. Nevertheless, non-neutral technological advances in self-efforts may conflict with the resultant effects of the other factors and hence may have ambiguous effects. By contrast, the effects of neutral technological advances are clear-cut. When either self-insurance or self-protection is considered individually, neutral technological advances decrease the optimal level of self-insurance but do not affect that of self-protection. When either self-insurance or self-protection is analyzed along with insurance, the optimal levels of self-insurance and insurance decrease, whereas those of self-protection and insurance increase, with neutral technological advances. Furthermore, there unambiguously appears to be a sort of symmetry that undermining technological advances decrease the optimal level of self-insurance, whereas enhancing technological advances increase that of self-protection.[1]

[1] The author thanks an anonymous referee for pointing out the symmetry.

References

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Appendix

$l^* = l(\eta^*(w_1, w_2, \beta, p, \tau\,); \tau);\ c^* = c\big(\eta^*(w_1, w_2, \beta, p, \tau\,)\big);$
$p^* = p(e^*(w_1, w_2, \beta, l, \theta\,); \theta);\ k^* = k\big(e^*(w_1, w_2, \beta, l, \theta\,)\big).$

A. Proof of Proposition 1

$$\frac{\partial U^*}{\partial \tau} = -c'^* u_1'^* \frac{\partial \eta^*}{\partial \tau} - \beta p u_{2L}'^* \left(l'^* \frac{\partial \eta^*}{\partial \tau} + \frac{\partial l^*}{\partial \tau} \right) = (-c'^* u_1'^* - \beta p u_{2L}'^* l'^*) \frac{\partial \eta^*}{\partial \tau} - \beta p u_{2L}'^* \frac{\partial l^*}{\partial \tau}$$
$$= -\beta p u_{2L}'^* \frac{\partial l^*}{\partial \tau} > 0 \quad \left(\because -c'^* u_1'^* - \beta p u_{2L}'^* l'^* = 0 \text{ and } \frac{\partial l^*}{\partial \tau} < 0. \right)$$

Technological advances in the other three cases in Section 3.2, 3.3, and 3.4 can also be proved advantageous for an agent in a similar manner.

B. Proof of Proposition 2

$$-c'^* u_1'^* - \beta p u_{2L}'^* l'^* = 0$$
$$\Rightarrow -c''^* \frac{\partial \eta^*}{\partial \tau} u_1'^* + (c'^*)^2 u_1''^* \frac{\partial \eta^*}{\partial \tau} + \beta p u_{2L}''^* \left(l'^* \frac{\partial \eta^*}{\partial \tau} + \frac{\partial l^*}{\partial \tau} \right) l'^* - \beta p u_{2L}'^* \left(l''^* \frac{\partial \eta^*}{\partial \tau} + \frac{\partial l'^*}{\partial \tau} \right) = 0$$
$$\Rightarrow [-c''^* u_1'^* + (c'^*)^2 u_1''^* + \beta p u_{2L}''^* (l'^*)^2 - \beta p u_{2L}'^* l''^*] \frac{\partial \eta^*}{\partial \tau} = \beta p u_{2L}'^* \frac{\partial l'^*}{\partial \tau} - \beta p u_{2L}''^* l'^* \frac{\partial l^*}{\partial \tau}$$
$$\Rightarrow U_{\eta\eta}^* \frac{\partial \eta^*}{\partial \tau} = \beta p u_{2L}'^* \frac{\partial l'^*}{\partial \tau} - \beta p u_{2L}''^* l'^* \frac{\partial l^*}{\partial \tau}$$
$$\Rightarrow \frac{\partial \eta^*}{\partial \tau} = \frac{\beta p u_{2L}'^* \frac{\partial l'^*}{\partial \tau} - \beta p u_{2L}''^* l'^* \frac{\partial l^*}{\partial \tau}}{U_{\eta\eta}^*} = \frac{\beta p u_{2L}'^* \frac{\partial l^*}{\partial \tau}}{U_{\eta\eta}^*} \left(\frac{\frac{\partial l'^*}{\partial \tau}}{\frac{\partial l^*}{\partial \tau}} - l'^* \frac{u_{2L}''^*}{u_{2L}'^*} \right)$$
$$\Rightarrow \operatorname{sgn}\left\{ \frac{\partial \eta^*}{\partial \tau} \right\} = \operatorname{sgn}\left\{ \frac{\frac{\partial l'^*}{\partial \tau}}{\frac{\partial l^*}{\partial \tau}} + l'^* R_{2L}^* \right\};\ \text{we then have } \frac{\partial l'^*}{\partial \tau} \geq 0 \Rightarrow \frac{\partial \eta^*}{\partial \tau} < 0.$$

