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Determination of the equilibrium expansion rate of money when money supply is driven by a time-homogeneous Markov modulated jump diffusion process

Yazmín V. Soriano-Morales
Escuela Superior de Economía del Instituto Politécnico Nacional

Francisco Venegas-Martínez
Escuela Superior de Economía del Instituto Politécnico Nacional

Benjamín Vallejo-Jiménez
Escuela Superior de Economía del Instituto Politécnico Nacional

Abstract

This paper is aimed at developing a general equilibrium model useful to determine the equilibrium expansion rate of money supply in a small open stochastic economy. The marginal change of money supply incorporates stylized facts in emerging economies reported in empirical literature such as regime switches in volatility and unexpected sudden jumps (interventions). To model these essentials, money supply will be driven by a time-homogeneous Markov modulated jump diffusion process. Under this framework, it is found that the expansion rate of money supply depends on the current exchange rate depreciation, the interest rate, the average size on the jump process, and the regime switching in volatility. The proposed model allows using the Monte Carlo method to simulate the average path of the equilibrium expansion rate of money.

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Contact: Yazmín V. Soriano-Morales - yvisoriano@gmail.com, Francisco Venegas-Martínez - fvenegas1111@yahoo.com.mx, Benjamín Vallejo-Jiménez - matematicastotales@gmail.com.
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1. Introduction

Literature concerned with the issue of determining the monetary expansion rate is extensive. Some of the pioneer papers are: Tobin (1968) and (1969) laying the foundations and illustrating a general framework for monetary analysis, and Friedman (1969) and (1971) that argues that government can serve best by limiting itself to essential government functions, keeping taxes of all kinds low, refraining from intervention into the economy, and providing a stable monetary. Subsequently, Turnovsky and Brock (1980) determine the optimal monetary and fiscal policies in perfect foresight equilibrium, McTaggart (1989) examines the optimal monetary policy rules in a two country game-theoretic setting, Turnovsky (1987) develops a model for a small open economy to determine the optimal monetary growth with an accommodating fiscal policy, Smith (1998) shows that there are likely many monetary policies (and inflation rates) consistent with a zero nominal interest rate, his main implication is that one ought to be careful in selecting a monetary policy to implement Friedman's proposal, and Richardson (1975) provides a method of comparing automatic and discretionary monetary policy with a well-defined optimal monetary policy

It is also argued that domestic monetary policy and exchange rate policy are not independent instruments. For instance, Gali and Monacelli (2005) and Clarida *et al*. (2002) proposed a model of a small open economy to examine monetary policy taking into account the exchange rate volatility and discussing a special case for which domestic inflation targeting constitutes the optimal policy, and Mathieson (1976) showed that the optimal exchange rate policy can be defined in terms of either a long-run secular policy or a short-run stabilization policy.

More recently, regarding the issue of optimal monetary policy, Khan *et al*. (2003) develop a monetary model that has a range of frictions-imperfect competition, sticky prices and the costly exchange of wealth for consumption useful to explore the nature of economic activity under optimal monetary policy. They show that the optimal monetary policy depends on the nature of frictions present in the economy. Moreover, Yun (2005) analyzes optimal monetary policy in a sticky price model with a Calvo-type staggered price setting and shows that the complete stabilization of the price level is optimal in the absence of initial price dispersion, while optimal inflation targets respond to changes in the level of relative price distortion in the presence of initial price dispersion. Giannoni (2007) presents a robust optimal policy rule in a simple forward-looking model when the policymaker faces uncertainty about model parameters and shock processes, and shows how this uncertainty may amplify the degree of "super-inertia" required by optimal monetary policy. Reis (2009) presents a dynamic stochastic general equilibrium (DSGE) model with sticky information to study monetary policy. Finally, Haider and Ramzi (2010) consider a closed economy version of DSGE model with various nominal frictions.[1]

The empirical literature regarding money supply in developing and emerging economies is wide and still growing; for instance: Haghighat (2011) carries out an empirical investigation from Iran concerning exogenous money, Kumarasamy (2011) finds empirical evidence of several relationships of money supply with fundamental nominal and real variables, and Lodha (2013) provides a review of empirical studies on money supply in

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[1] It is important to mention Moore's (1972) work in choosing the money stock as the control instrument. He obtains a monetary policy that reduces the variance of income.

developing economies. Some of the stylized facts of money supply reported in empirical literature are regime switches in volatility and unexpected sudden jumps in policy variables. Thus, it seems appropriated to model money supply through a time-homogeneous Markov modulated jump diffusion process.

