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Optimal policy under duopoly with environmental quality

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Abstract

This paper examines the optimal excise tax policy and the optimal environmental quality subsidy when consumers are willing to pay a price premium for environmentally friendlier variants of commodities with vertically differentiated environmental qualities. When high and low environmental quality producers exist in the market and the demand for both firms are positive, two policies are necessary to revise the firms' market power and the pollution externalities. Conversely, when all consumers demand high environmental quality products, only one policy is required to correct the pollution externality.

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1 Introduction

Nowadays, consumers tend to purchase environmentally conscious products and firms aim to develop environmentally friendly products. Additionally, government regulators are intervening to further improve the environment. For example, in countries around the world, such as Japan and Thailand, governments have cut excise taxes and provided subsidies to consumers who purchase environmentally friendly products like ecologically friendly cars and energy-saving electrical appliances. The primary question is as follows: Do governments have to intervene when both consumers and producers are environmentally conscious?

The purpose of this paper is to study the use of excise taxes and consumer subsidies when firms compete in environmental quality. We use a vertically differentiated product model with consumers who prefer environmentally superior products, as reflected in their willingness to pay more for ecologically friendly products.

A common theme in the literature is the imposition of a tax or a subsidy rate on economic activities that result in environmental damage (Arora and Gangopadhyay 1995; Cremer and Thisse 1999). Recent works, including those by Bansal and Gangopadhyay (2003) and Lombardini (2005), have introduced the concept of total pollution into the model. Lombardini (2005) studied the socially optimal emission and commodity tax policy. The first-best levels of quality can be obtained using a combination of a uniform ad valorem tax and an emission tax (or a subsidy for buying green products).

In this paper, we adopt Lombardini's (2005) basic model to analyze the use of an excise tax when firms compete in environmental quality contexts. However, Lombardini's model cannot be used to examine the effect of an excise tax on consumers or the investment in research and development (R&D). Hence, we have modified the model, as described below.

First, given that most pollution is caused by consumers, we assume that pollution emissions come from consumption. Therefore, we examine the effect of an excise tax on consumers because it directly affects consumer behavior and has not been previously studied. Second, we include the R&D of environmentally friendly products in the model. Firms have to spend capital on R&D, such as through laboratory and researcher funding, to invent new technologies capable of producing higher quality products. Thus, we assume that firms have to pay investment costs that are related to the emissions abatement level.

The main result is that the first-best allocation and necessary quality levels can be obtained by implementing an excise tax, a subsidy for green consumers, or a combination of both. When duopoly exists in a market, two policies are required to revise the firms' market power and pollution externalities. The optimal tax rate depends on the highest consumer's willingness to pay for environmental quality and the social valuation of the damage associated with polluting emissions. Meanwhile, the optimal subsidy rate hinges on the highest consumer's willingness to pay for environmental quality only. When only one firm exists, only an excise subsidy or a quality subsidy can induce first-best allocation. In this case, all consumers purchase high environmental quality products, and so the demand is fixed because of assumed full market coverage. The firm loses its market power, so only pollution externalities need to be corrected.

The remainder of this paper is organized as follows: the model is described in section 2, the duopoly equilibrium is solved in section 3, the social optimum is characterized and the optimal policy is examined in section 4, and the model is concluded in section 5.

2 The Model

We employ the duopoly model of vertical product differentiation under full information. Both firms, H and L, produce one variant of a good where each is vertically differentiated according to its environmental quality. We assume that there are no pollution emissions from production; rather, emissions are only discharged through consumption, such as in the case of the automobile industry. Moreover, both firms invest in emissions abatement through their R&D departments. We define e_i (with $i = H, L$) as the abatement effort of firm i to reduce the emission intensity of their product when the product is consumed by each consumer with unabated emissions, which is represented by $\bar{e}$. The emission intensity per unit of consumption is $(\bar{e} - e_i)$. We assume $\bar{e}$ to be equal for both variants. Additionally, we assume that the higher the emission intensity per unit of consumption $(\bar{e} - e_i)$, the lower the environmental quality of the product. *Without loss of generality we assume that* $e_H \geq e_L$, where H indicates the high environmental quality variant and L indicates the low environmental quality variant of the differentiated commodity.