C. Proof of Proposition 3

$$-k'^* u_1'^* + \beta p'^* (u_{2L}^* - u_{2N}^*) = 0$$

$$\Rightarrow -k''^* \frac{\partial e^*}{\partial \theta} u_1'^* + (k'^*)^2 u_1''^* \frac{\partial e^*}{\partial \theta} + \beta \left(p''^* \frac{\partial e^*}{\partial \theta} + \frac{\partial p'^*}{\partial \theta} \right) (u_{2L}^* - u_{2N}^*) = 0$$

$$\Rightarrow [-k''^* u_1'^* + (k'^*)^2 u_1''^* + \beta p''^* (u_{2L}^* - u_{2N}^*)] \frac{\partial e^*}{\partial \theta} = -\beta (u_{2L}^* - u_{2N}^*) \frac{\partial p'^*}{\partial \theta}$$

$$\Rightarrow U_{ee}^* \frac{\partial e^*}{\partial \theta} = -\beta (u_{2L}^* - u_{2N}^*) \frac{\partial p'^*}{\partial \theta} \Rightarrow \frac{\partial e^*}{\partial \theta} = \frac{-\beta (u_{2L}^* - u_{2N}^*) \frac{\partial p'^*}{\partial \theta}}{U_{ee}^*}$$

$$\Rightarrow \mathrm{sgn} \left\{ \frac{\partial e^*}{\partial \theta} \right\} = -\mathrm{sgn} \left\{ \frac{\partial p'^*}{\partial \theta} \right\}$$

D. Derivation of the Hessian in Subsection 3.3

Combining (13) with (14) yields

$$l'^* = -\frac{c'^*}{\pi'^*} < 0$$

$$\begin{aligned}
H_{\eta,q} &= U_{\eta\eta}^* U_{qq}^* - \left(U_{\eta q}^* \right)^2 \\
&= [-c''^* u_1'^* + (c'^*)^2 u_1''^* + \beta p u_{2L}''^* (l'^*)^2 - \beta p u_{2L}'^* l''^*][-\pi''^* u_1'^* + (\pi'^*)^2 u_1''^* + \beta p u_{2L}''^*] \\
&\quad - [c'^* \pi'^* u_1''^* - \beta p u_{2L}''^* l'^*]^2 \\
&= c''^* \pi''^* (u_1'^*)^2 - (c'^*)^2 \pi''^* u_1'^* u_1''^* - \beta p \pi''^* u_1'^* u_{2L}''^* (l'^*)^2 + \beta p \pi''^* u_1'^* u_{2L}'^* l''^* - c''^* (\pi'^*)^2 u_1''^* u_1'^* \\
&\quad + (c'^*)^2 (\pi'^*)^2 (u_1''^*)^2 + \beta p (\pi'^*)^2 u_1''^* u_{2L}''^* (l'^*)^2 - \beta p (\pi'^*)^2 u_1''^* u_{2L}'^* l''^* \\
&\quad - \beta p c''^* u_1'^* u_{2L}''^* + \beta p (c'^*)^2 u_1''^* u_{2L}''^* + (\beta p)^2 (u_{2L}''^*)^2 (l'^*)^2 - (\beta p)^2 u_{2L}'^* u_{2L}''^* l''^* \\
&\quad - (c'^*)^2 (\pi'^*)^2 (u_1''^*)^2 + 2 \beta p c'^* \pi'^* u_1''^* u_{2L}''^* l'^* - (\beta p)^2 (u_{2L}''^*)^2 (l'^*)^2 \\
&= c''^* \pi''^* (u_1'^*)^2 - (c'^*)^2 \pi''^* u_1'^* u_1''^* - \beta p \pi''^* u_1'^* u_{2L}''^* (l'^*)^2 + \beta p \pi''^* u_1'^* u_{2L}'^* l''^* - c''^* (\pi'^*)^2 u_1''^* u_1'^* \\
&\quad + \beta p u_1''^* u_{2L}''^* (\pi'^* l'^* + c'^*)^2 - \beta p (\pi'^*)^2 u_1''^* u_{2L}'^* l''^* - \beta p c''^* u_1'^* u_{2L}''^* \\
&\quad - (\beta p)^2 u_{2L}'^* u_{2L}''^* l''^* \\
&= c''^* \pi''^* (u_1'^*)^2 - (c'^*)^2 \pi''^* u_1'^* u_1''^* - \beta p \pi''^* u_1'^* u_{2L}''^* (l'^*)^2 + \beta p \pi''^* u_1'^* u_{2L}'^* l''^* - c''^* (\pi'^*)^2 u_1''^* u_1'^* \\
&\quad - \beta p (\pi'^*)^2 u_1''^* u_{2L}'^* l''^* - \beta p c''^* u_1'^* u_{2L}''^* - (\beta p)^2 u_{2L}'^* u_{2L}''^* l''^* \quad (\because \pi'^* l'^* = -c'^*) \\
&> 0
\end{aligned}$$

