Product market competition and fraud in a model of price competition with horizontally differentiated products

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Abstract
We investigate in a model of horizontally differentiated products the effect of product market competition on fraudulent behavior. We show that the relationship between fraud and competition depends on the source of variation of competition (number of firms, product substitutability and market size) as well as on the endogeneity or exogeneity of industry structure.
1 Introduction

Accounting scandals such as Enron in 2001, WorldCom in 2002, Healthsouth in 2003, Freddie Mac in 2003, American Insurance Group in 2005, Lehman Brothers in 2008, Satyam in 2009 and Tesco in 2014 show that fraudulent behavior remains a widespread and unfading practice. Among the reasons of the firm’s short-term market value manipulation are the need to meet short-term financial expectations, the attraction of funds (Teoh et al., 1998; Povel et al., 2007; Wang, et al. 2010), the stock price manipulation before share repurchases (Gong et al. 2008) and the accumulation of personal wealth (Burns and Kedia, 2006; Bergstresser and Philippon, 2006; Goldman, and Sleazak, 2006; Bruner et al., 2008). Product market competition (PMC) is often considered to have a disciplining effect on the management (see, for example, the discussion in Hart, 1983; Scharfstein, 1988; Hermalin, 1992; Schmidt, 1997; Raith, 2003) and thus raises the issue of how it affects fraudulent behavior.

In Andergassen (2010), where PMC is captured by the firm’s ability to collude, the shareholder’s solution to the trade-off between fraud and managerial effort leads to a negative relationship between competition and fraud. Markarian and Santaló (2014) show in a model of Cournot competition that stronger PMC leads to more fraud. In this paper we study a model with horizontally differentiated products (Salop, 1979) where firms’ marginal production costs can be either high or low and investigate how price competition affects firms’ incentives to engage in fraudulent behavior. A high cost firm may want to increase its short-term firm value by signaling untruthfully that its costs are low, which in our model consists in reporting profits and mimicking the pricing strategy of a low costs firm. Consequently, the more firms report fraudulently, the stronger competition. We show that the relationship between fraud and PMC depends on the source of variation of competition, that is, number of firms, product substitutability and market size, as well as on the exogeneity or endogeneity of industry structure. In particular, we find that if the market structure is exogenously determined, then an increase in the number of firms or an increase in product substitutability strengthens PMC and reduces fraudulent reporting, while an increase in market size strengthens PMC but increases fraudulent behavior. The reason for this result is that an increase in the number of competitors or in product substitutability, or a reduction in consumer mass, reduces profits of a firm signaling (truthfully or fraudulently) low production costs more than profits of a firm signaling high costs. Consequently, the market value of firms signaling low costs decreases more than those signaling high costs and thus fraudulent behavior declines. If the market structure is endogenously determined through a free entry condition, then in addition to the direct effect of a parameter change on firm profits, and hence on fraudulent behavior, there is an additional indirect effect through entry. An increase in product substitutability, by reducing firm profits, reduces entry and thus increases fraudulent behavior. We find that this indirect positive effect on fraudulent reporting is stronger than the direct negative one, and thus an increase in product substitutability increases PMC and fraudulent behavior.

The remaining part of the paper is organized as follows. In Section 2 the model is presented and the equilibrium fraud probability is calculated. The relationship between PMC and fraud in the case of an exogenously and an endogenously defined market structure is studied in Section 2.1 and Section 2.2, respectively. Section 3 contains some concluding remarks. Proofs are in the Appendix.