The main contribution of this paper is to determine the equilibrium expansion rate of money in a small open economy when money supply is driven by a time-homogeneous Markov modulated jump diffusion process and, subsequently, use Monte Carlo method to simulate the path of the equilibrium expansion rate of money in order to provide a set of recommendations on monetary policy. One outstanding characteristic of the proposed model is that it incorporates volatility components modulated by a time-homogeneous Markov chain, as well as Poisson Jumps in money supply. Following much of the current literature, the analysis is based on the assumption of perfect capital mobility, in the sense that the uncovered interest rate parity (UIP) hold, which leads to a domestic inflation rate depending only on the depreciation rate. In this case, the optimal monetary policy depends especially on the exchange rate of depreciation, the interest rate and the instantaneous volatility.

The outline of the paper is as follows: section 2 presents the structure of the economy; section 3 states the household's rational behavior; section 4 describes the firm's rational behavior and the labor market equilibrium; section 5 computes the household's economic welfare, and carry out comparative statics exercises on exogenous variables; section 6 incorporates Brownian fluctuations and Poisson jumps in the money supply with volatility modulated by a time-homogeneous Markov chain; section 7 determines the equilibrium expansion rate of money; section 8 generates Monte Carlo simulations of the equilibrium expansion rate of money; finally, section 9 provides conclusions and acknowledge limitations.

2. The Macroeconomic Framework

In this section the main characteristics of the economy are stated. The proposed framework considers a small open economy that produces and consumes a single perishable good. The economy has three sectors: consumers, firms and government. Both consumers and firms are assumed to be identical, that is, consumers have the same preferences and endowments and firms share the same technology, which leads to representative individuals for each group. Finally, the consumer is also a producer, that is, the consumer owns a firm.

Throughout this research, it will be assumed that the domestic economy is small, that is, it is a price-taking economy. Moreover, the economy produces and consumes a unique internationally traded good, which is, for the sake of simplicity, free of barriers to trade. Finally, it is also assumed that firms share the same technology and their technology have constant returns to scale.

Since the economy is small, the foreign price of the good can be taken as given. In what follows, it is assumed that the purchasing power parity (PPP) holds, that is, the domestic and foreign prices are related through the nominal exchange rate $P_t = P_t^* E_t$ where P_t is the general price level of the domestic economy, P_t^* is the general price level of the foreign economy, and E_t is the nominal exchange rate. In percentage instantaneous change terms the above three variables satisfied

$$\frac{\dot{P}_t}{P_t} = \pi , \quad \frac{\dot{P}_t^*}{P_t^*} = \pi^* \quad \text{and} \quad \frac{\dot{E}_t}{E_t} = \varepsilon$$

where π is the inflation rate in the domestic economy, π^* is the inflation rate in rest of the world, and ε is the actual rate of exchange depreciation, which under perfect foresight, it is also equals to the anticipated rate of exchange depreciation. Now then, from the PPP relationship, it follows that

$$\pi = \pi^* + \varepsilon. \tag{1}$$

Also, under perfect foresight, the uncovered interest rate parity (UIP) relationship is given by

$$r = i - \pi \tag{2}$$

where r is the domestic real interest rate and i is the domestic nominal interest rate. Since the economy is small, it can be written $r = r^*$ where r^* stands for the foreign interest rate. For the sake of simplicity, in what follows, it will be assumed that P_t^* is constant and equal to unity, that is, $P_t^* = 1$, this implies $\pi^* = 0,$ therefore, the domestic inflation rate depends only on the depreciation rate, that is, $\pi = \varepsilon.$ Hence, by substituting (1) into (2), it follows