E. Proof of Proposition 4

$$-c'^* u_1'^* - \beta p u_{2L}'^* l'^* = 0$$

$$\begin{aligned}
\Rightarrow -c''^* \frac{\partial \eta^*}{\partial \tau} u_1'^* - c'^* u_1''^* \left(-c'^* \frac{\partial \eta^*}{\partial \tau} - \pi'^* \frac{\partial q^*}{\partial \tau} \right) - \beta p u_{2L}''^* \left(-l'^* \frac{\partial \eta^*}{\partial \tau} - \frac{\partial l^*}{\partial \tau} + \frac{\partial q^*}{\partial \tau} \right) l'^* \\
- \beta p u_{2L}'^* \left(l''^* \frac{\partial \eta^*}{\partial \tau} + \frac{\partial l'^*}{\partial \tau} \right) = 0
\end{aligned}$$

$$\Rightarrow -c''^* u_1'^* \frac{\partial \eta^*}{\partial \tau} + (c'^*)^2 u_1''^* \frac{\partial \eta^*}{\partial \tau} + c'^* \pi'^* u_1''^* \frac{\partial q^*}{\partial \tau} + \beta p u_{2L}''^* (l'^*)^2 \frac{\partial \eta^*}{\partial \tau} + \beta p u_{2L}''^* l'^* \frac{\partial l^*}{\partial \tau}$$

$$- \beta p u_{2L}''^* l'^* \frac{\partial q^*}{\partial \tau} - \beta p u_{2L}'^* l''^* \frac{\partial \eta^*}{\partial \tau} - \beta p u_{2L}'^* \frac{\partial l'^*}{\partial \tau} = 0$$

$$\Rightarrow [-c''^* u_1'^* + (c'^*)^2 u_1''^* + \beta p u_{2L}''^* (l'^*)^2 - \beta p u_{2L}'^* l''^*] \frac{\partial \eta^*}{\partial \tau} + [c'^* \pi'^* u_1''^* - \beta p u_{2L}''^* l'^*] \frac{\partial q^*}{\partial \tau}$$

$$= -\beta p u_{2L}''^* l'^* \frac{\partial l^*}{\partial \tau} + \beta p u_{2L}'^* \frac{\partial l'^*}{\partial \tau}$$

$$\Rightarrow U_{\eta\eta}^* \frac{\partial \eta^*}{\partial \tau} + U_{\eta q}^* \frac{\partial q^*}{\partial \tau} = -\beta p u_{2L}''^* l'^* \frac{\partial l^*}{\partial \tau} + \beta p u_{2L}'^* \frac{\partial l'^*}{\partial \tau}$$