2 The model

Consider a unit circle populated by $n$ symmetrically distributed firms and a mass $m$ of uniformly distributed consumers. The utility function of a consumer positioned at $l$ and buying from producer $i$ positioned at $z_i$ is $V_i(l) = s - p_i - \tau (l - z_i)^2$. We assume that consumer’s gross surplus $s$ is sufficiently large such that the market is always covered. $\tau$ is the transportation cost: the lower is $\tau$, the greater the product substitutability. The marginal consumer positioned at $l_{i+}$ is indifferent between buying from producer $i$ or $i+1$ if $V_i(l_{i+}) = V_{i+1}(l_{i+})$. Since what matters for consumers (apart from prices $p_i$ and $p_{i+1}$) is the distance between producer $i$ and $i + 1$, we position producer $i$ in 0 and $i + 1$ in $\frac{1}{n}$ and thus $l_{i+}$ is implicitly defined by $s - p_i - \tau l_{i+}^2 = s - p_{i+1} - \tau (\frac{1}{n} - l_{i+})^2$; $l_{i-}$ is defined in a similar way. Since $m$ is the mass of consumers in a given position, demand of firm $i$ is

$$q_i = m \left[ \frac{E(p_i) - p_i}{\frac{1}{n}} + 1 \right]$$

(1)
where \( E(p_i) = \frac{1}{2}(p_{i+1} + p_{i-1}) \).

We assume that firms produce with constant marginal production costs \( c_t, t \in \{l, h\} \), where \( c_l < c_h \) and where with probability \( \frac{1}{2} \) costs are high \((c_h)\) and low \((c_l)\). Firms learn their type once they enter the market, but they do not know the type of their competitors. We define \( E(c_t) = \frac{1}{2}c_l + \frac{1}{2}c_h \) and assume that \( \Delta = c_h - c_l \) and guarantee that the equilibrium firm profits are always positive. The optimal pricing strategy of a firm of type \( t \in \{l, h\} \) is

\[
P_{i,t} = \frac{E(p_i) + c_t + \frac{\Delta}{n^2}}{2}
\]  

(2)

In order to inflate its short-term market value, a firm of type \( h \) may want to signal that it is of type \( l \), reporting fraudulently profits of a firm of type \( l \) and correspondingly price its good as if its costs were \( c_l \). Using (2),

\[
P_{i,\sigma} = \frac{E(p_i) + c_\sigma + \frac{\Delta}{n^2}}{2}
\]  

(3)

is the price charged by a firm signaling that its costs are \( \sigma \in \{l, h\} \). From (3) we observe that firms price goods more aggressively the more similar are products (the greater is \( n \)), the lower the transportation cost \( \tau \) (the higher the product substitutability), the lower the expected price of competitors and the lower the signaled cost.

We calculate the Bayesian Nash equilibrium where expectations are consistent with the behavior of firms, that is, \( E(p_i) = E(p_{i,\sigma}) \)

\[
E(p_i) = E(c_\sigma) + \frac{\tau}{n^2}
\]  

(4)

where \( E(c_\sigma) \) is the expected cost signaled at the equilibrium. Let \( \lambda_i \) be the probability that firm \( i \) of type \( h \) reports fraudulently inflated profits of type \( l \) and sells its goods at price \( p_{i,l} \). Let \( \lambda \) be the fraud probability at the aggregate level, then the expected signaled cost is

\[
E(c_\sigma) = \frac{1}{2}(1 + \lambda) c_l + \frac{1}{2}(1 - \lambda) c_h
\]  

(5)

The expected price (4) can be written as

\[
E(p_i) = E(c) - \frac{1}{2}\lambda\Delta + \frac{\tau}{n^2}
\]  

(6)

More fraudulent behavior leads to a lower signaled cost, to more aggressive pricing in (3) and thus to stronger PMC.

We assume that a fraudulently reporting firm gets fined once its misbehavior gets discovered. Let \( \Pi_{i,t,\sigma} \) be firm \( i \)'s profits if its true type is \( t \) and it signals \( \sigma \), and let \( P \) be the expected present value of the fine, then

\[
\Pi_{i,t,\sigma} = \frac{m n}{4} \left\{ E(p_i) - c_t + \frac{\tau}{n^2} \right\}^2 - (E\sigma - c_l)^2 \left\{ \sigma \neq t \right\} I_{\sigma \neq t} P
\]  

(7)

for \( \sigma, t \in \{h, l\} \), \( I_{\sigma \neq t} \) being an indicator function indicating 1 if \( t \neq \sigma \). It is easy to see that the signal that maximizes firm profits (7) is the true one, i.e. \( t = \sigma \). Nevertheless, as shown below, high cost firms have an incentive to mimic lower cost ones by reporting profits of type \( l \) and setting price at \( p_{l,t} \).