To develop a higher quality, firms spend capital investment on R&D. The R&D cost is assumed to be a quadratic function of the abatement level, ke_i^2 where $k > 0$ is an investment efficiency parameter. We assume this parameter is identical for both firms. We also assume that once the firms decide on their own abatement level e, they cannot change the quality level, and they do not have to pay variable costs related to e. However, both firms also have variable costs with linear quantities[1]. The total cost is represented by the following expression:

$$Total\ cost = c_i x_i + ke_i^2 \qquad\qquad i = H, L,$$

where c_i and x_i indicate the marginal cost and the output level, respectively.

To control the emissions from consumers' consumption, a government regulator introduces sanctions to address the total pollution. We consider an ad valorem excise tax and subsidy in this case. The government imposes a uniform excise tax t on the consumers when they purchase one unit of goods. In addition, the regulator also grants a quality subsidy S for consumers who purchase the high-quality variant H.

There is a continuum of consumers whose willingness to pay for environmental quality is measured by the parameter θ, which is uniformly distributed over $[\underline{\theta}, \bar{\theta}]$, with $\bar{\theta} - \underline{\theta} = 1$ and $\underline{\theta} > 0$. We define $\underline{\theta}$ and $\bar{\theta}$ as the lowest and the highest willingness to pay, respectively. We assumed that the market is fully covered, meaning each consumer buys one unit of the differentiated commodity.

Consumers are environmentally conscious; they are willing to pay a higher price if the goods on offer are more environmentally friendly. Consumers derive utility from consuming variant i, whereas their welfare is reduced by the price of the variant they bought and the total pollution produced. The indirect utility of a type θ consumer who buys the variant i of environmental quality e_i at an excise tax-included price p_i is given by the following:

$$U_i = \theta e_i - p_i + dS + T - \gamma E, \tag{1}$$

where θe_i measures the intrinsic utility of a type θ consumer when they consume one unit of variant i. Besides, a consumer who purchases variant H, has paid subsidy $S = s(e_H - e_L)$. Furthermore, d is a dummy variable with $d = 1$ if $i = H$ and $d = 0$ if $i = L$. Nevertheless, all consumers are paid a lump sum transfer from government T. Additionally, $\gamma \geq 0$ measures the social valuation of the damage associated with polluting emissions, meaning the marginal damage caused by emissions. E denotes the aggregate

[1] Lombardini(2005) assumed two things: one, the emissions came from production, and two, without R&D expenditure, the variable costs are convex in quality and linear in quantity.

emissions as $E = (\bar{e} - e_H)x_H + (\bar{e} - e_L)x_L$. Consequently, γE shows the damage from aggregate emissions.

We assume that the individual consumer treats E as a public bad. Hence, E does not affect individual actions or the duopoly equilibrium because it is constant in any individuals' maximization problem. However, this term affects social welfare.

We define a marginal consumer as a consumer who is indifferent between purchasing the high and low quality variant, i.e. $U_L = U_H$. Let $\hat{\theta}$ be the willingness to pay for environmental quality of the marginal consumer. As a consequence, then we have

$$\hat{\theta} = \frac{p_H - p_L}{e_H - e_L} - s, \tag{2}$$

where p_H and p_L represent tax-included consumer prices.

It follows that all consumers willing to pay (θ) higher than $\hat{\theta}$ will demand the high-quality variant. Conversely, all consumers whose θ is lower than $\hat{\theta}$ will demand the low-quality variant. Hence, the demand for the high-quality variant is $x_H = \bar{\theta} - \hat{\theta}$, and the demand for the low-quality variant is $x_L = 1 - (\bar{\theta} - \hat{\theta})$.

