Real-Exchange-Rate-Adjusted Inflation Targeting in an Open Economy: Some Analytical Results

Richard T Froyen  
*University of North Carolina*

Alfred V Guender  
*University of Canterbury*

Abstract

Under optimal policy from a timeless perspective, a central bank targeting an inflation measure which is adjusted for changes in the real exchange rate (REX inflation) has the ability to stabilize the output gap and inflation against demand disturbances in an open economy. This distinct advantage is lost if a central bank follows a Taylor-type rule. The bank has an incentive to add the real exchange rate to the Taylor rule because it duplicates the performance of the optimal policy for portfolio shocks. The Taylor-type rule becomes a Monetary Conditions Index (MCI) that outperforms Taylor-type rules which accord no weight at all or a higher weight to the real exchange rate. In the current environment of concern about sudden increases in U.S. interest rates, the properly designed MCI would have a considerable advantage.
1. Introduction

Policymakers in small open economies around the world are increasingly concerned about the undesirable consequences of sudden increases in U.S. interest rates. This paper demonstrates that in a small open economy a central bank can successfully ward off sudden changes in foreign interest rates and risk premium shocks—portfolio shocks—by choosing a core rate of inflation, real-exchange-rate-adjusted (REX) inflation, as its inflation objective and adding the real exchange rate to a Taylor-type rule. The Taylor-type rule with a prescribed exchange rate response becomes a Monetary Conditions Index (MCI). From a welfare perspective the Taylor-type rule, with (the MCI) or without the exchange rate response, delivers welfare losses that are substantially greater than under optimal policy with commitment. But the performance of the properly designed MCI relative to other Taylor type rules improves with portfolio shocks becoming a more important source of disturbances.

2. Model for a Small Open Economy

The open economy model is a variant of the New Keynesian type:

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa y_t + b(q_t - q_{t-1}) - \beta b(E_t q_{t+1} - q_t) + u_t \]  
\[ y_t = E_t y_{t+1} - a_1(R_t - E_t \pi^{CPI}_{t+1}) + a_2(q_t - E_t q_{t+1}) + a_3\left(\pi_t - E_t \pi^{CPI}_{t+1}\right) + v_t \]  
\[ R_t - E_t \pi_{t+1} = R^{f}_{t} - E_t \pi^{f}_{t+1} + E_t q_{t+1} - q_t + \varepsilon_t \]  
\[ \pi^{CPI}_{t+1} = \pi_t + \gamma \Delta q_t \]

\( \pi_t \) = the rate of domestic inflation  
\( E_t \pi^{CPI}_{t+1} \) = the expected rate of CPI inflation  
\( q_t \) = the real exchange rate  
\( y_t \) = the output gap  
\( R_t \) = the nominal rate of interest (policy instrument)  
\( R^{f}_{t} \) = the foreign nominal rate of interest  
\( E_t \pi^{f}_{t+1} \) = the expected foreign rate of inflation

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1 REX inflation is similar to the modified inflation measure suggested by Ball (1999) in the open-economy context.
\[ y_t^\prime = \text{the foreign output gap} \]

Lower case variables represent logarithms. All parameters are positive. The discount factor \( \beta \) is less than or equal to one. \( \gamma \) denotes consumption openness: \( 0 < \gamma < 1 \). We treat all foreign variables as exogenous random variables that are independent of each other. All shocks are white noise processes with constant variance.

Equation (1) represents an open economy Phillips curve. It is derived from an optimizing framework where price-setting domestic firms respond not only to changes in marginal cost but also to foreign price and exchange rate induced changes in their competitiveness vis-à-vis foreign firms. The “competitiveness effect” results in a direct real exchange rate channel in the Phillips curve which complicates the formulation of optimal monetary policy.\(^2\) Equation (2) is an open economy IS relation while the Uncovered Interest Rate Parity (UIP) condition is given by equation (3). Equation (4) represents the relationship between CPI inflation, domestic inflation and the real exchange rate under complete exchange rate pass-through.

