The Impact of the Population Age Structure on the Response to Negative Asset Shocks

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Abstract

The objective of this study is to examine how the population age structure and demographics might arbitrate the effects of an unexpected negative shock on household savings. The focus is on transitional dynamics that result due to changes in the economy's aggregate demand and the role played by the age distribution of households at the time of the shock and over the transitional period. Simulation experiments are performed using population series of different demographic scenarios.

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1. Introduction

Between 2007 and 2010, according to data from the U.S. Survey of Consumer Finances (SCF), mean American family net worth fell by 14.7% while median family net worth fell 38.8%. It is hardly surprising that a drop of this magnitude would have a significant impact on American consumer behaviour, and it is also reasonable to hypothesize that the impact of such a drop in wealth on aggregate economic behaviour would depend in part on the age distribution of the population. There are a number of reasons to expect the age structure of the population to matter: savings behaviour differs across age groups, and the impact of the asset shock varied across age groups. The SCF indicates that the 35-44 age group experienced a 54.4% drop in median net worth while the less than 35 age group saw a 25% drop in median net worth. Reported percentage changes can mask absolute changes in asset holdings: median family net worth for families with a head of household under 35 dropped from 12,400 to 9,300 between 2007 and 2010 while for families with a head of household aged 35-44 the drop in median net worth was from 92,400 to 42,100. For households with a head aged 55-64 years, median net worth dropped 33%, from 266,200 to 179,400. We would expect the impact of these drops on household behaviour to vary with the age of the household: a younger household has a much longer working life ahead of it in which to make up for the loss than does an older household.

At the individual level, asset accumulation is an inter-temporal optimization problem with current savings (or consumption) as the control variable and accumulated assets as the state variable. An asset shock can then be modelled as a negative shock to the state variable. In a stochastic optimal control framework, we could model the individual’s problem using the Poisson version of Ito’s Lemma, incorporating the probability of an asset shock from the beginning of the planning horizon and treating the occurrence of the shock as the beginning of a new problem. In this case, the initial value of the state variable is what is left of the individual’s stock of assets after the shock and the new planning horizon is the time left until the end of the planning horizon of the original optimal control problem. Looked at this way, it is clear that an individual’s response to an asset shock will depend both on the magnitude and the timing of the shock. Some young individuals, who still have a long planning horizon ahead of them will find it optimal to return virtually to the original intertemporal trajectory. Older individuals will find the fact that their remaining horizon is relatively short means that the optimal path involves at most a partial convergence on the original trajectory, settling on a new trajectory for the remainder of their planning horizon.

The objective of this paper is to examine how the age structure of the population can arbitrate the macroeconomic effects of a major asset shock, through its impact on savings and consumption behaviour and the composition of the aggregate demand. We model a small and open economy, populated by heterogeneous households with respect to their age, productivity and asset holdings where the dynamics of employment and production are set to depend strictly on the demand for goods and services. We impose an exogenous shock on the assets which all age groups have accumulated to date, and allow the consequences of that shock to work themselves out through the responses of inter-temporally optimizing consumers, where the response of any age group of consumers will depend in part on their age and in part on their remaining expected lifetime. In this note we present impulse responses for the dynamics of output in transition, following the occurrence of the shock, from a series
of preliminary simulations with different demographic scenarios.

Asset shocks and economic crises have, unsurprisingly, become the subject of considerable research interest among macroeconomists in recent years (Barro and Ursúa (2008a), (Barro and Ursúa, 2008b), (Barro and Jin, 2011) and Nakamura et al. (2013)). There has also been considerable empirical and theoretical research showing that demographic dynamics can have an impact on economic growth (Lindh and Malmberg (1999), Fougere and Merette (1999), Van Groezen et al. (2005), Madsen (2010), Marattin and Salotti (2011), Elgin and Tumen (2012), Gonzalez-Eiras and Niepelt (2012)), aggregate productivity (Beaudry et al. (2005), Feyrer (2007)) and business cycle fluctuations (DellaVigna and Pollet (2007), Lugauer (2012), Macunovich (2012)). In addition, Backus et al. (2014) investigate the role played by current demographic trends to determine the level of international capital flows. Our contribution to this literature is to investigate how the age distribution of a country’s population relates to its response to an asset shock, along the lines of recent experience, with a particular focus on age dependent asset accumulation behaviour and demand driven fluctuations. To preview our results, we find that economies with an aging population experience a greater negative effect from a shock to asset values. A sharp reduction in consumption among middle and older cohorts translates into a sharper reduction in labour demand. This in turn transmits the shock to younger, working cohorts who have no assets and reinforces the shock for middle cohorts who both work and have assets. However, since older economies have a higher number of retired workers, whose income does not depend on employment earnings, they converge at a faster rate to a new equilibrium.