The model consists of a three-stage game. In the first stage, the regulator establishes tax and subsidy policies to maximize the social welfare given the best response function of the firms. In the second stage, given the policy chosen by the government in the previous stage, duopolists simultaneously choose the level of pollution abatement e_i. In the third stage, they compete in price. As is customary, we solved the game backwards starting from the third stage.

3 The Duopoly Equilibrium

3.1 The Price Game

As a consequence of the excise tax, the producer price (p_i^p) is defined by $p_i^p = \frac{p_i}{\tau}$ where $\tau = 1 + t$. Accordingly, the profits of duopolists are described by

$$\pi_i = (p_i^p - c_i)x_i - ke_i^2 \qquad i = H, L. \tag{3}$$

We derive the equilibrium price in this stage by plugging (2) into x_H and x_L and substitute into (3), and then differentiating each firm's profit function with respect to p_i^p. When solving the equations for the equilibrium producer price, the equilibrium producer price can be expressed as a consumer tax-included price as[2]

$$p_H = \frac{1}{3}\left[(\bar{\theta} + 1 + s)(e_H - e_L) + \tau(2c_H + c_L)\right], \tag{4}$$

$$p_L = \frac{1}{3}\left[(2 - \bar{\theta} - s)(e_H - e_L) + \tau(c_H + 2c_L)\right]. \tag{5}$$

Substituting (4) and (5) into (2), we obtain the marginal consumer's willingness to pay in this stage as

$$\hat{\theta}_B = \frac{1}{3\Delta e}\left[(2\bar{\theta} - 1 - s)\Delta e + \tau\Delta c\right], \tag{6}$$

where Δe represents the quality dispersion, which is $e_H - e_L > 0$, and Δc represents the marginal cost dispersion, which is $c_H - c_L > 0$.

The marginal consumer's willingness to pay for environmental quality depends on the highest willingness to pay ($\bar{\theta}$), and the marginal cost dispersion (Δc). If these two parameters are extremely low, the marginal consumer's willingness to pay becomes as low

[2] See Appendix 1 for additional detail.

as the lowest willingness to pay for environmental quality $(\underline{\theta})$[3], then the corner solution will occur. This is because, if the difference between marginal costs is small, then the price gap is reduced. To increase firms' market power, both firms have to increase their quality differentiation level. Hence, the quality dispersion is expanded and then the marginal consumer's willingness to pay becomes lower. Accordingly, we can separate this into two cases as seen below.

Case 1: $\hat{\theta}_B > \underline{\theta}$

In case 1, we obtain the interior solution. From (6) we obtain,

$$x_H = \frac{1}{3\Delta e}\left[(\bar{\theta}+1+s)\Delta e - \tau\Delta c\right], \quad (7)$$

$$x_L = \frac{1}{3\Delta e}\left[(2-\bar{\theta}-s)\Delta e + \tau\Delta c\right]. \quad (8)$$

Case 2: $\hat{\theta}_B = \underline{\theta}$

In case 2, $\hat{\theta}_B$ becomes as low as $\underline{\theta}$. Hence, all consumers' willingness to pay for environmental quality is higher than $\hat{\theta}_B$. Therefore, there is no demand for the variant L, such that $x_H = 1$ and $x_L = 0$.[4]

3.2 The Quality Game

Knowing the solution of the price game, we move back to the second stage. In this stage of the game, given the equilibrium prices from the third stage, firms can maximize their profits with respect to quality levels. As a consequence, we may separate this into two cases.

