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Constant utility index and inter-month substitution

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Abstract

This note aims at measuring the cost-of-living thanks to a constant utility index derived from monthly nested CES preferences that account for substitution across months. We estimate empirically that the bias due to substitution across months lies between .07 and .33 pp per year in the clothing industry. A simulation evaluates the global inter-month substitution bias between .02 and .1 pp per year, which amounts to an important part of the global substitution bias comprised between .05 and .4 pp. To approximate the cost-of-living, implementation guidelines for statistical agencies include (i) weighting monthly price indexes with monthly budget shares, and (ii) computing a weighted Fisher or another superlative index instead of a Laspeyres index.

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1. Introduction

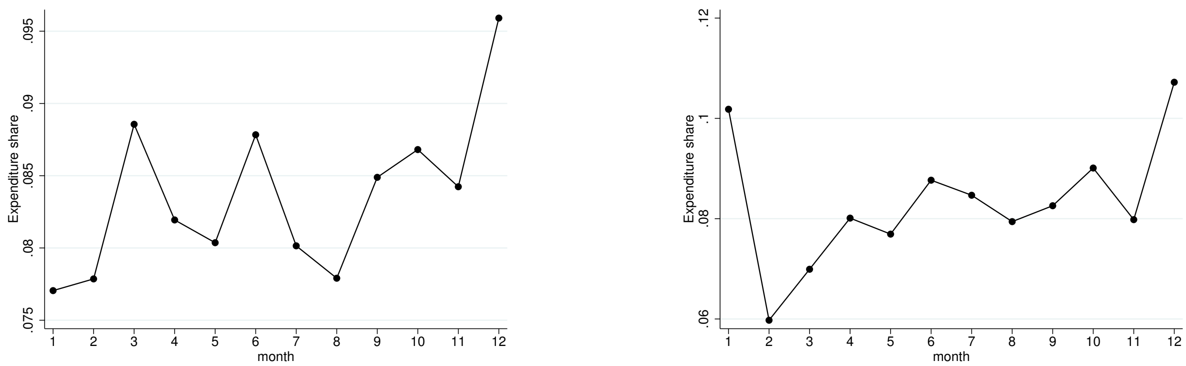
The Boskin Commission (Boskin, Dulberger, Gordon, Griliches, and Jorgenson, 1998) aimed at tracking any possible source of bias in the computation of the consumption price index (CPI). Its primary concern was not to overestimate inflation, a danger encountered by all indexes that neglect substitutions. The Commission concluded that the index computed by the Bureau of Labor Statistics (BLS) could have an upward bias of 1.1 pp per year. Since the US federal budget is indexed on the CPI, the Commission estimated that such a bias might be responsible for a supplementary burden worth 1000 billions dollars over a twelve-year period. Three sources of bias were put forward: the introduction of new goods and changes in product quality (about .6 pp per year); the substitution between products (about .4 pp); the substitution across retailers (about .1 pp).

This note focuses on another source of bias arising from a still unexplored dimension: intertemporal substitution, and more precisely substitution across months (hereafter inter-month substitution). Consumers may choose to delay their purchases within the year in order to benefit from promotions. For instance, peaks of demand are observed during sales in the French clothing industry, namely every January and every July. The computation of the CPI assumes so far that consumers' budget shares are identical across months. Yet this assumption is strongly rejected empirically: Figure 1a depicts the average pattern of households' monthly expenditures (proxied by retailers' revenues) in France from 2004 to 2011. The volatility may be even higher in specific sectors like the clothing industry (Figure 1b).

Figure 1: Monthly budget shares

(a) Retail sector

(b) Retail sector - clothing only



Source : Value-added tax data, French retail sector, 2004-2011.

