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Unbundling Truthful Revelation when Auctioning Bundled Goods

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Abstract

We study a seller who auctions bundles of goods and is interested in learning the buyer's valuations for each individual good. We show that truthful revelation is never the unique dominant strategy when each bundle contains multiple goods. We study the case with linear bundles and propose auction rules that eliminate all non-truthful reports for the Becker-Degroot-Marschak mechanism and the Vickrey auction.

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1 Introduction

This paper proposes auction rules to sell bundles of goods when the seller wants to learn the buyer's valuations for each good in the bundles. Truthful revelation is important in a number of situations. For example, the seller may wish to use the information obtained on the value of the goods in other markets, for internal transactions, policy evaluation, or to design subsequent auctions. The research question was in fact posed in the context of public timber sales where data from auctions is used to estimate the parameter values of forestry economics models, and provide benchmarks for other lots sold elsewhere at fixed prices. Similar needs exist in procurement auctions when a buyer outsources complex projects that involve multiple components (e.g. design, engineering, production and installation).

The literature on multiple-objects auctions focuses on direct and truthful mechanism but typically overlooks the fact that when multiple-objects are sold together, strong incentive compatibility is lost (see Jehiel et al. 2007). Truthful reporting gives the same payoff as some non-truthful reports that only reveal the value for the sum of the objects. This is relevant, for example, in auctions for bundles or packages of goods (Krishna 2009). Applied to the Becker-Degroot-Marschak (BDM) mechanism (Becker et al. 1964), this means that truthful reporting is not the unique weakly dominant strategy. Applied to a Vickrey auction (Vickrey 1961), it means that there exist multiple Nash equilibria in weakly dominant strategy.

We propose a simple solution to eliminate non-truthful reports. For the sake of exposition, we only apply this solution to the two mechanisms mentioned above (BDM and Vickrey auction) because these mechanisms are widely studied in economics and used in practice. We also consider a linear setup with independent private values interpreted as unit prices, which is relevant in timber sales and elsewhere. Specifically, we study sales to a single buyer using a generalization of the BDM, as well as to multiple buyers in a Vickrey auction. We show that with appropriate modifications, both mechanisms can achieve truthful revelation as a unique weakly dominant strategy in the case of BDM and in a unique Nash equilibrium in weakly dominant strategy in the case of a Vickrey auction. We point out that the results of this paper on truthful revelation applies more broadly to other extensions of the BDM and Vickrey auction.

2 Model

A seller auctions K bundles containing N goods each. Each bundle is denoted by a quantity vector $Q'_k = (q_{k,1},..,q_{k,N})$ for $k = 1..K$ and $n = 1..N$ where $q_{k,n} \geq 0$ is the quantity of good n in bundle k and the prime symbol denotes the transpose operator. We denote by $Q = (Q_1,..,Q_K)$ the matrix of all K quantity vectors. The composition of these bundles may be exogenously given or chosen by the seller. In both interpretations, the bundles are fixed for the auction and cannot be unbundled or divided by buyers. This is the case, for example, in many timber auctions where once the lot boundaries are determined, separation of species is impractical or too costly. Yet, lot boundaries could be exogenous (e.g. constrained by geography) or chosen by the seller.

A buyer has a private value vector $v' = (v_1, .., v_N)$ where v_n denotes the per-unit willingness to pay for good n. The buyer is thus willing to pay $\bar{v}_k \equiv Q'_k v$ for bundle Q_k. Although the analysis could be generalized beyond linear values, this assumption can be justified on the grounds that truthful revelation of individual values is less important when the information learned is about buyer-specific idiosyncratic components (for example, non-linear functions of q or complementarity between k and k') that has no use elsewhere. Vectors of private values are drawn from a distribution with full support over the $N-$dimension space $V = \Pi_{n=1}^N [v_{0,n}, v_{1,n}]$ where $v_0 \ll v_1$ and $v_i = (v_{i,0}, .., v_{i,N})$ for $i \in \{0,1\}$. $S = [Min_k Q'_k v_0, Max_k Q'_k v_1]$ is the resulting support for bundle values. The objective is to develop a mechanism that truthfully reveals the buyer's valuation vector v.