3.2.1 Case 1: $\hat{\theta}_B > \underline{\theta}$

The indirect profit functions as a function of quality are indicated in

$$\max_{e_H} \pi_H = \frac{\left(\Delta e(\bar{\theta}+1+s) - \tau\Delta c\right)^2}{9\tau\Delta e} - ke_H^2, \quad (9)$$

$$\max_{e_L} \pi_L = \frac{\left(\Delta e(2-\bar{\theta}-s) + \tau\Delta c\right)^2}{9\tau\Delta e} - ke_L^2. \quad (10)$$

The solutions to the above equations are indicated in

$$e_H{}' = \frac{2a^2(2D-3B^2)F^{1/3} - Y}{6a^3F^{1/3}}, \quad (11)$$

$$e_L{}' = \frac{2a^2(D-3B^2)F^{1/3} + Y}{6a^3F^{1/3}}, \quad (12)$$

where $a = 18k\tau$, $D = A^2 + B^2$, $A = \bar{\theta}+1+s$, $B = 2-\bar{\theta}-s$, $F = 27a^8(\tau\Delta c)^2 - a^3D^3 + 3\sqrt{81a^{16}(\tau\Delta c)^4 - 6a^{14}D^3}$ and $Y = F^{2/3} + a^4D^2$.

The quality dispersion is derived in,

$$e_H{}' - e_L{}' = \frac{a^2DF^{1/3} - Y}{3a^3F^{1/3}}. \quad (13)$$

[3]The parameter θ is uniformly distributed over $[\underline{\theta}, \bar{\theta}]$. Thus, $\hat{\theta}_B$ less than $\underline{\theta}$ will not exist.

[4]We assume that the firms have different marginal costs, and that $c_H > c_L$ and firm L can only produce a low quality variety. As a result, firm L earns negative profit and withdraws from the market.

We also witness the equilibrium marginal consumer's willingness to pay in this case as described in

$$\hat{\theta}'_B = \frac{(2\bar{\theta} - 1 - s)(a^2 D F^{1/3} - Y) + 3a^3 F^{1/3} \tau \Delta c}{3(a^2 D F^{1/3} - Y)}. \tag{14}$$

3.2.2 Case 2: $\hat{\theta}_B = \underline{\theta}$

In this case, all consumers demand a high environmental quality product. Therefore, the marginal consumer's willingness to pay is equal to the lowest willingness to pay: $\hat{\theta}_B = \underline{\theta}$. That is

$$\hat{\theta}_B = \underline{\theta} = \bar{\theta} - 1 = \frac{p_H}{e_H} - s.$$

Therefore, we get the inverse demand function for firm H in

$$p_H{}'' = e_H(\bar{\theta} - 1 + s). \tag{15}$$

Hence, profit maximization is found in

$$\max_{e_H} \pi_H = \frac{e_H(\bar{\theta} - 1 + s) - \tau c_H}{\tau} - k e_H^2. \tag{16}$$

The solution to the above equation is found in

$$e''_H = \frac{(\bar{\theta} - 1 + s)}{2k\tau}. \tag{17}$$

Determinately, the abatement level increases amid the subsidy, and the highest willingness to pay for environmentally friendly products. Even so, the excise tax and the investment efficiency parameter decrease the abatement level.

4 The Social Optimum and the Optimal Policy

4.1 The Social Optimum

This paper has not focused on the government budget; therefore, we assume that tax revenues are redistributed to consumers as a lump sum and that the quality subsidy is financed by a lump sum collected through consumer taxes. As a result, social welfare is defined as the sum of consumer surplus, producer surplus, and tax revenue minus social damage from total pollution. The optimum allocation can be obtained by solving the problem

$$\begin{aligned} \max_{\hat{\theta}, e_L, e_H} W = & \int_{\bar{\theta}-1}^{\hat{\theta}} (\theta e_L - p_L) d\theta + \int_{\hat{\theta}}^{\bar{\theta}} (\theta e_H - p_H + s(e_H - e_L)) d\theta - \pi_L - \pi_H \\ & + t\left(p_H^p(\bar{\theta} - \hat{\theta}) + p_L^p(\hat{\theta} - \bar{\theta} + 1)\right) - s(e_H - e_L)(\bar{\theta} - \hat{\theta}) - \gamma E. \end{aligned} \tag{18}$$

The optimal allocation of consumers between the two variants and the optimal level of quality are found in[5]

$$\hat{\theta}^* = \underline{\theta}, \tag{19}$$

$$e_H^* = \frac{2\bar{\theta} + 2\gamma - 1}{4k}, \tag{20}$$

$$e_L^* = 0. \tag{21}$$

[5]See Appendix 2 for additional detail.

