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Military Spending and Stochastic Growth: A Small Open Economy

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Abstract

In a stochastic endogenous growth model, the effects of military spending on economic growth are explored. In addition to the traditional crowding-out effect, we show that the portfolio effect also plays an importance role in determining growth rate in a small open economy. We find that there exists a non-linear relationship between the defense burden and the economic growth.

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1. Introduction

The traditional Keynes-viewpoint indicates that a rise in government expenditure would generate the crowding-out effect. Namely, an increase in military spending would crowd out private investment and in turn lead to a decrease in economic growth. In the empirical study, while Benoit (1978) shows that the impact of the defense spending on economic growth is positive, the reverse-U relation between defense spending and economic growth has been found in Landau (1993, 1996), DeRouen (1995). That is to say, the growth effects of the military spending are ambiguous. Therefore, the purpose of this paper is to construct a theoretical model to re-examine the link between military spending and growth rate.

In the recent development, by using the endogenous growth model, Shieh et al. (2002a, 2002b) and Shieh et al. (2007) show that the link between defense spending and the rate of economic growth may be positive or reverse-U. In the stochastic endogenous growth framework, Gong and Zou (2003) discuss the linkage between military spending and output growth. They show that the relationship between the military spending and economic growth depends on the intertemporal substitution elasticity in consumption. Lin and Lee (2006, 2012) argue that, in addition to the crowding-out effect, the spin-off effect and the resource mobilization effect derived from military spending also affect the growth rate. In sum, the existing studies consider the closed economy, that is, ignore the influence of the open economy.

This paper will construct a stochastic endogenous growth model to analyze the impact of military spending on economic growth. In addition to the traditional crowding-out effect, we show that the portfolio effect derived from military spending is an important factor affecting growth rate in a small open economy.[1] The portfolio

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[1] The implication of the crowding-out effect implies that an increase in military spending will reduce aggregate investment and hence has a negative effect on the expected growth rate. The portfolio effect argues that a rise in military spending affects the private sector's portfolio and then has an ambiguous effect on the expected growth rate.

effect is not found in Deger and Sen (1995), Shieh *et al*. (2002a), and Lin and Lee (2006, 2012), because of ignoring the influence of open economy.

The structure of the paper is as follows. Section 2 sets up a small open-economy stochastic endogenous growth model. Section 3 discusses the effects of military spending on economic growth. Section 4 concludes the paper.

2. The model

The small open economy produces and consumes a single traded good, which is taken to be *numeraire*. Output, Y, is produced by hiring human as well as physical capital, K, and then the production function can be expressed by the stochastic AK technology as follows:[2]

$$dY = \alpha K(dt + du_y), \tag{1}$$

where α is a positive constant that reflects the level of the technology. The stochastic term du_y represents the output flow shocks and is a temporally independent and normally distributed random variable with mean zero and variance, $\sigma_y^2 dt$.

Assume that the military spending (M) is a fixed fraction of output flow.[3] Hence, we have:

$$dM = \alpha K(hdt + h' du_y), \quad 0 < h < 1, \quad 0 < h' < 1. \tag{2}$$

The parameter h can be treated as an index of the defense burden. Therefore, an increase in h will lead to an expansion in military expenditure in a growing economy. The parameter h' denotes the defense burden on the stochastic component of military spending.

The military spending can provide a measure of security and hence increase the

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[2] For the same assumption, please see Turnovsky (2000) and Gong and Zou (2003).

[3] For the same assumption, please see Shieh *et al*. (2007), Shieh *et al*. (2002a, 2002b) and Lin and Lee (2012).

country's utility, that is, the marginal utility of military spending is positive. The preference of the small open economy is defined on the consumptions of consumption-good, C, and the military spending, M, and then the utility function can be specified as follows:[4]

$$U(C, M) = \ln C + \eta \ln M \ , \ \ \eta > 0 \, . \tag{3}$$

The assumption of positive marginal utility from consumption of military spending will lead to $\eta > 0$.