3. **Flexible Real-Exchange-Rate-Adjusted (REX) Inflation Targeting: Optimal Policy under Commitment\(^3\)**

An inflation measure which can serve as a central bank’s inflation target in an open economy is domestic inflation stripped of the effects of changes in the real exchange rate: REX inflation. The REX inflation measure is similar to Ball’s (1999) modified inflation measure. His measure, which he argues is “similar in spirit to calculations of ‘core’ or ‘underlying’ inflation by central banks”, also strips away transitory exchange rate effects.\(^4\) As shown below, the rationale for choosing REX inflation derives from its attractive stabilizing properties in the face of demand-driven shocks. Defining

\[ \pi_t^{REX} = \pi_t - b(q_t - q_{t-1}) \]

\(^2\) With the exception of equation (1) the model conforms to the standard open economy New Keynesian framework proposed by Gali and Monacelli (2005). In Gali and Monacelli (2005) firms set domestic prices as a mark-up over marginal cost only. The effect of competitiveness vis-à-vis foreign firms receives support from surveys [Greenslade and Parker (2012)] as well as from micro data [Brunn and Ellis (2012a), (2012b)]. The derivation of the open economy Phillips curve is explained in greater detail in Froyen and Guender (2014). Phillips curves with distinct exchange rate channels feature also in Ball (1999) and Svensson (2000).

\(^3\) The formation of optimal monetary policy is described in the context of a linear-quadratic framework which is rather standard in the New Keynesian literature (Clarida, Gali, and Gertler (1999), Woodford (2011) for a closed economy and Clarida, Gali, and Gertler (2002), Gali and Monacelli (2005) for an open economy).

\(^4\) Ball’s (1999) definition of “long-run inflation” in a simple backward-looking framework involves the level and not the change of the real exchange rate.
as the domestic rate of inflation purged of the real exchange rate effect allows us to rewrite the original open-economy Phillips curve as

$$\pi_t^{REX} = \beta E_t \pi_{t+1}^{REX} + \kappa y_t + u_t.$$  \hfill (6)

Except for the definition of the rate of inflation, equation (6) corresponds to the standard closed-economy Phillips curve where policy exerts its effect on inflation only through the output gap channel. As such neither the open-economy IS nor the UIP relation acts a binding constraint under optimal policy.

The central bank’s objective is to stabilize the output gap and REX inflation.\(^5\) Under optimal policy from a timeless perspective, the central bank minimizes the squared deviations from target in both target variables:

$$\min_{y_t, \pi_t} \frac{1}{2} E_t \left[ \sum_{i=0}^{\infty} \beta^i \left( y_{t+i}^2 + \mu \pi_{t+i}^{REX^2} \right) \right]$$

subject to

$$\pi_t^{REX} = \beta E_t \pi_{t+1}^{REX} + \kappa y_t + u_t$$

Barring the definition of the rate of inflation, the resulting target rule is the same as in the closed economy:

$$y_t - y_{t-1} + \mu \kappa \pi_t^{REX} = 0.$$  \hfill (8)

This is the celebrated result according to which optimal monetary policy in an open economy is isomorphic to that in a closed economy. It implies that

i. only a cost-push shock dislodges the output gap and REX inflation from their respective target levels;

ii. the central bank can shield both the output gap and REX inflation from the effects of both domestic and foreign disturbances arising in the goods market as well as portfolio shocks.

\(^5\) Ideally, the central bank maximizes the expected utility of a representative household. In this context, Kirsanova et al (2006) show that the objective function of a central bank must include not only the output gap and domestic inflation but also the real exchange rate (or terms of trade). By choosing REX inflation as a target variable, the central bank mitigates this concern. The target level for REX inflation is set to zero.
4. The Stabilizing Properties of a Taylor-Type Rule

Optimal policy is often dismissed as being infeasible. A prescriptive rule for monetary policy commonly used in policy analysis is a Taylor-type rule. While ad hoc in its conception a Taylor-type rule has considerable appeal because of its inherent simplicity and its robustness in a wide variety of macroeconomic models.6

In this section we evaluate Taylor-type rules in a small open economy where the central bank targets REX inflation. Our analysis addresses the following issues:

- How do Taylor-type rules compare to optimal policy?
- Should the rule respond to the real exchange rate?
- If so, what is the appropriate weight on the real exchange rate?