The optimal allocation is equal to the lowest willingness to pay, which means that all consumers purchase variant H. This is due to the assumption that the market is fully covered. Therefore consumption of only a high abatement level product is better for the environment and overall social welfare. The optimal abatement level increases in accordance with the highest willingness to pay for environmental quality and the social valuation of the damage associated with polluting emissions. Meanwhile it decreases in the investment efficiency parameter of R&D.

4.2 The Optimal Policy

4.2.1 Case 1: $\hat{\theta}_B > \underline{\theta}$

In this situation, allocation and quality diverge from the social optimum due to market power and pollution externality. Thus, we require the two policies to correct these externalities by solving the equations given below.[6]

$$\hat{\theta}^* = \hat{\theta}_B \; ; \; \bar{\theta} - 1 = \frac{1}{3\Delta e}\left[(2\bar{\theta} - 1 - s)\Delta e + \tau\Delta c\right], \tag{22}$$

$$e_H^* = e_H^{'} \; ; \; \frac{2\bar{\theta} + 2\gamma - 1}{4k} = \frac{(\Delta e)^2(1 + \bar{\theta} + s)^2 - (\tau\Delta c)^2}{18k\tau(\Delta e)^2}, \tag{23}$$

$$e_L^* = e_L^{'} \; ; \; 0 = \frac{(\tau\Delta c)^2 - (\Delta e)^2(2 - \bar{\theta} - s)^2}{18k\tau(\Delta e)^2}. \tag{24}$$

In solving these equations, we obtain the optimal policy shown in

$$[t, s] = [\frac{3 - 2\bar{\theta} - 2\gamma}{2\bar{\theta} - 1 + 2\gamma}, 2 - \bar{\theta}]. \tag{25}$$

The optimal excise tax is positive if the social valuation of the damage associated with polluting emissions is low enough, meaning if $\gamma < 3/2 - \bar{\theta}$. An excise tax reduces the quality level and increases the demand for variant L. If this marginal damage from emissions γ is large, governments should pay the excise subsidy to increase the abatement level and increase the demand for variant H to lessen damage from emissions. Interestingly, the optimal subsidy rate is independent of the marginal damage γ. As long as the highest willingness to pay for environmental quality is not extremely high, $\bar{\theta} < 2$, the quality subsidy rate remains positive. Conversely, if consumers have high conscientiousness regarding the environment, $\bar{\theta} > 2$, the government ought to collect a quality tax from consumers who purchase variant H, to reduce excess prices caused by the over-abatement levels of both firms. Therefore, the optimal combination can be summarized as follow: if $\bar{\theta} < 3/2 - \gamma$, then the optimal policies are an excise tax and a quality subsidy, and if $\bar{\theta} \in (\frac{3-2\gamma}{2}, 2)$, then the optimal policies are a quality subsidy and an excise subsidy. Regardless of the social valuation of the pollution, if $\bar{\theta} > 2$, then the optimal policy is to establish a quality tax and excise subsidy.

4.2.2 Case 2: $\hat{\theta}_B = \underline{\theta}$

Since the allocation in the duopoly equilibrium is identical to the first-best and the quality level is still sub-optimal, government intervention is required to further reduce

[6] Instead of (11), (12) and (14), we use (6) and the value of $e_H^{'}$ and $e_L^{'}$ which are derived from the first order condition of (9) and (10) as shown by $e_H^{'} = \frac{(\Delta e)^2(1+\bar{\theta}+s)^2-(\tau\Delta c)^2}{18k\tau(\Delta e)^2}$, $e_L^{'} = \frac{(\tau\Delta c)^2-(\Delta e)^2(2-\bar{\theta}-s)^2}{18k\tau(\Delta e)^2}$.