Monthly variations in consumption expenditures reflect both price changes (promotions, sales) and quantity changes. Most striking examples are seasonal items: these items are consumed during well-identified periods of the year (not only fruits, vegetables, but also heating, electricity, etc.). Even for non-seasonal items, peaks of demand can also be observed during (public) holidays, in December, not to be exhaustive. Besides, recent forms of dynamic pricing including revenue management consist in adjusting prices in real-time, and are widely used by airlines, railways, hotels, car rental dealers, etc. To maximize revenues, firms charge high (low) prices when the demand is high (low) –contrary to what happens during promotions and sales when prices are low and the demand is high. According to the CPI reference manual published by ILO, IMF, OECD, UNECE, Eurostat, and the World Bank (2004), seasonal items account for 20% to 33% of total expenditures –revenue management aside. However, two factors contribute to lower the share of items on sales resulting from strategic delaying of purchase. First, some goods and services cannot be delayed within the year: if a fridge breaks down, it is necessary to replace it as soon as possible, which prevents consumers from optimizing across months. Second, even though consumers may optimize across months, they will be less prone to do so if they are impatient, if they have a bounded rationality, but also because they can benefit from goods longer if they do not delay their purchase. Determining that share is an empirical and complicated issue, at least from market-level data; yet the lower this share, the lower the bias due to inter-month substitution.

Monthly variations of consumption expenditures may still require to adjust the computation of the CPI. In practice, the most widely used index –in Europe at least– is the Laspeyres index, an arithmetic mean of price ratios weighted by budget shares. However, such indexes are well-known for not taking any substitution into account –either across products or across stores/cities/months. In particular, monthly indexes provide higher frequency information that correct for seasonality but do not account for consumer's optimization across months within the year. From a microeconomic perspective, it will turn out that the approach proposed here rationalizes, and also goes further than a statistical weighting of monthly indexes. To deal with the issue of substitution in general, a first method consists in resorting to a constant utility index, that is, to a cost-of-living index (COLI). Following Konüs (1924), a COLI I^t is defined for year t with respect to some base year 0 as:

$$V(\mathbf{p}^0, R^0) = V(\mathbf{p}^t, I^t R^0), \tag{1}$$

where $V(\mathbf{p}, R)$ is the indirect utility of a representative consumer endowed with income R and facing a price vector $\mathbf{p}$. A second method is related to superlative indexes (Fisher, Törnqvist, etc.) as praised by Diewert (1976). Such indexes are approximations of COLI that become particularly good when preferences are homothetic. A last method has been proposed more recently by Reis (2009) and Aoki and Kitahara (2010), which consists in measuring a dynamic inflation by extending the concept of COLI to an intertemporal utility function. One advantage of this approach is to model explicitly (long-term) intertemporal substitution. However, this method puts much emphasis on the role played by price uncertainty, and its link with anticipations, which is not the main concern here.

To account for inter-month substitution, this note suggests to model the choice of a representative consumer whose preferences are encompassed by a monthly nested CES utility function. The novelty of the approach consists in considering months as a relevant dimension of consumer choice. The nested CES specification enables consumers to optimize their purchases in order to benefit from within-year price variations. On actual CPI data in the French clothing industry, we estimate that the inter-month substitution bias in that sector amounts to .07-.33 pp per year, depending on the value of the elasticity of intertemporal substitution. Our simulations indicate that the global inter-month substitution bias amounts to roughly .02-.10 pp per year, depending again on the value of the elasticity of intertemporal substitution. On US data over the 1959-1985 period, Manser and McDonald (1988) estimate that the bias due to substitution across goods of a Laspeyres index is about .18% of that index every year. Since the inter-month substitution bias might represent an important part of the total substitution bias, both researchers and statisticians should not disregard this issue. We provide national statistical agencies with two practical recommendations in the absence of demand estimation. First, an appropriate weighting of monthly indexes based on monthly budget shares does a good job in reducing the bias. Second, a weighted superlative index like the Fisher would almost eliminate this bias. It is however an empirical issue to determine whether the "weighting bias" is larger than the "formula bias".