$$-\pi'^* u_1'^* + \beta p u_{2L}'^* = 0$$

$$\Rightarrow -\pi''^* \frac{\partial q^*}{\partial \tau} u_1'^* - \pi'^* u_1''^* \left(-c'^* \frac{\partial \eta^*}{\partial \tau} - \pi'^* \frac{\partial q^*}{\partial \tau}\right) + \beta p u_{2L}''^* \left(-l'^* \frac{\partial \eta^*}{\partial \tau} - \frac{\partial l^*}{\partial \tau} + \frac{\partial q^*}{\partial \tau}\right) = 0$$

$$\Rightarrow -\pi''^* u_1'^* \frac{\partial q^*}{\partial \tau} + c'^* \pi'^* u_1''^* \frac{\partial \eta^*}{\partial \tau} + (\pi'^*)^2 u_1''^* \frac{\partial q^*}{\partial \tau} - \beta p u_{2L}''^* l'^* \frac{\partial \eta^*}{\partial \tau} - \beta p u_{2L}''^* \frac{\partial l^*}{\partial \tau} + \beta p u_{2L}''^* \frac{\partial q^*}{\partial \tau}$$

$$= 0$$

$$\Rightarrow [c'^* \pi'^* u_1''^* - \beta p u_{2L}''^* l'^*] \frac{\partial \eta^*}{\partial \tau} + [-\pi''^* u_1'^* + (\pi'^*)^2 u_1''^* + \beta p u_{2L}''^*] \frac{\partial q^*}{\partial \tau} = \beta p u_{2L}''^* \frac{\partial l^*}{\partial \tau}$$

$$\Rightarrow U_{\eta q}^* \frac{\partial \eta^*}{\partial \tau} + U_{qq}^* \frac{\partial q^*}{\partial \tau} = \beta p u_{2L}''^* \frac{\partial l^*}{\partial \tau}$$

$$\begin{bmatrix} U_{\eta\eta}^* & U_{\eta q}^* \\ U_{\eta q}^* & U_{qq}^* \end{bmatrix} \begin{bmatrix} \frac{\partial \eta^*}{\partial \tau} \\ \frac{\partial q^*}{\partial \tau} \end{bmatrix} = \begin{bmatrix} -\beta p u_{2L}''^* l'^* \frac{\partial l^*}{\partial \tau} + \beta p u_{2L}'^* \frac{\partial l'^*}{\partial \tau} \\ \beta p u_{2L}''^* \frac{\partial l^*}{\partial \tau} \end{bmatrix}$$

$$\Delta_{\eta/\tau} = \begin{vmatrix} -\beta p u_{2L}''^* l'^* \frac{\partial l^*}{\partial \tau} + \beta p u_{2L}'^* \frac{\partial l'^*}{\partial \tau} & U_{\eta q}^* \\ \beta p u_{2L}''^* \frac{\partial l^*}{\partial \tau} & U_{qq}^* \end{vmatrix}$$

$$= -\beta p u_{2L}''^* l'^* U_{qq}^* \frac{\partial l^*}{\partial \tau} + \beta p u_{2L}'^* U_{qq}^* \frac{\partial l'^*}{\partial \tau} - \beta p u_{2L}''^* U_{\eta q}^* \frac{\partial l^*}{\partial \tau}$$

$$= -\beta p u_{2L}''^* \left(l'^* U_{qq}^* + U_{\eta q}^*\right) \frac{\partial l^*}{\partial \tau} + \beta p u_{2L}'^* U_{qq}^* \frac{\partial l'^*}{\partial \tau}$$

$$= -\beta p u_{2L}''^* (-\pi''^* u_1'^* l'^* + (\pi'^*)^2 u_1''^* l'^* + \beta p u_{2L}''^* l'^* + c'^* \pi'^* u_1''^* - \beta p u_{2L}''^* l'^*) \frac{\partial l^*}{\partial \tau}$$

$$+ \beta p u_{2L}'^* U_{qq}^* \frac{\partial l'^*}{\partial \tau}$$

$$= \beta p \pi''^* u_1'^* u_{2L}''^* l'^* \frac{\partial l^*}{\partial \tau} + \beta p u_{2L}'^* U_{qq}^* \frac{\partial l'^*}{\partial \tau} \qquad (\because \pi'^* l'^* = -c'^*)$$