The study is organized as follows: Section 2 describes the model economy while model is calibrated in section 3. Our computational results are presented in section 4. Section 5 presents some concluding remarks.

2. The Model

To capture the role of demographics on the impact of the shock via changes in the economy’s aggregate demand, we build a model of a small open economy populated by overlapping generations of cohorts. Agents work a fraction of their life and make life-cycle consumption and savings decisions. Firms hire labour and capital to produce output and make all production decisions. In this economy, household savings are invested abroad and return the word interest rate while productive capital is hired from abroad and is paid at the same rate. This assumption assures that the asset shock affect only household assets and not the value of productive capital, allowing us to focus on the effects of the age structure on the demand side of the economy.

1In addition, recent literature that estimates the impact of changes in the demographic structure also includes an examination of the effects on stock returns (Krueger and Ludwig (2007), Poterba (2001) and Cheolbeom (2010), trade deficits (Ferrero (2010)), saving rates (Chen et al. (2007)), capital and labour taxation (Mateos-Planas (2010)), to only mention a few. Related to our work is also the study of Aizenman and Noy (2013) who incorporate population age distributions into an empirical investigation of the long term effects of economic shocks on public and private savings. Given the space restrictions of this journal, we omit a more detailed description of the literature.
2.1. Demographics

We assume that in each period $t$ a new cohort is born and a typical individual may live for a maximum of $J$ model periods. The conditional probability of surviving from age $j$ to age $j+1$ is $\psi_j \in (0,1)$, for $j = 1, \ldots, J-1$. Therefore, the unconditional probability of becoming $s$-years of age is $\prod_{j=1}^{s} \{\psi_j\}$. People are born and die with certainty should they live the entire lifespan, so $\psi_1 = 1$ and $\psi_J = 0$. Individual life is divided into a childhood period, which lasts for $J_0$ periods, and an adult period, which spans the remaining life-cycle. As children, individuals are taken care of by their parents and do not make any economic decisions or work in labour markets. Adult agents make all decisions and work until retirement which is mandatory at age $j_r$. Cohort shares in each time period, denoted by $\mu_{j,t}$, are then used as weights in the aggregation of households decisions.

2.2. Preferences, Income and Technology

In each time period, individuals derive utility from a composite consumption good, $c_{j,t}$. Since children cannot make any economic decisions, optimization choices are made when agents enter adulthood. A young adult born at time $b_0$ maximizes his/her expected discounted lifetime utility:

$$E_{b_0+1} \left\{ \sum_{j=J_0+1}^{J} \beta^{j-1} \prod_{i=1}^{j} \{\psi_i\} U \left( \frac{c_{j,t}}{\eta_j} \right) \right\}$$

where $\beta$ denotes the subjective discount factor. Given that adults are responsible for taking care and providing for children, we assume that there are age specific differences between the level of consumption spent and consumption enjoyed. For example, middle aged individuals may have a larger household compared to older individuals or younger adults. Vector $\eta_j$ indicates a weight that accounts for the family size of the individual household. We assume that any bequests left due to accidental death are taxed away by the government and redistributed evenly via a lump-sum transfer to all living agents.

Prior to the age of mandatory retirement, individuals are endowed in each period with one unit of time which they are willing to supply inelastically to work. Labour, however, may not be fully employed, depending on labour market conditions. Let $x \in (0,1)$ denote the fraction of time an agent spends at work. We assume that if people are less than fully employed, they derive no utility from leisure. One unit of time, if devoted to work, can be transformed to $\epsilon_j$ efficiency units of labour. Since retirement is mandatory, for $j_r < j < J$, $\epsilon_j = 0$. A unit of efficient labour receives $w$, the average wage rate in the economy.

Agents enter the market with zero asset holdings, $a_{J_0+1} = 0$. At early stages of life, earnings come mainly from work, although individuals may also borrow to finance consumption. Over time, agents accumulate assets to provide for old age and retirement and to smooth their life-cycle consumption profile. Investments consists of foreign assets which return the world interest $\bar{r}$. In addition, earnings are also augmented by interest payments on accumulated assets. No bequest motives link older with younger generations and all assets are consumed before people die, should they live the entire possible lifespan ($a_{J+1} = 0$).
In each period, the budget constraint of an agent of age \( j \) at time \( t \) is:

\[
c_{j,t} + a_{j+1,t+1} = a_{j,t}(1 + \tau) + w_t x_t \epsilon_j + beq_t
\]