pollution. Consequently, the optimal policy is identified by solving the equation given by

$$e_H^* = e_H^{''} \; ; \; \frac{2\bar{\theta} + 2\gamma - 1}{4k} = \frac{\bar{\theta} - 1 + s}{2k\tau}. \tag{26}$$

When we set $s = 0$ and solve (26), we will obtain the optimal excise tax as in

$$t = -\frac{2\gamma + 1}{2\bar{\theta} - 1 + 2\gamma}. \tag{27}$$

The optimal quality subsidy s is analogously calculated. We set $t = 0$ and solve (26), thereby producing

$$s = \frac{1 + 2\gamma}{2}. \tag{28}$$

In this case, the optimal policy is a subsidy: an excise subsidy or a quality subsidy. The excise subsidy produces the same effect as the quality subsidy because all consumers purchase high quality variants. Therefore, the total demand for firm H is fixed at 1. Since the firm cannot adjust the quantity, the market power is removed, though the pollution externality still exists. Hence, the government has to grant a subsidy to consumers to motivate firms to raise their environmental quality to indirectly correct pollution externalities.

The optimal excise subsidy rate and environmental quality subsidy increase the social valuation of the damage associated with polluting emissions γ. However, the highest willingness to pay for environmental quality only raises the optimal excise subsidy rate.

Given these data, the results are summarized in the following proposition.

Proposition: *When the market is fully covered, the social optimum can be obtained by using an excise tax, a quality subsidy, or a combination of both. If the marginal consumer's willingness to pay for environmental quality is higher than the lowest willingness to pay ($\hat{\theta}_B > \underline{\theta}$), then two policies are required to revise the market power in addition to the pollution externality. Conversely, if the marginal consumer's willingness to pay for environmental quality becomes as low as the lowest willingness to pay ($\hat{\theta}_B = \underline{\theta}$), only one policy is required.*

These findings are different from those revealed by Lombardini (2005). In this model, only one policy can induce the social optimum, while in Lombardini's (2005) case, a set of two policies is necessary.

5 Concluding Remarks

To study the optimal excise tax and quality subsidy policy, we formulated a model of vertical product differentiation with full market coverage. This assumption crucially affected the main result. The social optimum can be obtained using an excise tax, a subsidy for green consumers, or a combination of these policies. When two policies are required, the optimal tax rate will depend on the consumers' willingness to pay for the environmental and social valuation of the damage associated with polluting emissions. Conversely, the optimal subsidy rate hinges on the consumers' willingness to pay for the environment only. However, only an excise subsidy or a subsidy for quality can produce the first-best allocation. In this case, the low environmental quality producer disappears from the market due to consumers demand for high environmental quality. Under the assumption that the market is covered, firms cannot decrease or increase their output. As a result, market power is erased and only the pollution externality exists. Finally, the regulator requires only one policy to control pollution emissions.

Appendix

Appendix 1

Since the producer price is denoted as $p_i^p = \frac{p_i}{\tau}$ where $\tau = 1 + t$. Hence, from equation (2), we get the demand for each firm as

$$x_H = \bar{\theta} - \frac{\tau(p_H^p - p_L^p)}{e_H - e_L} + s, \tag{a1}$$

$$x_L = 1 - (\bar{\theta} - \frac{\tau(p_H^p - p_L^p)}{e_H - e_L} + s). \tag{a2}$$

Substitue (a1) and (a2) into (3), we get the profit mamization of each firm.

$$\max_{p_H^p} \pi_H = (p_H^p - c_H)[\bar{\theta} - \frac{\tau(p_H^p - p_L^p)}{e_H - e_L} + s] - ke_H^2, \tag{a3}$$

$$\max_{p_L^p} \pi_L = (p_L^p - c_L)[1 - (\bar{\theta} - \frac{\tau(p_H^p - p_L^p)}{e_H - e_L} + s)] - ke_L^2. \tag{a4}$$