The note is organized as follows. Section 2 presents a model of consumer choice which allows for substitutions both across products and across months. Section 3 derives a COLI from previous preferences. Section 4 is devoted to an estimation of the inter-month substitution bias; it also proposes two approximations of the COLI that can be viewed as implementation recommendations. Section 5 concludes.

2. Model of consumer behavior

As in most CPI settings, we consider a representative consumer in a certain environment under perfect information. Her annual preferences are encompassed by some monthly nested CES utility, which accounts for inter-month substitution. Nested CES production functions were considered first by Sato (1967) while Brown and Heien (1972) studied nested CES utility functions –Keller (1976) examined cases with more than two nests. The novelty of our approach resides in the definition of nests: an upstream level corresponding to months, and a downstream level corresponding to products. To the best of our knowledge, months have never been considered as a relevant dimension for optimization. By extension, the downstream level could concern varieties of goods, retail stores or even cities. The consumer optimizes her purchases by taking profit of monthly price changes. Denoting months by m and goods by i, the utility derived from consumption of the vector of goods $\mathbf{x}$ is defined as:

$$U(\mathbf{x}) = \left\{ \sum_{m=1}^{M} \left[\sum_{i=1}^{n_m} \alpha_{im} x_{im}^{\rho_m} \right]^{\frac{\rho}{\rho_m}} \right\}^{\frac{1}{\rho}}, \tag{2}$$

where the monthly discount factor is normalized to one, $\rho_m < 1$ is an intra-month factor of substitution across goods, $\rho < 1$ is an inter-month substitution factor and n_m is the number of goods available on month m. Coefficients α_{im} are structural parameters indicating consumer tastes for products. By definition, there are $M = 12$ months in a year. Finally, preferences are homothetic: U is homogeneous of degree 1.

Denoting by $U_m = (\sum_{i=1}^{n_m} \alpha_{im} x_{im}^{\rho_m})^{\frac{1}{\rho_m}}$ the utility in month m, the annual utility $U = \left(\sum_{m=1}^{M} U_m^{\rho}\right)^{\frac{1}{\rho}}$ is a CES aggregation of monthly utilities. Since U is weakly separable in vectors $\mathbf{x}_m = (x_{1m}, \ldots, x_{n_m m})$, its maximization under the annual budget constraint:

$$\sum_{m=1}^{M} \sum_{i=1}^{n_m} p_{im} x_{im} \leq R \tag{3}$$

is performed in two steps.[1]

The first step consists in optimizing every month the function U_m under a monthly budget constraint, hence in solving a CES program: $\forall m$,

$$\max_{(x_{i1},\ldots,x_{in_m})} \left[\sum_{i=1}^{n_m} \alpha_{im} x_{im}^{\rho_m} \right]^{\frac{1}{\rho_m}} \text{ s.t. } \sum_{i=1}^{n_m} p_{im} x_{im} \leq R_m. \tag{4}$$

Denoting by $\sigma_m = \frac{1}{1-\rho_m} > 0$ the elasticity of intra-month substitution, one has:

$$x_{im} = R_m \frac{\left(\frac{\alpha_{im}}{p_{im}}\right)^{\sigma_m}}{\sum_{i=1}^{n_m} p_{im} \left(\frac{\alpha_{im}}{p_{im}}\right)^{\sigma_m}}. \tag{5}$$

Let X_m be the denominator of (5) elevated to the power $\frac{1}{1-\sigma_m}$: X_m is then the invert of the Lagrange multiplier related to the budget constraint (4), and more interestingly the invert of the marginal utility of income in month m:

$$X_m = \frac{R_m}{U_m} = \left(\sum_{i=1}^{n_m} \alpha_{im}^{\sigma_m} p_{im}^{1-\sigma_m} \right)^{\frac{1}{1-\sigma_m}}. \tag{6}$$

The second step consists in determining optimal monthly budgets R_m in order to

[1]More generally, Strotz (1957) relies on weak separability to show that the consumer's optimum consists in maximizing independently each sub-utility U_m under a monthly budget constraint, since the latter constraint involves prices of the considered month only.