(2)

The level of bequests is given by: \( beq_t = \sum_{i=J_0}^J (1 - \psi_i) \mu_{i,t} a_{i,t} \), with \( i \) denoting the age of non-surviving individuals.\(^2\)

When the economy is in its long run equilibrium, a typical individual maximizes his\(\) her lifetime utility at the beginning of economic life. In transition paths, agents re-optimize in each time interval until the economy settles to its new steady state. We assume that over the transitional period, agents form myopic expectations and consider current levels of employment, income and prices, a good approximation of future outcomes. An alternative assumption would be perfect foresight for agents planning life-cycle consumption and asset profiles. We opt for the former assumption since perfect foresight would eliminate any short run dynamics and the occurrence of the shock would simply bring the economy from one steady state to another.\(^3\)

A firm produces a tradable commodity \( Y \) via a constant returns to scale production function, \( Y_t = A_t f(K_t, N_t) \), by employing capital services \( K \) and efficiency units of labour \( N \), where \( A \) denotes the current level of technology. This small open economy imports the required level of capital services from abroad while labour units are hired in the domestic market.\(^4\) Both capital and labour are paid their marginal product. Aggregate employment at time \( t \) depends on the fraction \( x \) of employed labour time, labour productivity and current level of the working population:

\[
N_t = x_t \sum_{j=J_0+1}^{J_t-1} \mu_{j,t} \epsilon_j
\]

(3)

The level of output produced is determined by consumer demand for commodity \( c \). A competitive firm hires capital services from abroad up to the point where capital’s marginal product equals the world price of capital: \( MP_K = A_t f_K(K_t, N_t) = \tau \). Similarly, the average wage rate in the economy is given by: \( w = MP_N = A_t f_N(K_t, N_t) \). The firm adjusts employment levels to accommodate the required level of output and the economy is in equilibrium if the goods market clears: \( \sum_{j=J_0+1}^J \mu_{j,t} c_{j,t} + nx_t = Y_t \), where \( nx \) denotes net exports traded at time \( t \). Imports include capital services in the production process while exports are constant and exogenous in each period.

\(^2\)A more detailed description of the standard life-cycle model can be found in Koka and Kosempel (2014).

\(^3\)Essentially, since we abstract from technological change and exogenous growth, we assume that each young individual assumes that when he reaches his parent’s current age he will be earning as much as his parent is currently earning. We also hold the parameters of the intertemporal utility function constant across age groups, so that differences in consumption behaviour will depend on current income, expected income, the interest rate and the individual’s remaining life expectancy.

\(^4\)We are dealing with an economy along the lines of, say, the Canadian province of Prince Edward Island rather than a self-sufficient national economy. This also means that we do not model the decision to invest in fixed capital in this exercise: the demand for capital services, like the demand for labour, is determined by the demand for goods.
2.3. Description of experiments and timing

We assume that the economy is initially in its long run equilibrium where labour is fully employed and the steady state is characterized by a constant age distribution of consumption and savings.\(^5\) We start simulations by allowing a one-time unexpected shock to reduce individual savings across agents of all ages. The effects are distributed equally among cohorts if individual savings are positive but the shock does not affect agents who are primarily net borrowers. Specifically, we assume that for each individual the shock reduces the value of her accumulated assets by half.\(^6\) The reduction in savings leads to a transitional period until the economy settles to a new equilibrium. The mechanism through which the shock impacts the aggregate economy is as follows. First, in response to the shock on accumulated assets, all affected individuals re-configure consumption and savings profiles for the remaining of their life-cycle. Following the decline in savings, agents plan to lower their consumption levels both in the present and in the future. While the impact of the shock is even across agents, consumption responses depend on their age at the time of the shock as well as their remaining working life until retirement.

Secondly, since individual decisions determine the aggregate demand for consumption of goods and services, a reduction of consumption across cohorts hit by the shock lowers total demand in the economy. In response to the fall in aggregate consumption demand, firms reduce the level of output they produce and employment levels for working agents.\(^7\) The underemployment of labour leads to lower payoffs from work and cause agents to re-adjust earlier decisions; previously chosen consumption and savings profiles are no longer optimal. By re-optimizing and determining new optimal paths, agents also determine a new level of aggregate consumption demand in the current period. The cycle continues and the economy is in disequilibrium until a new steady state is reached. The transition is complete when \(Y_t = Y_{t-1}\), in other words output converges to a new equilibrium level. Experiments include computing transitional outcomes with stationary and non-stationary population time series.