3 Case $K < N$: No Truthful Revelation

There could be a single bundle ($K = 1$) or multiple bundles ($K > 1$). In a direct mechanism denoted $(x(b), t(b))$, the buyer sends signal $b \in V$, pays transfer $t(b) \in \Re$ to the seller and receives bundle k with probability $x_k(b)$. Denote by $x() \in \Re^K$ the probability vector of attributing the bundles. Let $IC(v, \tilde{v})$ denote the incentive compatibility constraint ensuring that buyer type v truthfully reports $b = v$ instead of $b = \tilde{v} \in V$

$$x(v)Q'v - t(v) \geq x(\tilde{v})Q'v - t(\tilde{v}) \tag{1}$$

A mechanism is incentive compatible if (1) holds for all $(v, \tilde{v})$ and strong incentive compatible if the inequalities are strict.

Theorem 1. *There does not exist a strong incentive compatible mechanism.*

Proof: Take a pair of valuations $v \neq \tilde{v}$ such that $\tilde{v} = v + w$ where $w \in Ke(Q')$. Such a pair exists because the kernel of Q' has dimension at least $N - K > 0$ and the distribution of v has full support on V. Thus, there exists a $w \neq 0^N$ such that $\tilde{v} = v + w$. From (1), the incentive compatibility constraint $IC(v, \tilde{v})$ says $x(v)Q'v - t(v) \geq x(\tilde{v})Q'v - t(\tilde{v})$ and $IC(\tilde{v}, v)$ says $x(\tilde{v})Q'\tilde{v} - t(\tilde{v}) \geq x(v)Q'\tilde{v} - t(v)$, and using the fact that $Q'w = 0$, we rewrite this constraint as $x(\tilde{v})Q'v - t(\tilde{v}) \geq x(v)Q'v - t(v)$. We obtain

$$x(v)Q'v - t(v) = x(\tilde{v})Q'v - t(\tilde{v}).$$

Thus, buyer type v is indifferent between revealing v or $\tilde{v}$. In any direct and truthful mechanism, there are multiple dominant strategies. QED

To illustrate, take the case when a single bundle is offered ($K = 1$). Truthful revelation of v as a unique dominant strategy is not possible. The only relevant issue to the buyer is the value and price of the bundle as a whole. Nothing in the problem provides incentives to consider individual goods separately. Obviously, it is possible to reveal v as a weakly dominant strategy. We demonstrate this point using a modified BDM where the buyer bids vector $b \in V$ and the seller draws a random value r with differentiable CDF $G()$, PDF $g()$

and support R such that $S \subsetneq R$. For bid b, the buyer wins the bundle and pays r when $Q_1'b \geq r$. The buyer maximizes:

$$\int_0^{Q_1'b} (Q_1'v - r)\, dG(r).$$

Any bid b such that $Q_1'b \notin S$ can be ignored because there exist a b' that dominates b. For bid b such that $Q_1'b \in S$, the FOC for b_n gives:

$$Q_1'(v - b)q_n g(Q_1'b) = 0.$$

with $g(Q_1'b) > 0$. When $q_{1,n} > 0$ for some n, all FOC simplify to $Q_1'(v - b) = 0$. Any bid such that $Q_1'b = Q_1'v = \bar{v}$ is weakly dominant. As previously indicated, individual bids on goods do not matter as long as the total bid equals the value of the bundle. Therefore, the mechanism can only reveal the bidder's aggregate valuation $\bar{v}$; not the vector of valuations v.

An implication of Theorem 1 is that a necessary condition for strong incentive compatibility is that there be at least N bundles. The rest of this paper considers the case with $K \geq N$.

4 Truthful Revelation with Generalized BDM (GBDM)

We now consider an extension of the BDM in order to establish truthful revelation as a strictly dominant strategy.

Definition 1. *In a GBDM, the buyer bids $b \in V$. The seller draws a random price r with differentiable CDF $G()$, PDF $g()$ and support R such that $S \subsetneq R$. For $k = 1..K$, the bidder receives bundle k and pays r if $Q_k'b \geq r$.*

The same bid vector b and price r are used to allocate *all* bundles.