maximize the annual utility, which is equivalent to solving a CES program in the monthly budgets:

$$\max_{(R_1,\dots,R_M)} \left[\sum_{m=1}^{M} X_m^{-\rho} R_m^{\rho}\right]^{\frac{1}{\rho}} \quad \text{s.t.} \quad \sum_{m=1}^{M} R_m \leq R. \tag{7}$$

Denoting by $\sigma = \frac{1}{1-\rho} > 0$ the inter-month elasticity of substitution,[2] optimal monthly budgets are given by:

$$R_m = R \frac{X_m^{1-\sigma}}{\sum_{m=1}^{M} X_m^{1-\sigma}}. \tag{8}$$

From (5) and (8), one has:

$$x_{im} = R \frac{\left(\frac{\alpha_{im}}{p_{im}}\right)^{\sigma_m} X_m^{\sigma_m - \sigma}}{\sum_{m=1}^{M} X_m^{1-\sigma}}. \tag{9}$$

The latter expression provides with a demand equation that enables the econometrician to estimate the structural parameters α_{im}, σ_m and σ, provided that data on price and quantity (or expenditure) are available at a monthly frequency.

The limit cases of the model are well-known. When $\sigma = 0$ ($\rho = -\infty$), no inter-month substitution is authorized: the annual utility becomes Leontief in monthly utilities – months are perfect complements. When $\sigma = 1$ ($\rho = 0$), the utility function becomes a Cobb-Douglas. When $\sigma = +\infty$ ($\rho = 1$), months are perfect substitutes: the utility function is linear. Similar phenomena arise within a month for extremal values of σ_m (ρ_m). However, only the case when $\sigma > 1$ ($\rho > 0$) is considered by Feenstra (1994) and Melser (2006) since it is a necessary condition for monthly budgets to increase with income.

3. Cost-of-living index (COLI)

The monthly COLI derived from previous preferences and from definition (1) is equal to the ratio of marginal utilities of income evaluated in $\mathbf{p}_m^0$ (base year) and in $\mathbf{p}_m^t$ (current year):

$$I_m^t(\mathbf{p}_m^0, \mathbf{p}_m^t) = \frac{X_m(\mathbf{p}_m^t)}{X_m(\mathbf{p}_m^0)}. \tag{10}$$

This result stems directly from the linearity of the indirect utility function under CES preferences, the slope of which is precisely equal to the marginal utility of income. This index is a monthly Lloyd-Moulton index (Lloyd, 1975; Moulton, 1996). It accounts for intra-month substitutions across goods –contrary to most usual CPI. A monthly Laspeyres

[2] In Allen's sense. Sato (1967) distinguishes carefully among intra- or inter-month elasticities of substitution, on the one hand, and direct or Allen elasticities of substitution, on the other hand. The direct elasticity is equal to the intra-month elasticity σ_m for products of the same month m, but to an harmonic mean for products available in two distinct months. The Allen elasticity coincides with the inter-month elasticity for products available in two different months.

index corresponds to the limit case where goods are perfect complements, *i.e.* $\sigma_m = 0$ ($\rho_m = -\infty$). An harmonic mean of price ratios corresponds to the Cobb-Douglas case where the intra-month elasticity of substitution is equal to one, *i.e.* $\sigma_m = 1$ ($\rho_m = 0$).