$$= \beta p u_{2L}'^* U_{qq}^* \frac{\partial l^*}{\partial \tau} \left(\frac{\pi''^* l'^* u_1'^*}{U_{qq}^*} \frac{u_{2L}''^*}{u_{2L}'^*} + \frac{\frac{\partial l'^*}{\partial \tau}}{\frac{\partial l^*}{\partial \tau}} \right)$$

$$\frac{\partial \eta^*}{\partial \tau} = \frac{\Delta_{\eta/\tau}}{H_{\eta,q}} \Rightarrow \operatorname{sgn}\left\{\frac{\partial \eta^*}{\partial \tau}\right\} = \operatorname{sgn}\{\Delta_{\eta/\tau}\} = \operatorname{sgn}\left\{ \frac{\frac{\partial l'^*}{\partial \tau}}{\frac{\partial l^*}{\partial \tau}} - \pi''^* l'^* \left(\frac{u_1'^*}{U_{qq}^*}\right) R_{2L}^* \right\}$$

We then have $\frac{\partial l'^*}{\partial \tau} \geq 0 \Rightarrow \frac{\partial \eta^*}{\partial \tau} < 0$.

As a special case, $\pi(q) = \bar{\pi} q \Rightarrow \pi''^* = 0 \Rightarrow \operatorname{sgn}\left\{\frac{\partial \eta^*}{\partial \tau}\right\} = -\operatorname{sgn}\left\{\frac{\partial l'^*}{\partial \tau}\right\}$.

$$\Delta_{q/\tau} = \begin{vmatrix} U_{\eta\eta}^* & -\beta p u_{2L}''^* l'^* \frac{\partial l^*}{\partial \tau} + \beta p u_{2L}'^* \frac{\partial l'^*}{\partial \tau} \\ U_{\eta q}^* & \beta p u_{2L}''^* \frac{\partial l^*}{\partial \tau} \end{vmatrix}$$

$$= \beta p u_{2L}''^* U_{\eta\eta}^* \frac{\partial l^*}{\partial \tau} + \beta p u_{2L}''^* l'^* U_{\eta q}^* \frac{\partial l^*}{\partial \tau} - \beta p u_{2L}'^* U_{\eta q}^* \frac{\partial l'^*}{\partial \tau}$$

$$= \beta p u_{2L}''^* \left(U_{\eta\eta}^* + l'^* U_{\eta q}^*\right) \frac{\partial l^*}{\partial \tau} - \beta p u_{2L}'^* U_{\eta q}^* \frac{\partial l'^*}{\partial \tau}$$

$$= \beta p u_{2L}''^* (-c''^* u_1'^* + (c'^*)^2 u_1''^* + \beta p u_{2L}''^* (l'^*)^2 - \beta p u_{2L}'^* l''^* + c'^* \pi'^* u_1''^* l'^*$$
$$- \beta p u_{2L}''^* (l'^*)^2) \frac{\partial l^*}{\partial \tau} - \beta p u_{2L}'^* U_{\eta q}^* \frac{\partial l'^*}{\partial \tau}$$

$$= -\beta p u_{2L}''^* (c''^* u_1'^* + \beta p u_{2L}'^* l''^*) \frac{\partial l^*}{\partial \tau} - \beta p u_{2L}'^* U_{\eta q}^* \frac{\partial l'^*}{\partial \tau} \qquad (\because \pi'^* l'^* = -c'^*)$$

$$= -\beta p u_{2L}''^* (c''^* u_1'^* + \pi'^* u_1'^* l''^*) \frac{\partial l^*}{\partial \tau} - \beta p u_{2L}'^* U_{\eta q}^* \frac{\partial l'^*}{\partial \tau} \qquad (\because -\pi'^* u_1'^* + \beta p u_{2L}'^* = 0)$$

$$= -\beta p u_{2L}'^* U_{\eta q}^* \frac{\partial l^*}{\partial \tau} \left[(c''^* + \pi'^* l''^*) \left(\frac{u_1'^*}{U_{\eta q}^*}\right) \frac{u_{2L}''^*}{u_{2L}'^*} + \frac{\frac{\partial l'^*}{\partial \tau}}{\frac{\partial l^*}{\partial \tau}} \right]$$

$$\frac{\partial q^*}{\partial \tau} = \frac{\Delta_{q/\tau}}{H_{\eta,q}} \Rightarrow \operatorname{sgn}\left\{\frac{\partial q^*}{\partial \tau}\right\} = \operatorname{sgn}\{\Delta_{q/\tau}\} = -\operatorname{sgn}\left\{ \frac{\frac{\partial l'^*}{\partial \tau}}{\frac{\partial l^*}{\partial \tau}} - (c''^* + \pi'^* l''^*) \left(\frac{u_1'^*}{U_{\eta q}^*}\right) R_{2L}^* \right\}$$