Let us calculate the firm’s market value, assuming that the risk-free interest rate is zero. Given that the firm signals \( \sigma \), its market value (i.e. expected present value of its profit flow) is \( V_{i,\sigma} = E(\Pi_i | \sigma) \). Thus, the value of firm \( i \) signaling low (high) costs is \( V_{i,l} \) \((V_{i,h})\). A low cost firm has never an incentive to signal high costs. Therefore, if a firm reports profits of type \( h \), then the market infers that it is with certainty of type \( h \) and thus its firm value is

\[
V_{i,h} = E(\Pi_i | \sigma = h) = \Pi_{i,h,h}
\]  

(8)

1It is implicitly assumed that the firm never declares bankruptcy.
On the contrary, since a high cost firm has an incentive to mimic a low cost one, if a firm reports profits of type \( l \), then the market will use Bayes’ law to update its beliefs. In particular, given that firm \( i \) of type \( h \) signals with probability \( \lambda_i \) that it is of type \( l \), using Bayes’ law, the market value of firm \( i \) signaling \( l \) is

\[
V_{i,l} = E (\Pi_i | \sigma = l) = \frac{1}{1 + \lambda_i} \Pi_{i,t,l} + \frac{\lambda_i}{1 + \lambda_i} \Pi_{i,h,l}
\]  

(9)

An increase in the fraud probability produces two effects on the firm value (9). Firstly, it reduces the signaled expected firm value (5). Secondly, it increases the probability that the true type of an \( l \)-signaling firm is \( h \), reducing \( V_{i,l} \).

Note that for \( \lambda_i = 0 \) and a finite expected fine (\( P \)), from \( \Pi_{i,t,l} > \Pi_{i,h,h} \) it follows that \( V_{i,l} > V_{i,h} \), confirming that an \( h \) type firm has always an incentive to mimic an \( l \) type one since in this way it inflates its firm value from \( V_{i,h} \) to \( V_{i,l} \). Hence, since \( V_{i,l} \) is decreasing in \( \lambda_i \), an \( h \)-type firm will engage in fraud as long as \( V_{i,l} \geq V_{i,h} \). Using (6)-(9) and rearranging terms we obtain that at the equilibrium \( \lambda_i = \lambda = \lambda^* \), where

\[
\lambda^* = \min \left\{ \frac{2 \Delta \sigma^2}{2 \tau \frac{\tau}{m} P + \Delta^2}, 1 \right\}
\]  

(10)

Note that if \( P \) is nil, then \( \lambda^* = 1 \), while as long as \( P \) is finite, \( \lambda^* > 0 \). A finite expected fine \( P \) may be the result of a owner’s limited liability or finite endowment of wealth.

### 2.1 Exogenous market structure

Fraud probability (10) is increasing in \( \tau \) and \( m \) and decreasing in \( n \). A larger \( n \) and/or a lower \( \tau \) or \( m \) reduces \( \Pi_{i,t,l} \) and \( \Pi_{i,h,h} \) more than \( \Pi_{i,h,h} \). Consequently, \( V_{i,l} \) declines more than \( V_{i,h} \), leading to a lower \( \lambda^* \).

