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Implications of nonlinearity in environmental instrument choice

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Abstract

The classic paper by Weitzman (1974) on environmental instrument choice showed that relative efficiency of a price or quantity instrument depended on the elasticities of the marginal cost and benefit functions. Linear models with additive or multiplicative errors lent support for many of Weitzman's results. This paper investigates nonlinear functions and again shows Weitzman largely correct. Moreover it supports a conjecture he made about the relative efficacy of price versus quantity instruments.

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1. Introduction

The classic paper by Weitzman (1974) on environmental instrument choice has engendered a large literature on the factors that, in the face of ex post uncertainty about costs and/or benefits, would argue for either a quantity-based instrument or one that is price based. Weitzman concluded that the determining factor would be the relative elasticities of the marginal cost and benefit curves, a conclusion that has been supported by many subsequent analyses. Weitzman, and most who have followed him, have used linear models with additive errors terms, which do not affect the overall result. For example, although Laffont (1977) embeds Weitzman's model in a more general structure that clearly distinguishes between two types of uncertainty (information gaps versus genuine uncertainty), the functional form of the cost and benefit functions remain largely unchanged (i.e. linear model with additive error term).

Although these assumptions simplify the analysis, they may not accurately represent the true nature of the environmental choice problem A more realistic view is that as the level of pollution control increases, it becomes increasingly difficult to achieve greater levels of abatement, and, at the same time, the social gain associated with this additional abatement becomes minimal. We follow Watson and Ridker (1984, 310, emphasis added) who "*empirically derived* nonlinear functions and more realistic multiplicative error terms" in analyzing U.S. environmental policies. Weitzman speculated on the effect of nonlinearity—most especially how it might change the point elasticities depending on where the welfare maximizing quantity or price is set relative to the curvature of either the marginal benefit or marginal cost functions (or both). But he did not attempt to model formally the implications of nonlinearity, nor did he look at the impact of a multiplicative error term.

This gap has left questions of how nonlinearity might impact such models. Laffont (1977) considers how uncertainty in the curvature of the cost and benefit functions affects the choice of best instrument, but in the context of a special linear case in which $E((a(\theta_2)) = E(1/a(\theta_2))$. Watson and Ridker (1984) and Cole and Grossman (2002) addressed nonlinearity graphically but did not examine its formal properties. It is the intention of this paper to explore the formal implications of nonlinearity (along with multiplicative errors) with respect to environmental instrument choice when an informational gap exists at the agency level. It might seem that such clearly different properties of the functions would lead to radically different results from those Weitzman derived. But actually, our results tend to strengthen most of Weitzman's earlier insights.

2. The Model

2.1 Assumptions

We assume that the policymakers need to find the instrument that will provide the degree of pollution control (q') that will maximize expected social welfare, EW_t. In general we follow the approach found in Adar and Griffin (1976), (as well as Fishelson (1976), Baumol and Oates (1992), and Shrestha (2001)), explicitly modeling social marginal benefit and marginal cost functions. In contrast, however, we assume both functions are non-linear. For simplicity, we assume the benefit function $B(q)$ is known with certainty. The cost function, in contrast, is uncertain. That is, the cost function is given as $C(q, \mathrm{u})$, where the cost of producing q is determined both by the degree of control and by a random variable (or disturbance or error term) with expected value μ and known density $dF(\mathrm{u})$. Like the Weitzman case described by Laffont (1977), we assume u represents an information gap, in that u is known by the firm but random for the policymaker. Note that in contrast to Laffont (1977), uncertainty enters via a multiplicative error term, not the more traditional additive error term, in what is now a non-linear model. In particular, the marginal benefit associated with pollution control can be written as:

$$\mathrm{MB}(q) = a - bq^2 \tag{1}$$

and the marginal cost of pollution control can be written as:

$$\mathrm{MC}(q, u) = \alpha + \beta q^2 u \tag{2}$$

where a, b, α, and β are constants and u is a random variable with expected value of 1 and known density $dF(\mathrm{u})$.