We then have $\frac{\partial l'^*}{\partial \tau} \leq 0 \Rightarrow \frac{\partial q^*}{\partial \tau} < 0$.

F. Derivation of the Hessian in Subsection 3.4

$$H_{e,q} = U_{ee}^* U_{qq}^* - \left(U_{eq}^*\right)^2$$
$$= [-k''^* u_1'^* + (k'^*)^2 u_1''^* + \beta p''^*(u_{2L}^* - u_{2N}^*)][-\pi''^* u_1'^* + (\pi'^*)^2 u_1''^* + \beta p^* u_{2L}''^*] - [k'^*\pi'^* u_1''^* + \beta p'^* u_{2L}'^*]^2$$
$$= k''^*\pi''^*(u_1'^*)^2 - (k'^*)^2\pi''^* u_1'^* u_1''^* - \beta p''^*\pi''^* u_1'^*(u_{2L}^* - u_{2N}^*) - k''^*(\pi'^*)^2 u_1'^* u_1''^* + (k'^*)^2(\pi'^*)^2(u_1''^*)^2 + \beta p''^*(\pi'^*)^2 u_1''^*(u_{2L}^* - u_{2N}^*) - \beta p^* k''^* u_{2L}''^* u_1'^* + \beta p^*(k'^*)^2 u_1''^* u_{2L}''^* + \beta^2 p^* p''^* u_{2L}''^*(u_{2L}^* - u_{2N}^*) - (k'^*)^2(\pi'^*)^2(u_1''^*)^2 - 2\beta p'^* k'^*\pi'^* u_1''^* u_{2L}'^* - \beta^2(p'^*)^2(u_{2L}'^*)^2$$
$$= k''^*\pi''^*(u_1'^*)^2 - (k'^*)^2\pi''^* u_1'^* u_1''^* - \beta p''^*\pi''^* u_1'^*(u_{2L}^* - u_{2N}^*) - k''^*(\pi'^*)^2 u_1'^* u_1''^* + \beta p''^*(\pi'^*)^2 u_1''^*(u_{2L}^* - u_{2N}^*) - \beta p^* k''^* u_{2L}''^* u_1'^* + \beta p^*(k'^*)^2 u_1''^* u_{2L}''^* + \beta^2 p^* p''^* u_{2L}''^*(u_{2L}^* - u_{2N}^*) - 2\beta p'^* k'^*\pi'^* u_1''^* u_{2L}'^* - \beta^2(p'^*)^2(u_{2L}'^*)^2$$

The sign of $H_{e,q}$ is ambiguous and assumed to be positive.

G. Proof of Proposition 5

Combining (22) with (23) yields

$$\frac{u_{2L}'^*}{u_{2L}^* - u_{2N}^*} = \frac{p'^*\pi'^*}{p^* k'^*} < 0$$

$$U_e^* = -k'^* u_1'^* + \beta p'^*(u_{2L}^* - u_{2N}^*) = 0$$
$$\Rightarrow -k''^*\frac{\partial e^*}{\partial\theta}u_1'^* - k'^* u_1''^*\left(-k'^*\frac{\partial e^*}{\partial\theta} - \pi'^*\frac{\partial q^*}{\partial\theta}\right) + \beta\left(p''^*\frac{\partial e^*}{\partial\theta} + \frac{\partial p'^*}{\partial\theta}\right)(u_{2L}^* - u_{2N}^*) + \beta p'^* u'^*_{2L}\frac{\partial q^*}{\partial\theta} = 0$$
$$\Rightarrow [-k''^* u_1'^* + (k'^*)^2 u_1''^* + \beta p''^*(u_{2L}^* - u_{2N}^*)]\frac{\partial e^*}{\partial\theta} + [k'^*\pi'^* u_1''^* + \beta p'^* u'^*_{2L}]\frac{\partial q^*}{\partial\theta} = -\beta(u_{2L}^* - u_{2N}^*)\frac{\partial p'^*}{\partial\theta}$$
$$\Rightarrow U_{ee}^*\frac{\partial e^*}{\partial\theta} + U_{eq}^*\frac{\partial q^*}{\partial\theta} = -\beta(u_{2L}^* - u_{2N}^*)\frac{\partial p'^*}{\partial\theta}$$