The specification of the Taylor-type rule takes the following form:

\[ R_t = \tau_{\pi} \pi_t^{\text{REX}} + \tau_{\gamma} y_t + \tau_q q_t \]  

(9)

\( \tau_i \) = policy parameter chosen by the central bank; \( i = \pi, y, q \).

The central bank adjusts the policy setting in response to movements in REX inflation, the output gap, and, if warranted, the level of the real exchange rate.7

4.1 Optimal Policy versus Taylor-Type Rules: A Theoretical Assessment

Combining equations (1) – (4) with equation (8) and equation (9), respectively, yields the solutions for the variables of the model under optimal policy from a timeless perspective and a Taylor-type rule. The analytical solutions appear in Table I.

Inspection of the results in the bottom half of Table I confirms that under optimal policy both target variables are fully insulated from demand-side shocks and are displaced only by a cost-push disturbance.8

The top half of the table shows the stabilization response of the four endogenous variables under a Taylor-type rule. It is evident that REX inflation and the output gap are susceptible to all shocks of the model. The inferior stabilization response of a Taylor-type rule relative to optimal policy comes about because the policy instrument cannot respond

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7 For the two target variables and the real exchange rate the respective target level is zero.
8 To save space, we do not report the solutions for the policy instrument and the real exchange rate.
directly to the shocks of the model. A close look at the coefficients on the shocks reveals further why a Taylor rule that responds arbitrarily to real exchange rate movements may lead to inferior stabilization outcomes compared to a conventional Taylor rule. First, adding the real exchange rate to a Taylor rule worsens the stabilization response of both target variables to demand-side disturbances. The response parameter $\tau_q$ appears in both the numerator and denominator of the coefficient on all demand-side disturbances in the solution for the output gap and REX inflation. For demand-side shocks, best stabilization of both target variables under a Taylor type rule is achieved if $\tau_q = 0$. Second, for REX inflation a distorted stabilization response occurs also in case of a cost-push shock if the real exchange rate appears in a Taylor-type rule, i.e. if $\tau_q > 0$. The only gain (for the target variables) from adding the real exchange rate to the instrument rule is better output gap stabilization in the wake of a cost-push shock. If REX inflation rises because of a cost-push shock, the ensuing appreciation of the real exchange rate implies that the monetary tightening necessary to bring inflation to heel need be less severe. The real exchange rate response softens the impact of the cost-push shock and leads to a smaller contraction of the output gap. This is clearly evident in Table I: $\tau_q$ appears only in the denominator of the coefficient on the cost-push shock in the solution for the output gap.

4.2 The Real Exchange Rate as a Stabilization Tool

The foregoing analysis shows that a Taylor-type rule that provides for an arbitrary real exchange rate response can be problematic for stabilization policy in an open economy. In this section attention focuses on whether a central bank can make the policy instrument respond to the real exchange rate so as to nullify the effect of disturbances.

Inspection of the coefficient on $R_t^f + \varepsilon_t$ in the top half of Table I makes it clear that a central bank can engineer an optimal response to portfolio shocks. By choosing $\tau_q = \frac{a_2}{a_1} - (b + \gamma)$, the central bank can shield the output gap and REX inflation from this source of disturbance. Adopting this setting for $\tau_q$ amounts to following a special Monetary Conditions Index where the relative weight on the real exchange rate is composed of the ratio of two demand-side parameters adjusted for the degree of openness in the Phillips curve and consumption openness.\(^\text{10}\)

\(^{10}\) Previous investigations of MCIs are by Ball (1999) and Gerlach and Smets (2000).
4.3 Flexible REX Inflation Targeting under Taylor-Type Rules

To measure overall performance, we examine the stabilizing properties of four specifications of a Taylor-type rule, each of which assigns a specific weight (ranging from 0 to 0.5) to the real exchange rate. A standard loss function consisting of the weighted variances of REX inflation and the output gap is used to gauge the performance of each Taylor-type rule. To complete the analysis, we compare the performance of the Taylor type rules to the benchmark case of optimal policy from a timeless perspective.