Let us characterize the average equilibrium price (6). Using (10) \( E (p_i) \) can be written as

\[
E (p_i) = \begin{cases} 
E (c) + \frac{2 \Delta \sigma^2}{2 \tau \frac{\tau}{m} P + \Delta^2} \frac{P}{\tau^2} & \text{if } \lambda^* < 1 \\
E (c) + \frac{\tau}{\tau^2} & \text{if } \lambda^* = 1 
\end{cases}
\]  

(11)

Parameters \( \tau \) and \( n \) affect the average equilibrium price and hence PMC directly by affecting the optimal pricing strategy of firms (3) and, as long as \( \lambda^* < 1 \), indirectly through the signaled cost (5). In particular, a larger \( n \) and/or lower \( \tau \) leads for a given expected signaled cost to more aggressive pricing (3). But since it also reduces \( \lambda^* \), it increases the expected signaled cost, which leads to a less aggressive pricing strategy and hence to a weaker PMC. The former effect dominates the latter one and thus a larger \( n \) and/or lower \( \tau \) reduces the average price. An increase in \( m \) affects the average price only indirectly if \( \lambda^* < 1 \) through the expected signaled costs. In particular, an increase in \( m \) increases \( \lambda^* \) and reduces the expected signaled cost, leading to a more aggressive pricing strategy and to a lower average price. These results are summarized in Table 1.

### 2.2 Endogenous market structure

Given that entrepreneurs do not know their marginal production costs before entering the industry, the expected firm value is \( E (V_i) = \frac{1}{2} (1 + \lambda_i) V_{i,l} + (1 - \lambda_i) \frac{1}{2} V_{i,h} \). Entrepreneurs enter as long as \( E (V_i) \) is larger than sunk entry costs \( F \), i.e. \( E (V_i) \geq F \). In the following we assume that \( \lambda^* < 1 \). Using the expressions (9) and (8) we obtain that \( E (V_i) = \Pi_{i,h,h} \). Neglecting the integer problem, the free entry condition reads

**Table 1: PMC and fraud probability \( \lambda^* \) with exogenous market structure**

| \( n \) \( \uparrow \) | PMC \( \downarrow \) | \( \lambda^* \) \( \uparrow \) |
| \( \tau \) \( \downarrow \) | PMC \( \uparrow \) | \( \lambda^* \) \( \downarrow \) |
| \( m \) \( \uparrow \) | PMC \( \uparrow \) | \( \lambda^* \) \( \uparrow \) |
consequently to a lower $\lambda^*$ endogenously determined industry structure.

Proof. Appendix

that the relationship between PMC and fraud depends on the source of variation of PMC as well as on the firms’ incentives to engage in fraudulent reporting aimed at boosting their short term value. We have shown in the number of competitors, the transportation costs (product substitutability) and the consumer mass on

We studied in a model of price competition with horizontally differentiated products the effect of a change in $T$able 2.

thus, in addition to the direct negative effect on $\lambda^*$, it follows that, after rearranging terms, $\frac{d\lambda^*}{d\tau} < 0$ if and only if

$E(V_i) = \frac{1}{2x} \left[ -\frac{1}{2} \Delta + \frac{1}{2} \frac{2xP + \Delta^2 m}{n} \right]^2 = F$ (12)

where $2\frac{\tau^*}{m^*} = x$. The derivative of $E(V_i)$ with respect to $n$ is negative while the derivatives with respect to $\tau$ and $m$ are positive. These results are intuitive: the greater $n$, or the lower $\tau$, the stronger PMC and the lower $E(V_i)$; an increase in consumer mass $m$ increases PMC but increases also the demand of a firm; the latter effect dominates the former.

We consider the effect a parameter change has on equilibrium fraud probability $\lambda^*$. A decline in sunk entry costs $F$ increases $n$, thereby increasing PMC and reducing $\lambda^*$. A reduction in $\tau$ reduces $E(V_i)$ and thus, in addition to the direct negative effect on $\lambda^*$ (seen in the previous section), it also leads to an indirect positive one through reduced entry. It can be shown that the indirect effect dominates the direct one, and thus a reduction in $\tau$ leads to an increase in PMC and in $\lambda^*$. An increase in $m$ increases firm profits and hence entry, producing a direct positive effect on $\lambda^*$ (seen in the previous section) and an indirect negative one through increased entry. The direct positive effect dominates the indirect negative one and thus an increase in $m$ increases PMC and $\lambda^*$. These results are formally proven in the Appendix and summarized in Table 2.