2.2 A Quantity Instrument

The policymaker's goal is to set q in order to maximize expected welfare, which can be expressed as:

$$EW_q = \underset{u}{E} \int_0^q \left[\left(a - bq^2\right) - \left(\alpha + \beta q^2 u\right) \right] dq \tag{3}$$

Let q* be the solution to this problem, which can be expressed as:

$$q^* = \left(\frac{a - \alpha}{b + \beta} \right)^{\frac{1}{2}} \tag{4}$$

Thus, expected welfare under a quantity instrument is:

$$EW_q = \underset{u}{E} \int_0^{q^*} \left[\left(a - bq^2\right) - \left(\alpha + \beta q^2 u\right) \right] dq \tag{5}$$

Integrating, we obtain:

$$EW_q = \underset{u}{E} \left[(a - \alpha) q^* - \frac{(b + \beta u)}{3} {q^*}^3 \right] \tag{6}$$

Substituting (4) into (6), it is straightforward to show:

$$EW_q = \frac{2(a-\alpha)^{\frac{3}{2}}}{3(b+\beta)^{\frac{1}{2}}} = \frac{2(a-\alpha)}{3} q^* \tag{7}$$

Note that in order to highlight the similarity between (7) and previous results in the literature, we assume, without loss of generality, μ is standardized to 1. Although the specific functional form differs, this result is similar to that obtained by Weitzman and others. In particular, under a quantity instrument expected welfare depends on parameters a, α, b, β, and μ, but is independent of the variance of u. Thus, Weitzman's characterization of expected welfare under a quantity instrument can be extended to the non-linear case with multiplicative error.

2.3 A Price Instrument

Under a price instrument, policymakers set a price, P. For a given P, the firm chooses the level of pollution control q' such that P is equal to marginal cost, or:

$$P = \alpha + \beta q'^2 u \tag{8}$$

Solving (8) for q′ yields:

$$q' = \left(\frac{P-\alpha}{\beta u}\right)^{\frac{1}{2}} \tag{9}$$

Thus, we note that under a price instrument, the level of pollution control is a random variable.

The policymaker sets P in order to maximize expected welfare, W_t , where:

$$EW_t = \underset{u}{E} \int_0^{q(P)} \left[\left(a - bq^2\right) - \left(\alpha + \beta q^2 u\right)\right] dq \tag{10}$$

Thus, the optimal price, P^*, must satisfy the first order condition:

$$\frac{d}{dP} EW_t = 0, \text{ that is:} \tag{11}$$

$$\frac{d}{dP} \underset{u}{E}\left[(a-\alpha)q - \frac{(b+\beta u)}{3} q^3 \Big|_0^{q'(P^*,u)} \right] = 0 \tag{12}$$

Substituting (9) into (12), differentiating, then rearranging terms, it is straightforward to show that the condition for the optimal price, P^*, is:

$$\underset{u}{E}\left[\frac{(a-\alpha)}{u}\right]=\underset{u}{E}\left[q'(P^*,u)^2\cdot\frac{(b+\beta u)}{u^{\frac{1}{2}}}\right] \tag{13}$$

Substituting (9) and (13) into (10) and evaluating, it is possible to derive the expected net social benefit under the price instrument, which can be expressed as:

$$EW_t=\frac{\alpha b}{3\beta}\underset{u}{E}\left[\frac{q'(P^*,u)}{u}\right]+\frac{2a-\alpha}{3}\underset{u}{E}[q'(P^*,u)]-\frac{\alpha(a-\alpha)}{3\beta}\underset{u}{E}\left[\frac{1}{uq'(P^*,u)}\right] \tag{14}$$

The key point of this equation is that the expected net social welfare gain from a price instrument will depend on the parameters a, α, b, β—as one would expect. But now what one also finds is that the frequency distribution of the error term as well as the manner in which it enters the marginal cost function also will explicitly determine the outcome.