The variances of all endogenous variables appear in Table II. The numerical evaluation of the performance of the four Taylor-type rules suggests that the MCI (rule (2)), which accords a very low weight (0.0333) to the real exchange rate, produces the lowest loss score at 1.8197. The differences in performance among the four specifications are minute if each shock has unit variance. A striking difference, however, exists between the loss scores of the Taylor-type rules and the loss score of optimal policy from a timeless perspective. The bottom row of Table II records welfare losses for the Taylor-type rules (vis-à-vis optimal policy) ranging from 99.3% to 102.2%. As explained in Section 2, under optimal policy, the central bank can shield both REX inflation and the output gap from IS, foreign output, and portfolio shocks by aggressive use of the policy instrument. A Taylor-type rule can at best protect both target variables from portfolio shocks if a central bank follows an MCI. The performance of the MCI relative to the other Taylor-type rules listed in Table II improves further with portfolio shocks becoming the dominant source of disturbances (e.g. for $\sigma_{\tau/r}^2 = \sigma_{\hat{e}}^2 = 4$ and the variances of all other shocks remaining unity).

5. Conclusion

In an open economy where the central bank practices flexible REX inflation targeting welfare losses from following a Taylor-type rule relative to optimal policy are substantial. These losses arise primarily because a Taylor-type rule cannot stabilize the output gap as well as optimal policy. There is a gain from including the real exchange rate in a Taylor-type rule as such a rule (MCI) offers the central bank the opportunity to fashion a response to the real

11 Earlier contributions that have studied Taylor-type rules are Ball (1999), Taylor (2001), Batini et al (2003), to name but a few. We use representative values for the model parameters, policy and preference parameters as well as the variances of the shocks to compute the variances of the endogenous variables. See the bottom of Table II for further details.

12 The main focus here is on overall performance rather than individual variables. The variance of the real exchange rate is considerably higher than the variance of the policy instrument due to the acute sensitivity of the real exchange rate to portfolio shocks.
exchange rate which eliminates the effect of a portfolio shock on both REX inflation and the output gap. The optimal weight on the real exchange rate is small but the welfare gains from its inclusion in the rule increase with the importance of portfolio shocks relative to the other shocks. In view of the uncertainty about the future path of monetary policy in the US, this advantage is considerable.
References:


Table I: Taylor-Type Rule with Response to Real Exchange Rate and Optimal Policy from a Timeless Perspective

<table>
<thead>
<tr>
<th></th>
<th>( v_t )</th>
<th>( R_t^{f} + \varepsilon_t )</th>
<th>( y_t^{f} )</th>
<th>( u_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>( \tau )</strong></td>
<td>( \frac{1 + b + \tau_q}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
<td>( \frac{-(\tau_q + b + \gamma)\alpha_1 - \alpha_2}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
<td>( \frac{a_3(1 + b + \tau_q)}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
<td>( \frac{-(1 - \gamma)\alpha_1 + \alpha_2}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
</tr>
<tr>
<td><strong>( \pi_t^{REX} )</strong></td>
<td>( \frac{\kappa(1 + b + \tau_q)}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
<td>( \frac{-\kappa(\tau_q + b + \gamma)\alpha_1 - \alpha_2}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
<td>( \frac{\kappa a_3(1 + b + \tau_q)}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
<td>( \frac{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)\tau_y}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
</tr>
<tr>
<td><strong>( q_t )</strong></td>
<td>( \frac{-(\tau_y + \kappa \tau_P)}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
<td>( \frac{1 + a_1(\tau_y + \kappa \tau_P)}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
<td>( \frac{-a_3(\tau_y + \kappa \tau_P)}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
<td>( \frac{-\tau_p}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
</tr>
<tr>
<td><strong>( R_t )</strong></td>
<td>( \frac{(1 + b)(\tau_y + \kappa \tau_P)}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
<td>( \frac{(1 - b)\tau_q - (\tau_y + \kappa \tau_P)(\beta + \gamma)\alpha_1 - \alpha_2}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
<td>( \frac{a_3(1 + b)(\tau_y + \kappa \tau_P)}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
<td>( \frac{(1 + b)\tau_P}{1 + b + \tau_q + ((1 - \gamma)\alpha_1 + \alpha_2)(\tau_y + \kappa \tau_P)} )</td>
</tr>
</tbody>
</table>