3. Calibration

Demographic parameters are obtained from statistics of the Canadian population while the reference economy is set to have a stationary population. It is assumed that the model period amounts to one year in real time. Individuals enter the labour market at the age of 21, while time before this age is spent as children. The mandatory retirement date, \(jr\), is set at age 65. The length of individual life varies but each individual may live for a maximum of 90 years, therefore \(J = 90\). Conditional survival probabilities \(\psi_j\) are an average of males and females probabilities and are taken from the Life Tables (2011) of Statistics Canada. We assume that survival probabilities rates are time invariant and represent long run values for these parameters. The stationary age distribution is derived using a constant population growth

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\(^5\)For a detailed description of the solution algorithm see Heer and Maußner (2005).

\(^6\)Note that the shock size does not change the pattern of the results for the different demographic assumptions. We picked a 50 percent reduction in asset values for exposition, in the last section we also show outcomes for other shock sizes.

\(^7\)They will also reduce demand for the capital input, which is imported from abroad.
rate, \( n \), of 2.1 percent. Cohort shares are computed as follows: \( \mu_{j+1} = (1 + n)^{-1} \mu_j \psi_j \) where \( \mu_1 = (1 + \sum_{j=1}^{J-1} (1 + n)^{-j} \Pi_{i=1}^j \psi_i)^{-1} \). Current demographic data representing a non-stationary population are also obtained from Statistics Canada.

The instantaneous utility function is CRRA, has a constant elasticity of substitution and takes the functional form: \( U(c) = \left( \frac{\bar{c}}{c} \right)^{1-\sigma} - \frac{1}{1-\sigma} \). Values for family weights, \( \eta_j \), are computed from OECD household data and obtained from Ríos-Rull (2007). These too are held constant over time and assumed to represent long run values. \(^8\)

Since we are interested in transitional outcomes of the shock working through its effect on consumption demand, we target the calibration of the age dependent distribution of consumption expenditure. In particular, we try to match the simulated consumption outcomes in the steady state of the model with Canadian consumption expenditure obtained from the Survey of Household Spending (Statistics Canada). We estimate relative differences of distinct age cohorts, and compare model outcomes to four years of cross-sectional data (2010 to 2013). Thus we jointly set the discount factor \( \beta \), the coefficient of relative risk aversion \( \sigma \), the interest rate \( r \) and the depreciation rate to 0.96, 4, 5.2 percent and 4 percent, respectively, to get as close as possible to the ratios from the data of the average consumption of individuals in each age group to the average consumption expenditure of individuals aged less than 30. These ratios are displayed in figure 1.

As expected from the use of the standard life-cycle model, the age-dependent consumption profile obtained from the model is smoother than the data, particularly for older cohorts. Without considering income or lifetime uncertainty, incomplete markets and other factors, this is the closest that we can capture age differences in consumption across cohorts. Also, the estimated consumption profile is hump-shaped, similar to profiles observed in empirical studies while its peak is reached when agents are at the age of 50.\(^9\) In this case, attempts to get any closer to the relative variation across ages in the data would require sacrificing the attained peak of the simulated profile.

The production function takes the form: \( Y = AN^\alpha K^{1-\alpha} \). Age dependent efficiency units \( \epsilon_j \) are taken from Chen (2010). The share of labour income to total income is set to 0.7, an average of common values used in the literature. Finally, the technology parameter \( A \) is set equal to 1.

4. Transitional outcomes

To demonstrate the role played by the population age structure we examine transitional outcomes which differ only in demographic parameters. The age composition of the population used to aggregate individual decisions is displayed in figure 2 for a stationary population and a non-stationary population. The latter represents the age distribution of population

\(^8\)These weights are computed assuming two parent households.

\(^9\)Some of these studies estimate the peak of the consumption profile when individuals are in the age bracket 45 to 55 (see Hansen and Imrohoroglu (2008) for a discussion). For example, Fernández-Villaverde and Krueger (2002) estimate the peak of the age consumption profile using expenditure data from the US at the age of 52, while Gourinchas and Parker (2002) estimate that the maximum of the consumption profile is reached when individuals are 45 years old.
for Canada in 2014. Given the demographic data described in section 3, the stationary population has a higher share of cohorts under the age of 30 while the non-stationary population has a higher share of cohorts in every other age group. In addition, we run experiments with varying shock sizes and examine differences on short run demand and output responses.