$$r^* = i - \pi = i - \varepsilon. \tag{3}$$

3. Households rational behavior

The economy produces and consumes one generic good. The representative individual maximizes utility from consumption, c_t, and gets an income from labor given by wn_t^s where n_t^s is the number of hours that he/she allocates to work and w is real wage per unit of time. It is supposed that the consumer demands real monetary balances

$$m_t^d = \frac{M_t}{P_t} \tag{4}$$

where M_t is the nominal stock of money held by the individual. The agent may also hold bonds, in real terms, issued by the domestic government, $b_t = B_t / P_t,$ where B_t is the nominal price of the asset. The bonds pay the real interest rate r^*. The consumer is also a producer and he owns his firm obtaining a profit Π_t. Thus the consumer's budget constraint is given by

$$\dot{m}_t^d + \dot{b}_t = r^* b_t - \pi m_t^d + wn_t^s - c_t + \Pi_t \tag{5}$$

The representative rational consumer wishes to maximize his/her total discounted utility from consumption, c_t, real monetary balances m_t, and leisure $l_t^d = 1 - n_t^s$; without loss of generality, the number of total available hours of the individual has been restricted to unity. Thus, the agent wishes to maximize his/her total discounted utility

$$\int_0^\infty u(c_t, m_t, l_t^d) e^{-\rho t} \mathrm{d}t \tag{6}$$

where $u = u(c_t, m_t^d, l_t^d) = \ln(c_t) + \varphi_1 \ln(m_t^d) + \varphi_2 \ln(1 - n_t^s)$ is the utility index. Therefore, the representative individual solves the following intertemporal optimization problem

$$\text{Maximize} \int_0^\infty [\ln(c_t) + \varphi_1 \ln(m_t^d) + \varphi_2 \ln(1 - n_t^s)] e^{-\rho t} \mathrm{d}t \tag{7}$$

subject to

$$\dot{m}_t^d + \dot{b}_t = r^* b_t - \pi m_t^d + w n_t^s - c_t + \Pi_t$$

where ρ is the subjective discount rate, and m_0^d and b_0 are both given. From now on, for the sake of simplicity, it will be assumed that the subjective discount rate is equal to the real interest rate, that is, $\rho = r^*$. It is worth noticing that the assumption of this equality means that it is just attained by coincidence (just by chance) since ρ is an intertemporal preference parameter and r^* is the international price of the bond market. This will help to obtain simple paths of the optimal decision variables. For the time being, it will be assumed that $\Pi_t = 0$; firm's behavior will be introduced later. From the above assumptions, it can be easily shown that the budget constraint becomes

$$\dot{m}_t^d + \dot{b}_t = r^* b_t - \pi m_t^d + w n_t^s - c_t \tag{8}$$

and it can be rewritten in the following way

$$b_0 = \int_0^\infty (c_t + \dot{m}_t^d + \pi m_t^d - w n_t^s) e^{-r^* t} \mathrm{d}t\,. \tag{9}$$

The Lagrangian, $L(c_t, m_t, l_t^d, \lambda)$, associated with maximizing (7) subject to (9) is given by

$$L = (\ln(c_t) + \varphi_1 \ln(m_t^d) + \varphi_2 \ln(1 - n_t^s))e^{-r^* t} - \lambda e^{-r^* t}(c_t + \dot{m}_t^d + \pi m_t^d - w n_t^s)$$

where λ is the Lagrange multiplier. The first order conditions (necessary conditions) for an interior solution of the utility maximization problem are:

$$\frac{\partial L}{\partial c_t} = 0, \quad \frac{\partial L}{\partial m_t^d} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{m}_t^d}\right) = 0, \quad \text{and} \quad \frac{\partial L}{\partial l_t^d} = 0.$$