Equity investment is the real investment opportunity represented by this technology. Hence, the real rate of return on equity (capital) is:

$$dR_K = \frac{dY}{K} = \alpha dt + \alpha du_y \, . \tag{4}$$

There are two securities in the portfolio of the small open economy. These include the above equity claims on human as well as physical capital, K, and the foreign bonds, B. The price of foreign bonds, E, is assumed to be generated by the geometric Brownian motion process:

$$\frac{dE}{E} = \varepsilon dt + du_e \, , \tag{5}$$

where ε is the instantaneous expected rate of change in the price of foreign bonds. The stochastic term du_e represents the foreign-bonds price shocks and is a temporally independent and normally distributed random variable with mean zero and variance, $\sigma_e^2 dt$. The real rate of return on foreign bonds is:

$$dR_F = (i^* + \varepsilon) dt + du_e \, , \tag{6}$$

where i^* represents the foreign interest rate which is exogenously given. For simplicity, we suppose that the two stochastic terms, du_y and du_e, are uncorrelated.

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[4] The similar specification of the utility function can also be seen in Zou (1995), Chang et al. (1996), and Shieh et al. (2007).

The total wealth satisfies W=K+EB. The stochastic wealth accumulation equation can be expressed as:

$$dW = W(n_K dR_K + n_F dR_F) - Cdt - dM \ , \tag{7}$$

where $n_K = K/W$ is the share of capital on the total wealth, $n_F = (EB)/W$ is the share of foreign bond on the total wealth. Substituting Equations (4), (6), and (2) into Equation (7), we have:

$$\frac{dW}{W} = [\alpha(1-h)n_K + (i^* + \varepsilon)n_F - \frac{C}{W}]dt + du_w \, , \tag{8}$$

where

$$n_K + n_F = 1 \, , \tag{9}$$

and

$$du_w = \alpha(1-h')n_K du_y + n_F du_e \quad \text{and} \quad \sigma_w^2 = \alpha^2(1-h')^2 n_K^2 \sigma_y^2 + n_F^2 \sigma_e^2 \, . \tag{10}$$

Based on the Equations (3), (8) and (9) and the given initial conditions (i.e., W_0), the objective of the small open economy is to maximize the expectation of the discounted sum of future utilities as follows:

$$\max \mathrm{E}_0 \int_0^\infty U(C, M)e^{-\beta t} dt \, ,$$

$$\text{s.t.} \ \frac{dW}{W} = \psi \, dt \ + \ du_w, \tag{11a}$$

$$n_K + n_F = 1 \, ,$$

where the operator E_0 is a mathematical condition expectation on date 0. The parameter β is a constant rate of time preference, and $0 < \beta < 1$. In addition, we define the expected growth rate ψ as:

$$\psi \equiv \alpha(1-h)n_K + (i^* + \varepsilon)n_F - \frac{C}{W} \, . \tag{11b}$$

3. Comparative dynamics

The effects of the military spending on the expected growth rate will be analyzed in this section. By using the methods of stochastic dynamic program, we can derive:[5]

$$\frac{C}{W} = \beta . \tag{12a}$$

$$n_K = \frac{\alpha(1-h) - (i^* + \varepsilon) + \sigma_e^2}{[\alpha^2 (1-h')^2 \sigma_y^2 + \sigma_e^2]} . \tag{12b}$$

$$n_F = 1 - n_K . \tag{12c}$$

$$\sigma_w^2 = \alpha^2 (1-h')^2 n_K^2 \sigma_y^2 + n_F^2 \sigma_e^2 . \tag{12d}$$

The equilibrium values of C/W, n_K, n_F, and σ_w^2, are determined by Equations (12a)-(12d). We can also find the expected growth rate as follows:[6]

$$\psi = \alpha(1-h) n_K + (i^* + \varepsilon) n_F - \beta . \tag{13}$$

From Equations (13) and (12b), we can derive:

$$\frac{\partial \psi}{\partial h} = -\alpha n_K + [\alpha(1-h) - (i^* + \varepsilon)] \frac{\partial n_K}{\partial h}$$

$$= \frac{-2\alpha}{\alpha^2 (1-h')^2 \sigma_y^2 + \sigma_e^2} [\alpha(1-h) - (i^* + \varepsilon) + \frac{\sigma_e^2}{2}] . \tag{14}$$

As exhibited in the first line in Equation (14), we argue that a rise in the defense burden affects the expected growth rate through two channels.[7] The first is “the crowding-out effect” which implies that an increase in military spending will reduce aggregate investment and hence has a negative effect on the expected growth rate. The second is “the portfolio effect” which implies that a rise in military spending

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[5] For the proof, please see Appendix which states the analytical process of macroeconomic equilibrium.