We expect the fall in consumption demand to vary with age since age determines the level of agents’ accumulated assets. Initially, we examine the immediate response of consumption by the age of the household at the time of the shock, which we denote by $t_0$. Relative changes to steady state consumption are displayed in figure 3. We observe that the initial negative effect on consumption increases with age and it continues to do so until retirement, while remaining constant to the magnitude of the shock thereafter. Since retiree income consists of interest payments on accumulated assets, the optimal response of retired agents is to proportionally reduce consumption to the fall in assets. It follows that the immediate effect of the shock is a greater if the economy is comprised of larger groups of middle aged and older cohorts.

The initial response to the asset shock and the subsequent reduction in consumption demand prompts firms to lower output production and demand for effective labour and capital, as described in the model section. Underemployment of labour and declining incomes in transition will then affect all working members of the model economy. Further decisions and responses of individual households will depend on the degree to which their incomes derive from labour relative to asset returns, their level of accumulated savings, and the length of the time span remaining until retirement.

Young households are primarily net borrowers and initially not affected by the economic shock. Being at the beginning stages of life-cycle asset accumulation, their main source of income comes from labour earnings. It follows that when moving into the transition, their consumption demand subsequently falls due to declining work incomes experienced in this period. Middle aged households are affected by both the initial drop in savings and the lower labour income in transition. Their consumption expenditure is also reduced, although a higher level of accumulated assets serves as a buffer to prevent large drops in consumption. To illustrate, figure 4 depicts age dependent consumption responses of two individuals aged 25 and 50 at the time of the shock. Simulations involve the case when the population is stationary and the age distribution remains constant over the transitional period. We observe that the negative response of an individual aged 50 is higher than an individual aged 25, but that this relationship reverses itself half way through the transition.

Finally, agents over the age of 65, who initially reduce consumption by an equal proportion to the asset shock, are not affected by the decline in employment income since at this age they enter retirement.

While the contribution of each household to the aggregate demand varies across age cohorts, a smaller reduction in the consumption of a middle aged household who may consume more might be higher than a larger reduction in the consumption of a young or old household who may consume less. Distinguishing individuals by age at the time of the shock, we compute transitional consumption outcomes for specific age groups prior to retirement and compare them with equivalent steady state values.\(^{10}\) For example, for an individual aged

\(^{10}\)After the retirement age the family weights equal 1 and family sizes are assumed to be equal.
35 years old when the economy is hit by the asset shock, we evaluate the reduction in total consumption of this individual over the transitional period and compute its impact on the reduction of aggregate consumption demand. The exercise is done for agents aged 25 to 65 at the time of the shock, with an interval of 5 years in between. Figure 5 presents these estimates. It is shown that the impact initially increases with age, peaks at age group 40-45 and then it declines.

The next sets of simulations depict output impulse responses and convergence rates driven by age dependent demand changes. The response to the asset shock of the reference economy with a stationary population is compared to that of an economy with an ageing population, with an age structure similar to that of the Canadian economy. Household decisions in each case are aggregated using cohort shares presented in figure 2. Figure 6 depicts the relative changes of output per person, compared to the respective steady state values. The impulse responses of output reflect the cumulative effect of all factors mentioned above and capture the influence of the demographic structure on the effects of the shock. We first observe that the stationary population, which has a much younger population, results to a less severe decline of aggregate output over the transitional period. In terms of the speed of output convergence to the new steady state, depicted in figure 7, we observe an interesting result. While initially the fall in output is higher for the older population, the fact that older and retired cohorts are not affected by further reductions of labour income allows for a faster convergence of this economy to its new equilibrium.

Finally, we perform a set of simulations similar to the previous example, except that we vary the size of the asset shock. Experiments are performed using the stationary population. Figure 8 shows output impulse responses when the shock reduces household assets by 10, 20 and 30 percent, respectively. As it is expected, the more severe macro shocks reduce output and consequently employment to a much greater degree. The convergence rate, depicted in figure 9, is faster for economies hit by the shock the hardest, although these economies remain in transition for longer periods of time.

5. Conclusion

In this paper we investigate how the age composition of the population determines the impact of a shock on household assets. We employ a dynamic life-cycle model with overlapping generations and focus on a transitional analysis of the effects of the shock. Comparative experiments are performed using different scenarios with respect to the age structure of the population. We show that the age structure is not only important on the effects immediately following the macroeconomic shock, but also on the adjustment of the economy to a new equilibrium. Our results indicate that the impact of the asset shock on output per person is more pronounced in economies with a higher proportion of middle aged and older individuals. This finding is important to take into account when implementing expansionary fiscal policy at the onset of economic crises.

In future extensions we intend to relax some of the model assumptions. For example, we have simplified the model by assuming a small open economy, overlooking capital investment decisions and taking the price of capital as fixed. Finally, our objective is to test some of the model insights empirically.
References