Notice that logarithm utility is concave, thus necessary conditions are also sufficient (see Venegas-Martínez, 2006). The above second equation is known as the Euler-Lagrange equation. After computing the partial derivatives, it is obtained that

$$c_t = \lambda^{-1}\,, \tag{10}$$

$$m_t^d = \varphi_1 / \lambda(r^* + \varepsilon) = \text{constant}, \tag{11}$$

$$\dot{m}_t^d = 0\,, \tag{12}$$

$$n_t^s = 1 - \frac{\varphi_2}{\lambda}\left(\frac{1}{w}\right). \tag{13}$$

In order to determine λ, equations (10)-(13) are substituted into constraint (9), then

$$b_0 = \int_0^\infty \left(\frac{1}{\lambda} + \frac{\pi\varphi_1}{\lambda(r^* + \varepsilon)} - w_t + \frac{\varphi_2}{\lambda}\right) e^{-r^* t}\mathrm{d}t = \frac{1}{r^*}\left(\frac{1}{\lambda} + \frac{\pi\varphi_1}{\lambda(r^* + \varepsilon)} - w + \frac{\varphi_2}{\lambda}\right)$$

which implies

$$\lambda = \left(1 + \frac{\pi\varphi_1}{(r^* + \varepsilon)} + \varphi_2\right) / \left(r^* b_0 + w\right). \tag{14}$$

4. Firms behavior and labor market equilibrium

For the sake of simplicity, it is supposed that the firm's production function has constant marginal returns, that is, $f(n_t^d) = A n_t^d$, where A is a constant standing for the marginal product of labor. The firm's profit is given by

$$\Pi_t = f(n_t^d) - w n_t^d . \tag{15}$$

Profit maximization requires $f'(n_t^d) - w$, or $w = A$.[2] Therefore, equilibrium in the market labor leads to

$$n_t = 1 - \frac{\varphi_2\left(r^* b_0 + A\right)}{A\left(1 + \frac{\pi\varphi_1}{(r^* + \varepsilon)} + \varphi_2\right)} . \tag{16}$$

5. Household's economic welfare

The household's economic welfare (indirect utility), *W*, is obtained by substituting the optimal decisions, given in (10), (11) and (13) in the direct utility stated in (6), thus

$$W = \int_0^\infty \left(\ln\left(\frac{\left(r^* b_0 + w\right)}{\left(1 + \frac{\pi\varphi_1}{(r^* + \varepsilon)} + \varphi_2\right)} \right) + \varphi_1 \ln\left(\frac{\left(r^* b_0 + w\right)\varphi_1}{\left(1 + \frac{\pi\varphi_1}{(r^* + \varepsilon)} + \varphi_2\right)\left(r^* + \varepsilon\right)} \right) + \varphi_2 \ln\left(\frac{\varphi_2\left(r^* b_0 + A\right)}{A\left(1 + \frac{\pi\varphi_1}{r^* + \varepsilon} + \varphi_2\right)} \right) \right) e^{-r^* t} \mathrm{d}t$$

By using the result $\int_0^\infty r^* e^{-r^* t} \mathrm{d}t = 1$ in the above integral, it follows that

$$W = \frac{1}{r^*} \left(\ln\left(\frac{\left(r^* b_0 + w\right)}{\left(1 + \frac{\pi\varphi_1}{(r^* + \varepsilon)} + \varphi_2\right)} \right) + \varphi_1 \ln\left(\frac{\varphi_1\left(r^* b_0 + w\right)}{\left(1 + \frac{\pi\varphi_1}{(r^* + \varepsilon)} + \varphi_2\right)\left(r^* + \varepsilon\right)} \right) + \varphi_2 \ln\left(\frac{\varphi_2\left(r^* b_0 + A\right)}{A\left(1 + \frac{\pi\varphi_1}{r^* + \varepsilon} + \varphi_2\right)} \right) \right)$$