[6] The expected growth rate has been derived in Appendix.

[7] Deger and Sen (1995), Shieh *et al.* (2002a), and Lin and Lee (2006, 2012) stress the importance of the crowding-out effect, the spin-off effect, and the resource mobilization effect. The spin-off effect argues that a rise in military spending has positive externalities on private production, and hence has a positive effect on economic growth. The resource mobilization effect indicates that a rise in military spending affects the private sector’s consumption propensity.

affects the private sector's portfolio. That is, in a small open economy, the military spending influences the relative return to domestic and foreign investment and thus by affecting the private sector's portfolio can have an impact on economic growth. This channel has an ambiguous effect on the expected growth rate. In summary, the net effect which depends on two channels is ambiguous.

Next, we let the second line of Equation (14) be equal to zero, and in turn we can obtain the expected growth-maximizing defense burden, $\bar{h}$, as follows:

$$\bar{h} = \frac{1}{\alpha}[\alpha - (i^* + \varepsilon) + \frac{\sigma_e^2}{2}]. \tag{15}$$

Finally, we will explore the relationship between the expected growth rate and the stochastic term of military spending. From Equations (13) and (12b), differentiating ψ with respect to h' can obtain:

$$\frac{\partial \psi}{\partial h'} = [\alpha(1-h) - (i^* + \varepsilon)]\frac{\partial n_K}{\partial h'}$$

$$= [\alpha(1-h) - (i^* + \varepsilon)]\frac{2\alpha^2(1-h')\sigma_y^2[\alpha(1-h) - (i^* + \varepsilon) + \sigma_e^2]}{[\alpha^2(1-h')^2\sigma_y^2 + \sigma_e^2]^2} > 0,$$

$$\text{if} \quad \alpha(1-h) > (i^* + \varepsilon). \tag{16}$$

As shown in the first line in Equation (16), we find that the h' can influence the economic growth by affecting the private sector's portfolio. As exhibited in the second and third lines in Equation (16), we show that the link between h' and economic growth depends on the real returns of domestic capital and foreign bonds. If the real return of domestic capital ($\alpha(1-h)$) exceeds the real return of foreign bonds ($i^* + \varepsilon$), then the stochastic term h' has a positive effect on economic growth due to a rise in the investment of domestic capital.

4. Conclusions

By extending the Gong and Zou (2003) and Lin and Lee (2006) model, this paper analyzes the impact of military spending on economic growth in a stochastic endogenous growth model. In addition to the traditional crowding-out effect, we show that the portfolio effect also plays an importance role in determining growth rate in a small open economy. We find that there exists a non-linear relationship between the defense burden and the economic growth, as shown in Landau (1993, 1996), DeRouen (1995), and Lin and Lee (2006). We also find that there exists an optimal defense burden that maximizes the economic growth rate, as proved by Shieh et al. (2002b) and Lin and Lee (2006).

Appendix

To solve the problem described by Equations (11a) and (11b), we apply the methods of stochastic dynamic program. First, we define the differential generator of the value function $V(W,t)$ as follows:

$$L[V(W, t)]= \lim_{dt\to 0} \mathrm{E}\,[\frac{dV}{dt}]= V_t + V_W W[\,\alpha(1-h)n_K + (i^* + \varepsilon)n_F - \frac{C}{W}\,]+ \frac{1}{2}\, V_{WW} W^2 \sigma_w^2,$$

where

$$\sigma_w^2 = \alpha^2 (1-h')^2 n_K^2 \sigma_y^2 + n_F^2 \sigma_e^2 .$$

Based on the exponential time discounting, i.e., exp $(-\beta t)$, the value function, V, can be assumed to be the time separable form as:

$$V(W, t) = X(W)\, e^{-\beta t} .$$

Therefore, the small open economy chooses C, n_K, and n_F to maximize the following Lagrangean expression:[8]

$$U(C,M)e^{-\beta t} + L[X(W)\, e^{-\beta t}\,] + \ \lambda e^{-\beta t}(1-n_K - n_F) =$$

$$e^{-\beta t}\,\{\, \ln C + \eta \ln M \ - \beta\, X + X_W W[\,\alpha(1-h)n_K + (i^* + \varepsilon)n_F - \frac{C}{W}\,]$$