3 Conclusion

We studied in a model of price competition with horizontally differentiated products the effect of a change in the number of competitors, the transportation costs (product substitutability) and the consumer mass on firms’ incentives to engage in fraudulent reporting aimed at boosting their short term value. We have shown that the relationship between PMC and fraud depends on the source of variation of PMC as well as on the endogeneity or exogeneity of industry structure.

Appendix

Proof. In this Appendix we study formally the relationship between fraud and PMC in the case of an endogenously determined industry structure.

It is easy to see that $\frac{\partial \Pi_{i,h,h}}{\partial n} < 0$ and thus a reduction of entry costs leads to an increase in $n$ and consequently to a lower $\lambda^*$ and a stronger PMC. Taking the derivative of $\lambda^*$ with respect to $\tau$ we obtain $\frac{d\lambda^*}{d\tau} = \frac{\partial \lambda^*}{\partial \tau} + \frac{\partial \lambda^*}{\partial n} \frac{\partial n}{\partial \tau}$. Applying the implicit function theorem we obtain $\frac{dn}{d\tau} = \frac{-\frac{\partial \Pi_{i,h,h}}{\partial n}}{\frac{\partial \Pi_{i,h,h}}{\partial \tau}} > 0$, where $\frac{\partial \Pi_{i,h,h}}{\partial n} < 0$ and $\frac{\partial \Pi_{i,h,h}}{\partial \tau} > 0$. Collecting $\frac{\partial \lambda^*}{\partial \tau}$ and since $\frac{\partial \lambda^*}{\partial n} = -\frac{\partial \lambda^*}{\partial \tau} n - \frac{1}{n} \lambda^*$, we can write $\frac{d\lambda^*}{d\tau} = \frac{\partial \lambda^*}{\partial \tau} \left( 1 - \frac{\partial n}{\partial \tau} n - \frac{\partial \lambda^*}{\partial \tau} \right)$. Since

$$\frac{dn}{d\tau} \tau = \frac{\partial \Pi_{i,h,h} x}{\partial x} n + \frac{\partial \Pi_{i,h,h} \frac{x}{n}}{\partial x} n + \frac{1}{2} \left[ \frac{-1}{2} \Delta + \frac{1}{2} \frac{2xP + \Delta^2 m}{n} \right] \frac{2xP + \Delta^2 m}{n^2}$$ (13)

it follows that, after rearranging terms, $\frac{d\lambda^*}{d\tau} < 0$ if and only if

$$\frac{\partial \Pi_{i,h,h} x}{\partial x} n > \frac{1}{2} \left[ \frac{-2xP + \Delta^2 m}{xP + \Delta^2 n} \right] \frac{2xP + \Delta^2 m \partial \lambda^* \tau}{xP + \Delta^2 n^2 \partial \lambda^*}$$ (14)
Since $\frac{\partial \lambda^*}{\partial x} = \frac{\Delta^2}{xP + \Delta^2}$ and

$$\frac{\partial \Pi_{i,h,h}}{\partial x} = -\frac{1}{2x^2} \left[-\frac{1}{2} \Delta + \frac{1}{2} \frac{2xP + \Delta^2}{xP + \Delta^2} \frac{m}{n} \right]^2 + \frac{1}{2x} \left[ -\frac{1}{2} \Delta + \frac{1}{2} \frac{2xP + \Delta^2}{xP + \Delta^2} \right] \frac{2x^2P^2 + 4xP\Delta^2 + \Delta^4 m}{(xP + \Delta^2)^2}$$

after rearranging terms, inequality (14) can be written as

$$\frac{2x^2P^2 + xP\Delta^2 - \Delta^4 m}{(xP + \Delta^2)^2} x > -\Delta$$

It is easy to see that for $\lambda^* \leq 1$, which implies that $P \geq \frac{1}{2} (\Delta \frac{n}{m} - \Delta^2)$, inequality (16) is satisfied, thus proving the result.