**Timeless Perspective**

<table>
<thead>
<tr>
<th></th>
<th>( v_t )</th>
<th>( R_t^{f} + \varepsilon_t )</th>
<th>( y_t^{f} )</th>
<th>( u_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>( \tau )</strong></td>
<td>0</td>
<td>( \phi_{12} )</td>
<td>( \frac{-\mu k}{1 + \beta(1 - \phi_{12}) + \mu k^2} )</td>
<td>( \frac{1}{1 + \beta(1 - \phi_{12}) + \mu k^2} )</td>
</tr>
<tr>
<td><strong>( \pi_t^{REX} )</strong></td>
<td>0</td>
<td>( \frac{-(\phi_{12} - 1)}{\mu k} )</td>
<td>( \frac{1}{1 + \beta(1 - \phi_{12}) + \mu k^2} )</td>
<td>( \frac{1}{1 + \beta(1 - \phi_{12}) + \mu k^2} )</td>
</tr>
</tbody>
</table>

Note: \( \phi_{12} = \frac{1 + \beta + \mu k^2 - \sqrt{(1 + \beta + \mu k^2)^2 - 4 \beta}}{2 \beta} \)
### Table II: Evaluating Taylor-Type Rules

<table>
<thead>
<tr>
<th></th>
<th>(1) Taylor Rule</th>
<th>(2) MCI</th>
<th>(3) Taylor Rule +0.25$q_t$</th>
<th>(4) Taylor Rule +0.5$q_t$</th>
<th>Timeless Perspective (TP)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V(y)$</td>
<td>0.9193</td>
<td>0.9150</td>
<td>0.9053</td>
<td>0.9167</td>
<td>0.0456</td>
</tr>
<tr>
<td>$V(\pi^{REX})$</td>
<td>0.9023</td>
<td>0.9047</td>
<td>0.9181</td>
<td>0.9300</td>
<td>0.8675</td>
</tr>
<tr>
<td>$V(R)$</td>
<td>1.5963</td>
<td>1.5261</td>
<td>1.2046</td>
<td>1.0161</td>
<td>5.0124</td>
</tr>
<tr>
<td>$V(q)$</td>
<td>2.9496</td>
<td>2.8169</td>
<td>2.1377</td>
<td>1.6202</td>
<td>5.8572</td>
</tr>
<tr>
<td><strong>Loss</strong></td>
<td>1.8216</td>
<td>1.8197</td>
<td>1.8233</td>
<td>1.8467</td>
<td>0.9131</td>
</tr>
<tr>
<td><strong>Welfare Loss</strong></td>
<td>99.5%</td>
<td>99.3%</td>
<td>99.7%</td>
<td>102.2%</td>
<td>-</td>
</tr>
</tbody>
</table>

**Definitions:**

Taylor Rule: $R_t = 1.5\pi_t^{REX} + 0.5y_t$

MCI: $R_t = 1.5\pi_t^{REX} + 0.5y_t + (\frac{a_2}{a_1} - (b + \gamma))q_t$

Loss = $V(y_t) + \mu V(\pi_t^{REX})$

Welfare Loss = $\frac{Loss_1^{Rule}x - Loss_1^{TP}}{Loss_1^{TP}} \times 100 \quad x = (1), (2), (3), (4).$

**Numerical Values:**

Parameters: $a_1 = 0.45, a_2 = 0.195, a_3 = 0.27, \beta = 1, b = 0.1, \kappa = 0.1, \gamma = 0.3, \mu = 1.$

$$\tau_q = \frac{a_2}{a_1} - (b + \gamma) = 0.0333$$

The variance of each shock is $1.$