[8] For the same notion, please refer to Dixit and Pindyck (1994, p. 107) and Kamien and Schwartz (1991, p. 269).

$$+\frac{1}{2}X_{WW}W^2\sigma_w^2+\lambda(1-n_K-n_F)\ \}. \tag{A.1}$$

The parameter λ is the Lagrangean multiplier. Taking partial derivatives with respect to C, n_K, n_F, and λ, of Equation (A.1) and canceling exp $(-\beta t)$ can obtain:

$$C^{-1}=X_W, \tag{A.2a}$$

$$\alpha(1-h)WX_W+\alpha^2(1-h')^2 n_K\sigma_y^2W^2X_{WW}=\ \lambda, \tag{A.2b}$$

$$(i^*+\varepsilon)WX_W+n_F\sigma_e^2W^2X_{WW}=\ \lambda, \tag{A.2c}$$

$$n_K+n_F=1. \tag{A.2d}$$

Equation (A.2a) states that the marginal utility of consumption-good must be equal to that of the total wealth. From Equations (A.2a), (A.2b), (A.2c), and (A.2d), we can derive the optimal solutions of $\tilde{C}$, $\tilde{n}_K$, $\tilde{n}_F$, and $\tilde{\lambda}$, which will be affected by X_W and X_{WW}. In addition, the value function must satisfy the Bellman equation, which can be shown as follows:

$$\max_{C,n_K,n_F}\ \{U(C,M)e^{-\beta t}+L[X(W)\,e^{-\beta t}]\}=0.$$

Substituting the optimal solutions derived from Equations (A.2a)-(A.2d) into the Bellman equation, we can find:

$$\ln\tilde{C}+\eta\ln M\ -\beta X+X_WW[\alpha(1-h)\tilde{n}_K+(i^*+\varepsilon)\tilde{n}_F-\frac{\tilde{C}}{W}]+\frac{1}{2}X_{WW}W^2\tilde{\sigma}_w^2=0,$$

where $\tilde{}$ denotes optimal value. To solve the Bellman equation, we conjecture that a solution for the value function is the following form:

$$X(W)=\delta\ln W\ , \tag{A.3}$$

where the coefficient δ is going to be determined. From Equation (A.3), we get:

$$X_W=\delta W^{-1}, \tag{A.4a}$$

$$X_{WW}=-\ \delta W^{-2}. \tag{A.4b}$$

Substituting Equation (A.4a) into Equation (A.2a) yields:

$$\frac{C}{W} = \delta^{-1}. \tag{A.5}$$

Substituting Equation (A.4a) and (A.4b) into Equations (A.2a)-(A.2d) obtains:

$$n_K = \frac{\alpha(1-h) - (i^* + \varepsilon) + \sigma_e^2}{[\alpha^2 (1-h')^2 \sigma_y^2 + \sigma_e^2]}. \tag{A.6}$$

Substituting Equations (A.4a), (A.4b), and (A.5) into the Bellman equation gets:

$$\frac{\tilde{C}}{W} X - \beta X = 0.$$

We in turn can find the consumption-wealth ratio shown in Equation (12a) as follows:

$$\frac{C}{W} = \beta. \tag{A.7}$$

From Equations (A.5) and (A.7), the coefficient δ can be determined and in turn the value function can also be determined.

Next, substituting Equation (A.7) into (11b), we can derive the expected growth rate shown in Equation (13) as follows:

$$\psi = \alpha(1-h)n_K + (i^* + \varepsilon)n_F - \beta. \tag{A.8}$$

Finally, we have the dynamic equations for the wealth accumulation

$$\frac{dW}{W} = \psi\, dt + du_w.$$

The solutions to the dynamic equations, starting from initial wealth $W(0)$ at time 0, is:

$$W(t) = W(0)e^{(\psi - \frac{1}{2}\sigma_w^2)t + u_w(t) - u_w(0)}.$$

The transversality condition,

$$\lim_{t\to\infty} \mathrm{E}[X(W)e^{-\beta t}] = 0,$$

will be met if and only if

$$-\beta < 0,$$

which is equivalent to $C/W > 0$.

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