As it can be seen, economic welfare depends on the exchange rate of depreciation, the interest rate, preference parameters, and initial endowments. We now assess the impact of once-for-all changes in the independent variables on the welfare will be assessed. Notice first that

$$W = \frac{1}{r^*} \Bigg\{ \ln\left(r^* b_0 + w\right) - \ln\left(1 + \frac{\pi\varphi_1}{(r^* + \varepsilon)} + \varphi_2\right) + \varphi_1\left(\ln\left(r^* b_0 + w\right) - \ln\left(1 + \frac{\pi\varphi_1}{(r^* + \varepsilon)} + \varphi_2\right)\left(r^* + \varepsilon\right) \right)$$
$$+ \varphi_2\left(\ln\left(\varphi_2\left(r^* b_0 + A\right)\right) - \ln\left(A\left(1 + \frac{\pi\varphi_1}{r^* + \varepsilon} + \varphi_2\right)\right)\right) \Bigg\} \tag{17}$$

Notice that

$$\frac{\partial W}{\partial r^*} > 0 ,$$

that is, if the foreign interest rate increase the household's economic welfare increase. Also

$$\frac{\partial W}{\partial \varepsilon} < 0 .$$

In other words, if the actual rate of exchange depreciation increases, the welfare will be reduced. Finally observe that

[2] See also Palafox-Roca and Venegas-Martínez (2014), and Venegas-Martínez (1999).

$$\frac{\partial W}{\partial b_0} > 0$$

that is, if nominal price of the asset increase welfare, the increase the bonds.

6. Money supply

Money supply will be driven by a geometric Brownian motion (or geometric Wiener process) combined with a Poisson process. The volatility of the geometric Brownian motion is modulated by a time-homogeneous Markov chain. That is, money supply evolves according to the following stochastic differential equation.

$$\mathrm{d}m_t^s = (\theta_t - \varepsilon)m_t^s \mathrm{d}t + \sigma_i m_t^s \mathrm{d}U_t + v m_t^s \mathrm{d}N_t \tag{18}$$

where U_t is a standard Brownian motion defined on a fixed probability space $(\Omega_U, F_U, \mathbb{P}_U)$ equipped with augmented filtration $F_t \subset F_s$ with $t < s$. The Poisson process is defined in the second space probability, associated with jumps $(\Omega_N, F_N, \mathbb{P}_N)$, where is defined a Poisson process $\mathrm{d}N_t$ with intensity parameter ϕ, so that

$$\mathbb{P}_N\{\text{one unit jump during } \mathrm{d}t\} = \mathbb{P}_N\{\mathrm{d}N_t = 1\} = \phi\, \mathrm{d}t$$

and

$$\mathbb{P}_N\{\text{more than one jump during } \mathrm{d}t\} = \mathbb{P}_N\{\mathrm{d}N_t > 1\} = o(\mathrm{d}t).$$

Thus, $\mathbb{P}_N\{\text{no jump during dt}\} = 1 - \phi \mathrm{d}t + o(\mathrm{d}t)$, where $o(\mathrm{d}t)$ is such that $o(\mathrm{d}t)/\mathrm{d}t \to 0$ when $\mathrm{d}t \to 0$. Note that the expected mean time between two jumps is given by $1/\phi$. It can also easily show that $\mathrm{E}_N[\mathrm{d}N_t] = \mathrm{Var}_N[\mathrm{d}N_t] = \phi\, \mathrm{d}t$. Finally, σ_i is a continuous-time Markov chain with finite state space E, and $Q = (q_{ij})_{i,j \in E}$ is a matrix providing transition probability. In what follows, we suppose that $\sigma_i : E \to \mathbb{R}^+$ for all $i \in E$, allowing a regime switching in the volatility σ_i. All stochastic processes entering the subsequent analysis is adapted to the product involve filtration, $F_t \equiv F_{tW} \times F_{tN}$, in the probability space generated $\mathbb{P} \equiv \mathbb{P}_W \otimes \mathbb{P}_N$. Also, it is assumed that all stochastic processes involving equalities are fulfilled $\mathbb{P}$-almost surely (that is, with probability one). In what follows, it is assumed that all the processes are well defined, without explicitly stating regularity conditions to ensure this. The solution to equation (18) is given by