From consumer behavior and using again definition (1), we derive the annual COLI between annual price vectors $\mathbf{p}^t = (\mathbf{p}_1^t, \ldots, \mathbf{p}_M^t)$ and $\mathbf{p}^0 = (\mathbf{p}_1^0, \ldots, \mathbf{p}_M^0)$ as an aggregation of previous monthly COLI:

$$I^t(\mathbf{p}^0, \mathbf{p}^t) = \left[\sum_{m=1}^{M} w_m^0(\mathbf{p}_m^0) I_m^t(\mathbf{p}_m^0, \mathbf{p}_m^t)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{11}$$

where

$$w_m^0(\mathbf{p}_m^0) = \frac{X_m^{1-\sigma}(\mathbf{p}_m^0)}{\sum_{m=1}^{M} X_m^{1-\sigma}(\mathbf{p}_m^0)} = \frac{R_m}{R} \tag{12}$$

represents the share of month m in the annual budget of year 0. This index accounts for substitutions, not only across goods (within a month) but also across months –contrary to most usual CPI. When $\sigma = 0$, *i.e.* in the absence of inter-month substitution, the annual COLI degenerates into an arithmetic mean (or a Laspeyres aggregation) of monthly price indexes, weighted by monthly budget shares; this particular case corresponds to the statistical approach recommended by Diewert, Armknecht, and Nakamura (2009).

4. Estimation of the inter-month substitution bias

In this section, we estimate the inter-month substitution bias which designates the bias of the actual CPI with respect to our previous "ideal" COLI. This bias depends crucially on the value of the inter-month elasticity of substitution σ, the estimation of which is a challenging empirical research project *per se* which requires monthly scanner data. By lack of appropriate data in France, the structural estimation of the previous consumer behavior model (including parameters α_{im}, σ_m and σ) is left for further research. Nevertheless, we propose first to estimate the bias in the clothing sector for different values of σ by resorting to actual CPI data. Second, we simulate a stylized, calibrated version of the previous model in a two-goods economy in order to compute the range of the bias of a Laspeyres index when σ varies. Third, we seek to approximate the ideal COLI, and give some implementation recommendations at the address of statistical agencies.

First, we restrict our attention to the French clothing sector over the period 2004-2011. Indeed, providing an estimation of the global bias on actual data would require to aggregate over all the sectors, hence to compile and mix information from several sources (the French CPI is computed from different sources, including separate ones for fresh products and rents), which would make the computation barely transparent, and would not permit to identify the pure effect of inter-month substitution. Focusing on a single sector allows us to produce a reasonable estimate of that bias in this sector. We consider 2004 as the reference year, so that the value of the index in 2004 is 100, and compute annually chained indexes. We find a value of 86.2 for the CPI in 2011, which means that prices fell by 2.09% per year on average during this period (Table 1). The corresponding value for the COLI amounts to 85.4 when $\sigma = 2$, i.e., an average annual decrease of 2.22%. The inter-month substitution bias in this sector is thus positive as expected, and amounts to .13 pp per year. When σ varies in the range $[1; 5]$, we obtain the range [.07;.33] pp per year. Since the clothing industry is potentially most concerned by inter-month substitution, the global bias should be an attenuation of the latter, which must therefore be viewed as an upper bound.

Table 1: Estimation of the inter-month substitution bias (French clothing industry, 2004-2011)

average annual variation (%)	CPI	COLI	bias
$\sigma = 1$	-2.16	-2.09	.07
$\sigma = 1.5$	-2.19	-2.09	.10
$\sigma = 2$	-2.22	-2.09	.13
$\sigma = 3$	-2.29	-2.09	.20
$\sigma = 5$	-2.42	-2.09	.33

Source: French CPI data, 2004-2011.
Lecture: The CPI indicates that prices have fallen by 2.09% per year in the clothing industry from 2004 to 2011. The average annual bias with respect to the COLI amounts to [.07;.33] pp when σ varies from 1 to 5.