2.4 Quantity Instrument vs Price Instrument

Now comparisons may be made with alternatives. In this case we posit that policy will choose between a quantity instrument and a price instrument. From (7) and (14), a price instrument is preferred to a quantity instrument if:

$$\frac{\alpha b}{3\beta}\underset{u}{E}\left[\frac{q'(P^*,u)}{u}\right]+\frac{2a-\alpha}{3}\underset{u}{E}[q'(P^*,u)]-\frac{\alpha(a-\alpha)}{3\beta}\underset{u}{E}\left[\frac{1}{uq'(P^*,u)}\right]>\frac{2(a-\alpha)^{\frac{3}{2}}}{3(b+\beta)^{\frac{1}{2}}} \tag{15}$$

Substituting (4) and (9) into (15) and rearranging terms, we obtain:

$$\frac{\alpha b}{3\beta}\underset{u}{E}\left[\frac{q'(P^*,u)}{u}\right]+\frac{2a-\alpha}{3}\underset{u}{E}[q'(P^*,u)]-\frac{a(a-\alpha)}{3\beta}\underset{u}{E}\left[\frac{1}{uq'(P^*,u)}\right]-\frac{2(a-\alpha)}{3}\cdot q^*>0 \tag{16}$$

Note that the preferred instrument depends on parameters $a, \alpha, b, and\ \beta$, as well as the expected value of the level of pollution control under each instrument, $q'(P^*,u)$ and q^*. Once again the distribution of u comes into play.

3. Discussion

We demonstrate in Section 2 that in a nonlinear quantity model, even with multiplicative errors, the variance of the error term does not appear in the maximization result, and only the slope and intercept of the marginal cost and marginal benefit functions, as well as the expected value of the error, determine the q^* that maximizes expected social welfare. At the same time, as others (Adar and Griffin 1976, Fishelson 1976) who used multiplicative errors found, the variance of the error term is a factor when a price instrument is used. It may be that where the variance of the uncertainty term is large, a price instrument might produce not merely lower net social gains, but in fact, net losses. Thus, relaxing the restrictive, and perhaps unrealistic, assumption of a linear model with an additive error term does, under certain conditions, change the results.

Weitzman argued that a quantity regime was "more conservative" (486) than a price regime, and given circumstances where the magnitude of u is highly uncertain, we find the expected value of net social welfare EW_t can be more reliably ascertained (even in the presence of multiplicative errors) with a quantity regime. Our result supports Weitzman's observation that a quantity instrument might often have an advantage over a price instrument in that the latter is more likely to be a "*disastrous* choice of instrument far more often than quantities " (Weitzman 1974, 486, emphasis in the original). At the same time, nothing in our results (or Weitzman's) would argue that quantity instruments are always to be preferred over alternatives. The point elasticities are still likely to provide the most reliable measure of which type of instrument will provide the most socially beneficial result.

4. Conclusion

Weitzman's 1974 paper on the choice of environmental instruments is justly considered a classic. In this paper we take his basic question of environmental instrument choice, price or quantity based instruments, and extend it to circumstances where costs and benefits are nonlinear and subject to multiplicative errors. Our results confirm some of the basic insights of the original paper and, thus, demonstrate their robustness.

But Weitzman himself observed that there are unlikely to be any first-best instruments, that the best such models can do is provide direction for finding a second best. Indeed, there remain many factors—for example, market structures and institutions—that are not represented in models such as the one above—that might impact the relative effectiveness of any instrument choice. As Goulder and Parry (2008, 171) remind us, "significant challenges" as to economic analyses of environmental instrument choice remain.

In addition, we note that in many instances, the relevant comparison is between two distinct quantity instruments or two distinct price instruments, not between a quantity and a price instrument. For example, for carbon dioxide emissions one can choose a tax on the output of production or on the emissions themselves and it is not immediately clear which price instrument will be more effective given differing monitoring and enforcement costs (Gjerde et. al 2014). As Cole and Grossman (1999) point out the choice of a cap-and-trade sulfur dioxide program was only feasible when real time monitoring became available; when the U.S. Clean Air Act was passed in 1970, they argue that the command-and-control quantity targets were a more sensible choice. Clearly, the range of instruments and the factors determining choice are greater than a simple comparison of a price and quantity instrument might suggest. These broader issues seem a promising area of research interest going forward.

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