$$m_t^s = m_0^s \exp\left\{ \{\theta_t - \varepsilon - \tfrac{1}{2} \sum_{j \in E} q_{ij} [g(\sigma_j^2, t) - g(\sigma_i^2, t)]\} t + \sigma_i U_t + \ln(1+v) N_t \right\} \tag{19}$$

where function $g(\sigma_i^2, t)$ satisfies

$$\frac{\partial g(\sigma_i^2, t)}{\partial t} - F[g(\sigma_i^2, t)] + \sum_{j \in E} q_{ij} \left[g\left(\sigma_j^2, t\right) - g\left(\sigma_i^2, t\right) \right] = 0, \tag{20}$$

with

$$F(g(\sigma_i, t)) = (1/2)\{\sigma_i^2 - \sum_{j \in E} q_{ij} [g(\sigma_j^2, t) - g(\sigma_i^2, t)]\},$$

that is, F is linear in σ_i^2. This follows from Ito's lemma applied to $J(m_t^s, \sigma_i, t) = \ln(m_t^s) + g(\sigma_i, t)$ with (18) as the underlying process (see Appendix I). Notice also that (19) extends the classical deterministic framework, in which $m_t^s = m_0^s \exp\{(\theta_t - \varepsilon)t\}$. Finally, It is noteworthy that (20) was defined with the purpose to get an adequate format of equation useful for finding approximate numerical solutions.

7. Equilibrium Expansion Rate of Money

In this section, the trend parameter $\theta_t - \varepsilon$ in (19) will be determined in the equilibrium. Notice first that in equilibrium $m_t^d = m_t^s$, which leads to

$$\frac{\varphi_1\left(r^* b_0 + A\right)}{\left(1 + \dfrac{\pi\varphi_1}{(r^* + \varepsilon)} + \varphi_2\right)} = m_0^s \exp\left\{ \{\theta_t - \varepsilon - \tfrac{1}{2}\sum_{j\in E} q_{ij}[g(\sigma_j^2, t) - g(\sigma_i^2, t)]\}t + \sigma_i\sqrt{t}Z + \ln(1+\nu)N_t \right\}$$

The optimal monetary policy is obtained by solving the above equation for θ_t, that is,

$$\hat{\theta}_t = \frac{1}{t}\ln\left(\frac{1}{m_0^s} \frac{\varphi_1\left(r^* b_0 + A\right)}{\left(1 + \dfrac{\pi\varphi_1}{(r^* + \varepsilon)} + \varphi_2\right)} \right) + \tfrac{1}{2}\sum_{j\in E} q_{ij}[g(\sigma_j^2, t) - g(\sigma_i^2, t)] - \frac{1}{t}\left(\sigma_i\sqrt{t}Z - \varepsilon + \ln\left(1+\nu\right)N_t\right) \tag{21}$$

8. Monte Carlo simulation of the Equilibrium Expansion Rate of Money

In order to simulate the equilibrium expansion rate of money, we start from generating a random number to estimate a possible value θ_1 at time $t = 1/360$, then another random number Z is generated to obtain a second possible value θ_2 at time $t = 2/360$, and so on until T = 10 years. In this case, it will be assumed, for illustrative purposes, that $j = 1, 2$, $q_{ij} = 1/2$, and $g(\sigma_i^2, t) = \ln(\sigma_i^2)e^{-r^* t}$. We state the functional form of g following Venegas-Martínez (2006). The dynamics of the money supply is simulated on the basis of the parameter values in Table 1.