$$U_q^* = -\pi'^* u_1'^* + \beta p^* u_{2L}'^* = 0$$
$$\Rightarrow -\pi''^*\frac{\partial q^*}{\partial\theta}u_1'^* - \pi'^* u_1''^*\left(-k'^*\frac{\partial e^*}{\partial\theta} - \pi'^*\frac{\partial q^*}{\partial\theta}\right) + \beta\left(p'^*\frac{\partial e^*}{\partial\theta} + \frac{\partial p^*}{\partial\theta}\right)u_{2L}'^* + \beta p^* u_{2L}''^*\frac{\partial q^*}{\partial\theta} = 0$$
$$\Rightarrow -\pi''^* u_1'^*\frac{\partial q^*}{\partial\theta} + k'^*\pi'^* u_1''^*\frac{\partial e^*}{\partial\theta} + (\pi'^*)^2 u_1''^*\frac{\partial q^*}{\partial\theta} + \beta p'^* u_{2L}'^*\frac{\partial e^*}{\partial\theta} + \beta u_{2L}'^*\frac{\partial p^*}{\partial\theta} + \beta p^* u_{2L}''^*\frac{\partial q^*}{\partial\theta} = 0$$
$$\Rightarrow [k'^*\pi'^* u_1''^* + \beta p'^* u_{2L}'^*]\frac{\partial e^*}{\partial\theta} + [-\pi''^* u_1'^* + (\pi'^*)^2 u_1''^* + \beta p^* u_{2L}''^*]\frac{\partial q^*}{\partial\theta} = -\beta u_{2L}'^*\frac{\partial p^*}{\partial\theta}$$

$$\Rightarrow U_{eq}^{*}\frac{\partial e^{*}}{\partial\theta}+U_{qq}^{*}\frac{\partial q^{*}}{\partial\theta}=-\beta u_{2L}^{\prime *}\frac{\partial p^{*}}{\partial\theta}$$

$$\begin{bmatrix} U_{ee}^{*} & U_{eq}^{*} \\ U_{eq}^{*} & U_{qq}^{*} \end{bmatrix}\begin{bmatrix} \frac{\partial e^{*}}{\partial\theta} \\ \frac{\partial q^{*}}{\partial\theta} \end{bmatrix}=\begin{bmatrix} -\beta(u_{2L}^{*}-u_{2N}^{*})\frac{\partial p^{\prime *}}{\partial\theta} \\ -\beta u_{2L}^{\prime *}\frac{\partial p^{*}}{\partial\theta} \end{bmatrix}$$

$$\Delta_{e/\theta}=\begin{vmatrix} -\beta(u_{2L}^{*}-u_{2N}^{*})\frac{\partial p^{\prime *}}{\partial\theta} & U_{eq}^{*} \\ -\beta u_{2L}^{\prime *}\frac{\partial p^{*}}{\partial\theta} & U_{qq}^{*} \end{vmatrix}=-\beta(u_{2L}^{*}-u_{2N}^{*})U_{qq}^{*}\frac{\partial p^{\prime *}}{\partial\theta}+\beta u_{2L}^{\prime *}U_{eq}^{*}\frac{\partial p^{*}}{\partial\theta}$$

$$=-\beta(u_{2L}^{*}-u_{2N}^{*})U_{qq}^{*}\frac{\partial p^{*}}{\partial\theta}\left[\frac{\frac{\partial p^{\prime *}}{\partial\theta}}{\frac{\partial p^{*}}{\partial\theta}}-\frac{u_{2L}^{\prime *}U_{eq}^{*}}{(u_{2L}^{*}-u_{2N}^{*})U_{qq}^{*}}\right]$$