The first order condition from (a3) and (a4) are

$$(p_H^p - c_H)\frac{\tau}{e_H - e_L} = \bar{\theta} - \frac{\tau(p_H^p - p_L^p)}{e_H - e_L} + s, \tag{a5}$$

$$(p_L^p - c_L)\frac{\tau}{e_H - e_L} = 1 - (\bar{\theta} - \frac{\tau(p_H^p - p_L^p)}{e_H - e_L} + s). \tag{a6}$$

Therefore we obtain the equilibrium producer price as,

$$p_H^p = \frac{1}{3\tau}\left[(\bar{\theta} + 1 + s)(e_H - e_L) + \tau(2c_H + c_L)\right], \tag{a7}$$

$$p_L^p = \frac{1}{3\tau}\left[(2 - \bar{\theta} - s)(e_H - e_L) + \tau(c_H + 2c_L)\right], \tag{a8}$$

which can be rewritten to consumer tax-included price as (4) and (5).

Appendix 2

The equation (18) can be written as

$$\begin{aligned}
\max_{\hat{\theta}, e_L, e_H} W =& \int_{\bar{\theta}-1}^{\hat{\theta}} (\theta e_L - c_L)d\theta + \int_{\hat{\theta}}^{\bar{\theta}} (\theta e_H - c_H)d\theta - ke_H^2 - ke_L^2 \\
& - \gamma\left[(\bar{e} - e_H)(\bar{\theta} - \hat{\theta}) + (\bar{e} - e_L)\left(\hat{\theta} - (\bar{\theta} - 1)\right)\right]. \\
=& \frac{\hat{\theta}^2 - (\bar{\theta} - 1)^2}{2}e_L - c_L[\hat{\theta} - (\bar{\theta} - 1)] + \frac{\bar{\theta}^2 - \hat{\theta}^2}{2}e_H - c_H[\bar{\theta} - \hat{\theta}] - ke_H^2 - ke_L^2 \\
& - \gamma\left[(\bar{e} - e_H)(\bar{\theta} - \hat{\theta}) + (\bar{e} - e_L)\left(\hat{\theta} - (\bar{\theta} - 1)\right)\right]
\end{aligned} \tag{a9}$$

Partially differentiate (a9) with respect to $\hat{\theta}$, e_H and e_L, we get the first order condition as follow.

$$\hat{\theta} = \frac{c_H - c_L}{e_H - e_L} - \gamma, \tag{a10}$$

$$e_H = \frac{\bar{\theta}^2 - \hat{\theta}^2 + 2\gamma(\bar{\theta} - \hat{\theta})}{4k}, \tag{a11}$$

$$e_L = \frac{\hat{\theta}^2 - (\bar{\theta} - 1)^2 + 2\gamma\left(\hat{\theta} - (\bar{\theta} - 1)\right)}{4k}. \tag{a12}$$

From (a11) and (a12), we get

$$e_H - e_L = \frac{A - 2\hat{\theta}(\hat{\theta} + 2\gamma)}{4k}, \tag{a13}$$

where $A = \bar{\theta}^2 + (\bar{\theta} - 1)^2 + 2\gamma[\bar{\theta} + (\bar{\theta} - 1)] > 0$. Substitute (a13) into (a10) and solve for $\hat{\theta}$, we get

$$\begin{aligned}\hat{\theta}^* = & -\gamma + \frac{A^2 + 2\gamma^2}{(-216k\Delta c + \sqrt{(216k\Delta c)^2 - 216(A + 2\gamma^2)^3})^{1/3}} \\ & + \frac{(-36k\Delta c + \sqrt{(36k\Delta c)^2 - 6(A + 2\gamma^2)^3})^{1/3}}{6^{2/3}}.\end{aligned} \tag{a14}$$

The denominator of the second term and numerator of the third term are negative, therefore $\hat{\theta}^* < 0$. However, the parameter θ is uniformly distributed over $[\underline{\theta}, \bar{\theta}]$, hence

$$\hat{\theta}^* = \underline{\theta}. \tag{a10'}$$

By Substituting (a10') into (a11') and (a12'), we get (20) and (21).

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