Table 1. Parameter values

σ_1^2	σ_2^2	ε	Δt	m_0	$\varphi_1 = \varphi_2$	b_0	A	π	r^*
0.01	0.04	0.0255	1/360	1	1	1	1	0.04	0.0025

Source: Author's own elaboration

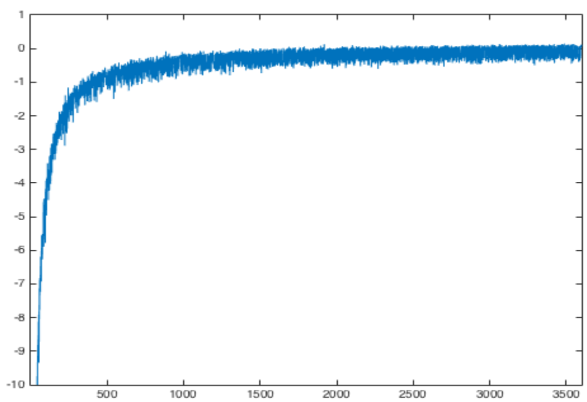
Notice that according to equation (18), the instantaneous expansion rate of money satisfies

$$\frac{\mathrm{d}m_t^s}{m_t^s} = (\theta_t - \varepsilon)\mathrm{d}t + \sigma_i \mathrm{d}U_t + \nu \mathrm{d}N_t.$$

Here, rates θ_t and ε are annualized, and quantities $\theta_t \mathrm{d}t$ and $\varepsilon \mathrm{d}t$ provide instantaneous rates; if instead of $\mathrm{d}t$, we use $\Delta t = 1/360$, we obtain daily rates, $\theta_t \Delta t$ and $\varepsilon \Delta t$. Regarding the values of parameters ε and r^*, it is important to indicate that under the current currency global crisis in 2015, the rate of depreciation of the annualized exchange rate in Mexico in early 2015 was approximately 0.025, in June it showed an increase to 0.041, and in August reached 0.051 (Source: Banxico).

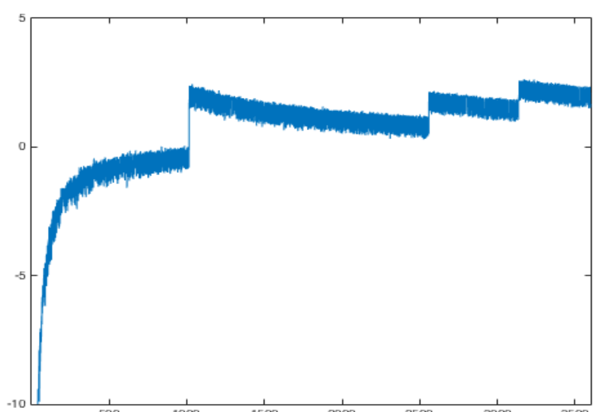
The accuracy of the results depends on the quality of random numbers. Hence, Kolmogorov-Smirnov test for randomness is used. Usually, uniform random variables are used to generate a standard normal random variable Z through the Box-Muller method: $Z=\sqrt{-2\ln U_1}\cos\left(2\pi U_2\right)$ or $Z=\sqrt{-2\ln U_1}\sin\left(2\pi U_2\right)$. Under this scheme, 10,000 possible paths are generated, and the average path is calculated. Figures 1 and 2 show typical paths of the equilibrium expansion rate of money without and with Poisson jumps and regime switching in volatility, respectively. Figures 3 and 4 show the average path of the equilibrium expansion rate of money without and with Poisson jumps and regime switching in volatility.[3] Finally, notice that the average path with jumps looks smooth and is higher than that without jumps.

Figure 1. Typical path without Poisson jumps and regime switching



Source: authors' own calculations

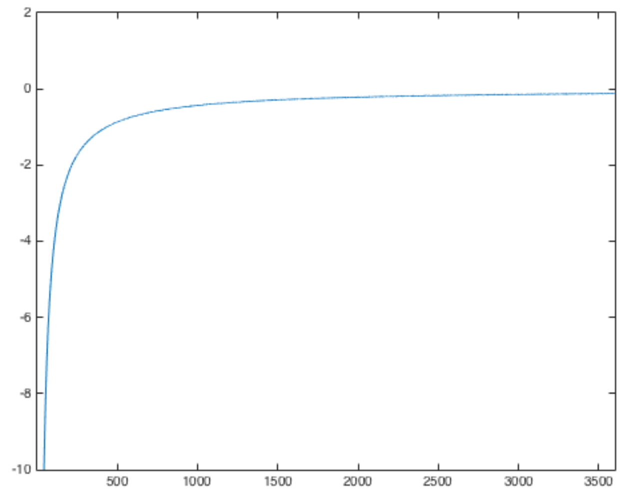
Figure 2. Typical path with Poisson jumps and regime switching