$$=-\beta(u_{2L}^{*}-u_{2N}^{*})U_{qq}^{*}\frac{\partial p^{*}}{\partial\theta}\left[\frac{\frac{\partial p^{\prime *}}{\partial\theta}}{\frac{\partial p^{*}}{\partial\theta}}-\frac{p^{\prime *}\pi^{\prime *}U_{eq}^{*}}{p^{*}k^{\prime *}U_{qq}^{*}}\right]\quad\left(\because\frac{u_{2L}^{\prime *}}{u_{2L}^{*}-u_{2N}^{*}}=\frac{p^{\prime *}\pi^{\prime *}}{p^{*}k^{\prime *}}\right)$$

$$\frac{\partial e^{*}}{\partial\theta}=\frac{\Delta_{e/\theta}}{H_{e,q}}\Rightarrow\operatorname{sgn}\left\{\frac{\partial e^{*}}{\partial\theta}\right\}=\operatorname{sgn}\{\Delta_{e/\theta}\}=\operatorname{sgn}\left\{\frac{\frac{\partial p^{\prime *}}{\partial\theta}}{\frac{\partial p^{*}}{\partial\theta}}-\frac{p^{\prime *}\pi^{\prime *}U_{eq}^{*}}{p^{*}k^{\prime *}U_{qq}^{*}}\right\}$$

We then have $\frac{\partial p^{\prime *}}{\partial\theta}\leq 0\Rightarrow\frac{\partial e^{*}}{\partial\theta}>0$.

$$\Delta_{q/\theta}=\begin{vmatrix} U_{ee}^{*} & -\beta(u_{2L}^{*}-u_{2N}^{*})\frac{\partial p^{\prime *}}{\partial\theta} \\ U_{eq}^{*} & -\beta u_{2L}^{\prime *}\frac{\partial p^{*}}{\partial\theta} \end{vmatrix}=-\beta u_{2L}^{\prime *}U_{ee}^{*}\frac{\partial p^{*}}{\partial\theta}+\beta(u_{2L}^{*}-u_{2N}^{*})U_{eq}^{*}\frac{\partial p^{\prime *}}{\partial\theta}$$

$$=-\beta(u_{2L}^{*}-u_{2N}^{*})U_{eq}^{*}\frac{\partial p^{*}}{\partial\theta}\left[-\frac{u_{2L}^{\prime *}U_{ee}^{*}}{(u_{2L}^{*}-u_{2N}^{*})U_{eq}^{*}}+\frac{\frac{\partial p^{\prime *}}{\partial\theta}}{\frac{\partial p^{*}}{\partial\theta}}\right]$$

$$=-\beta(u_{2L}^{*}-u_{2N}^{*})U_{eq}^{*}\frac{\partial p^{*}}{\partial\theta}\left[-\frac{p^{\prime *}\pi^{\prime *}U_{ee}^{*}}{p^{*}k^{\prime *}U_{eq}^{*}}+\frac{\frac{\partial p^{\prime *}}{\partial\theta}}{\frac{\partial p^{*}}{\partial\theta}}\right]\quad\left(\because\frac{u_{2L}^{\prime *}}{u_{2L}^{*}-u_{2N}^{*}}=\frac{p^{\prime *}\pi^{\prime *}}{p^{*}k^{\prime *}}\right)$$

$$\frac{\partial q^{*}}{\partial\theta}=\frac{\Delta_{q/\theta}}{H_{e,q}}\Rightarrow\operatorname{sgn}\left\{\frac{\partial q^{*}}{\partial\theta}\right\}=\operatorname{sgn}\{\Delta_{q/\theta}\}=\operatorname{sgn}\left\{\frac{\frac{\partial p^{\prime *}}{\partial\theta}}{\frac{\partial p^{*}}{\partial\theta}}-\frac{p^{\prime *}\pi^{\prime *}U_{ee}^{*}}{p^{*}k^{\prime *}U_{eq}^{*}}\right\}$$

We then have $\frac{\partial p^{\prime *}}{\partial\theta}\leq 0\Rightarrow\frac{\partial q^{*}}{\partial\theta}>0$.