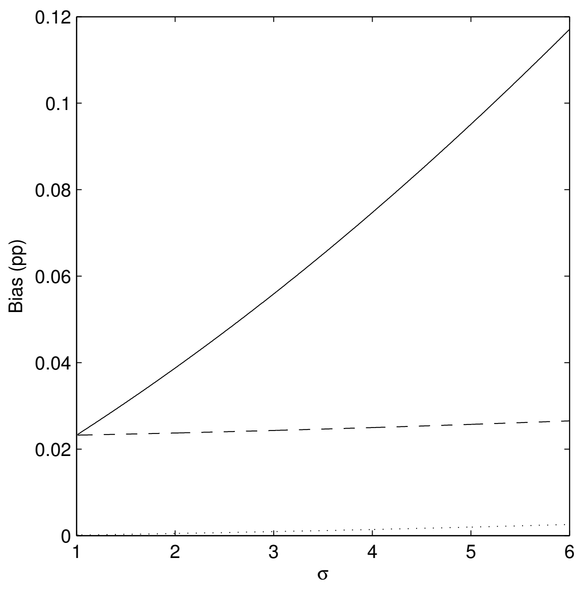
Second, we propose a stylized simulation of the model in order to quantify the bias of a Laspeyres index due to inter-month substitution. The calibration of parameters is based on the econometric estimation by Melser (2006) of σ_m, namely 3.9.[3] Two representative goods are considered over a two-year period (years 0 and t): a non-seasonal good (prices of which raise at a regular monthly pace of .2% from an initial price of 100 dollars) and a seasonal good (with a regular price of 50 dollars, except in January and in July where the price is discounted by 30%). Taste parameters α_1 and α_2 are set respectively to 1 and to .45, so that the annual budget share in good 2 amounts roughly to 1/3. This share raises mechanically every January and every July because the relative price of good 1 increases –the same holds for year t with respect to year 0.

Monthly Laspeyres indexes are computed, and then aggregated according either to a simple mean, or to a weighted mean, where weights correspond to monthly budget shares. The annual COLI is a CES aggregation of the 12 monthly COLIs that takes these weights into account, but that also depends explicitly on the elasticity of inter-month substitution σ, as shown by Figure 2 (plain line). A simple mean of monthly Laspeyres has a bias

[3] His paper considers a (non-nested) CES utility function and estimates elasticities of substitution across products on monthly data which may be compared with σ_m. For most products his estimations lie between 2 and 5, the average being 3.9.

which varies between .02 and .1 pp per year. While those values may sound small, they compare with the total substitution bias evaluated by the Boskin Commission to .4 pp in the USA and by Manser and McDonald (1988) to roughly .18 pp. Hence the inter-month substitution could represent a significant part of this bias. Moreover, these figures are consistent with our previous upper-bound estimates.

Figure 2: Simulation of the global inter-month substitution bias and approximations of the COLI



Lecture: Reference: COLI. Plain: difference between (simple) mean of monthly Laspeyres and the reference. Dashed: difference between (weighted) mean of monthly Laspeyres and the reference. Dots: difference between (weighted) mean of monthly Fisher and the reference. Weights: monthly budget shares.

Third, we propose two practical recommendations which can be easily followed by national statistical agencies in order to approximate the COLI. A first recommendation consists in weighting monthly price indexes by monthly budget shares, as praised by Diewert, Armknecht, and Nakamura (2009) for purely statistical reasons. Figure 2 (dashed line) shows that weighting diminishes significantly the bias even with a Laspeyres formula. Moreover, after weighting, this remaining formula bias seems almost independent from the value of σ. The second recommendation consists in resorting to a superlative index like the Fisher index: Figure 2 (dot line) shows that the residual bias of weighted monthly Fishers is negligible. The quality of approximation by a Fisher index is not surprising since preferences are homothetic, but nothing guaranteed *a priori* the good quality of weighting. Interestingly, the weighting bias exceeds the formula bias for high values of σ, namely when $\sigma > 2.5$.

5. Conclusion

This note suggests to take inter-month substitution into account in the computation of a CPI. It provides an ideal COLI derived from monthly nested CES preferences. Some implementable guidelines can be addressed to statistical agencies: weighting monthly price indexes by monthly budget shares, and resorting to superlative indexes as much as possible –independently from chaining issues. The next step of the research agenda consists in conducting the corresponding empirical analysis from monthly scanner data including prices and purchase expenditures.

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