We briefly show that a reduction in $\tau$ leads to a reduction in the average price-cost margin and hence to an increase in PMC. From (11), the average price-cost margin for $\lambda^* < 1$ reads

$$E(p_i) - E(c) = \frac{xP}{xP + \Delta^2} \frac{\tau}{n^2}$$

Taking the derivative of the price-cost margin with respect to $\tau$ we observe that it is positive if and only if

$$\frac{\partial \Pi}{\partial \tau} = \frac{\partial \Pi}{\partial \tau} = -\frac{xP}{xP + \Delta^2} \frac{\tau}{n^2}$$

which is always true.

Consider next the derivative of $\lambda^*$ with respect to $m$, which is $\frac{\partial \lambda^*}{\partial m} = \frac{\partial \lambda^*}{\partial m} + \frac{\partial \lambda^*}{\partial \tau} \frac{\partial \tau}{\partial m}$, and where, using the implicit function theorem, $\frac{\partial \tau}{\partial m} = -\frac{\partial \lambda^*}{\partial \tau} \frac{\partial \lambda^*}{\partial m}$. Straightforward calculus shows that $\frac{\partial \lambda^*}{\partial \tau} \frac{\partial \lambda^*}{\partial m} > 0$, and thus $\frac{\partial \tau}{\partial m} > 0$. A consequence of this result is that an increase in $m$ reduces the average price-cost margin (17) directly and indirectly (through an increase in $n$) and hence a larger $m$ leads to a stronger PMC.

The derivative $\frac{\partial \lambda^*}{\partial m}$ can be written as

$$\frac{d\lambda^*}{dm} = \frac{\partial \lambda^*}{\partial m} \left( 1 + \frac{\partial \lambda^*}{\partial m} \frac{n}{m} \frac{\partial n}{\partial m} \right) = \frac{\partial \lambda^*}{\partial m} \left( 1 + \frac{\partial \lambda^*}{\partial m} \frac{m - 2\lambda^* \frac{\partial \tau}{\partial m}}{n} \frac{\partial n}{\partial m} \right) = \frac{\partial \lambda^*}{\partial m} \left[ 1 - \left( 1 + 2 \frac{\Delta^2}{xP} \frac{\tau}{m} \right) \frac{\partial n}{\partial m} \right]$$

Hence, to show that $sign \left( \frac{d\lambda^*}{dm} \right) = sign \left( \frac{\partial \lambda^*}{\partial m} \right)$ it is sufficient to show that $\frac{m}{n} \frac{\partial n}{\partial m} < \frac{xP}{xP + \Delta^2 \tau}$. For convenience we rewrite expected profits in (12) as

$$\hat{\Pi}_{i,h,h} = \frac{1}{2x} \left[ -\frac{1}{2} \Delta + \frac{2xP + \Delta^2}{xP + \Delta^2} \frac{\tau}{n^2} \right]^2$$

Our assumption on parameters assures that $\hat{\Pi}_{i,h,h}$ is decreasing in $x$ and thus $\Pi_{i,h,h}$ in (12) is increasing in $m$. Using the implicit function theorem we obtain

$$\frac{m}{n} \frac{\partial n}{\partial m} = -\frac{\partial \Pi_{i,h,h}}{\partial x} \frac{x}{xP + \Delta^2} \left[ -\frac{1}{2} \Delta + \frac{2xP + \Delta^2}{xP + \Delta^2} \frac{\tau}{n^2} \right] \frac{2xP + \Delta^2}{xP + \Delta^2} \frac{\tau}{n^2}$$

and calculating $\frac{\partial \Pi_{i,h,h}}{\partial x}$, inequality $\frac{m}{n} \frac{\partial n}{\partial m} < \frac{xP}{xP + \Delta^2 \tau}$ is true if $-\frac{1}{2} \Delta + \frac{2xP + \Delta^2}{xP + \Delta^2} \frac{\tau}{n^2} < 0$, which, using (10), can be written as $-2xP\lambda^* < (1 - \lambda^*) \Delta^2$ and hence is always true.