Source: authors' own calculations

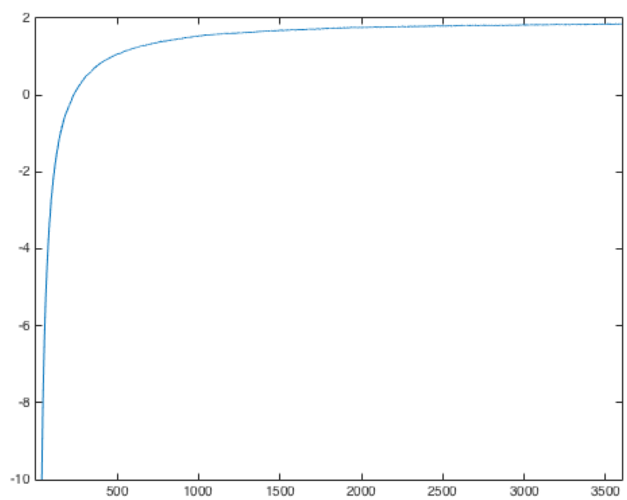
[3] The code in MATLAB is available under request.

Figure 3. Monte Carlo simulation without Poisson jumps and regime switching



Source: authors' own calculations

Figure 4. Monte Carlo simulation with Poisson jumps and regime switching



Source: authors' own calculations

8. Conclusions

Departing from previous works on determining the optimal monetary policy in small open stochastic economies, this research contributes to the literature on monetary policy by assuming, a more realistic approach incorporating stylized facts in emerging economies as regime switches in volatility with sudden jumps or interventions. Thus money supply was driven by a time-homogeneous Markov modulated jump diffusion process. Under this framework, the equilibrium expansion rate of money depends, particularly, on the current rate of exchange depreciation, the foreign interest rate, the expected size of the (Poisson) jump and the regime switching in volatility. Monte Carlo method was applied to simulate the equilibrium expansion rate of money. A recommendation for monetary policy is that the identification and quantification of the risk factors affecting the supply of money is indispensable for a consistent risk management to reach monetary targets.

Appendix I

If $\mathrm{d}m_t^s = \mu_m m_t^s \mathrm{d}t + \sigma_i m_t^s \mathrm{d}U_t + v m_t^s \mathrm{d}N_t$ and $J(m_t^s, \sigma_i, t)$ is the value function, then

$$\begin{aligned}\mathrm{d}(J(m_t^s,t,\sigma_i)) = {} & \left(\frac{\partial J(m_t^s,t,\sigma_i)}{\partial t} + \frac{\partial J(m_t^s,t,\sigma_i)}{\partial m_t^s} m_t^s \mu_m + \frac{1}{2}\frac{\partial^2 J(m_t^s,t,\sigma_i)}{\partial m_t^{s2}} m_t^{s2}\sigma_i^2\right)\mathrm{d}t \\ & + \frac{\partial J(m_t^s,t,\sigma_i)}{\partial m_t^s} m_t^s \sigma_i \mathrm{d}W_t + \left(\sum_{j\in E} q_{ij}\left[J\left(m_t^s,t,\sigma_j\right) - J\left(m_t^s,t,\sigma_i\right)\right]\right)\mathrm{d}t \\ & + \left[J\left(m_t^s(1+\nu),t,\sigma_i\right) - J\left(m_t^s,t,\sigma_i\right)\right]\phi\,\mathrm{d}t\end{aligned}$$

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