Proposition 1. *The GBDM achieves truthful revelation of v in strictly dominant strategy as long as there are $K \geq N$ linearly independent bundles.*

Proof: A buyer who bids b obtains bundle k if $Q_k'b \geq r$. As above, we ignore any b such that $Max_k Q_k'b \notin S$. The bidder's profits from placing bid $b \in S$ is

$$\sum_k \int_0^{Q_k'b} (Q_k'v - r) dG(r)$$

The first order condition with respect to b_n gives

$$\sum_k Q_k'(v - b)q_n^k g(Q_k'b) = 0$$

which can be written together in matrix form as

$$\left(\sum_k g(Q_k'b)Q_kQ_k'\right)(v-b)=0 \tag{2}$$

Denote $\tilde{Q}_k = g(Q_k'b)^{1/2}Q_k$ and $X = (\tilde{Q}_1, .., \tilde{Q}_K)$ so that (2) becomes

$$XX'(v-b)=0.$$

Since $Max_kQ_k'b \in S$, we have $g(Q_k'b) > 0$ and matrix X is of rank N if there are $K \geq N$ linearly independent bundles. When this is the case, matrix XX' is also of rank N and the unique solution to the N first order conditions is $b = v$. QED

The GBDM is strongly incentive compatible because it is always optimal to submit a total bid equal to the total value for each bundle, and with $K \geq N$ linearly independent bundles, the unique solution achieving this goal is to submit a truthful bid for each good.

The statement in Proposition 1 can be generalized in a number of ways. First, if there are only $K < N$ linearly independent bundles, the seller can reveal K linear combinations of the goods' prices. This matters to an auctioneer who cares only about the value of a few relevant bundles. Second, the proposition follows under the alternative scheme where the auctioneer draws a random vector of per-unit prices $r \in R'$ such that $V \subsetneq R'$ (instead of a single bundle price) and allocates bundle k if $Q_k'b \geq Q_k'r$. In doing so, the auctioneer can set random prices that better describe the assessed value of each good and bundle. Finally, the auctioneer does not always have to actually proceed with the sale of all of the K bundles. The bundles could be presented to the buyer with arbitrarily small probabilities that the sale will proceed. This may be a useful design feature, for example, if the physical quantities in the bundles are not mutually exclusive quantities. For instance, a seller interested in the value of two tree species on a single piece of land could offer two bundles, each allowing for the harvesting of different quantities of the two types of trees. Once the bid is received, the seller would then randomly select which of the two bundles is actually auctioned off.

5 Truthful Revelation with Vickrey Auctions

We turn to a competitive situation with multiple bidders. $M > 1$ bidders have i.i.d. valuation vectors v_m, for $m = 1, .., M$. Each bidder submits a bid $b_m' = (b_{1,m}, .., b_{N,m})$. Denote $\hat{B}$ the highest bid value, that is, $\hat{B} = Max_mQ'b_m$, and $\hat{\hat{B}}$ the second highest value. In a Vickrey auction, the bidder who bids $\hat{B}$ wins the bundle and pays $\hat{\hat{B}}$. An argument similar to the one presented in Theorem 1 shows that it is a weakly dominant strategy for bidder m to reveal any b such that $Q'b = Q'v_m$. In contrast with the one-good dimensional case, there is a continuum of Nash equilibria in weakly dominant strategy. We extend the analysis to multiple bundles to demonstrate the possibility of eliminating all Nash equilibria in weakly dominant strategy other than the truthful one.

Definition 2. *In a generalized Vickrey auction, each buyer bids vector* $b \in V$*. The bidder with bid* $\hat{B}_k$ *receives bundle* k *and pays* $\hat{\hat{B}}_k$*. Ties are broken by random allocation.*

Proposition 2. *Assume there are* $K \geq N$ *linearly independent bundles. In a random Vickrey auction, truthful revelation is the unique weakly dominant strategy for all bidders.*

Proof: Consider a bidder with valuation v_m who bids b_m. Take bundle k. Let $\bar{v}_{k,m} = Q'_k v_m$ and $V_{k,m} = \{x : Q'_k x = \bar{v}_{k,m}\}$. The standard argument that truthful revelation is a weakly dominant strategy in the scalar case (see, for example, Proposition 1 in Levin (2004)) applies here in the following sense: the set of weakly dominant strategies in auction k is $V_{k,m}$. This holds for all $k = 1..K$. The set of bidder m's weakly dominant strategy is $\bigcap_{k=1}^{k=K} V_{k,m}$. When there are $K \geq N$ linearly independent bundles, we have $v_m = \bigcap_{k=1}^{k=K} V_{k,m}$. QED

The Proposition implies that there exists a unique Nash equilibrium in weakly dominant strategy.[1]

6 Summary

In this paper, we derived three results:

1. An impossibility theorem stating that when the seller offers multiple bundles containing multiple goods, strong incentive compatibility is *not* possible when there are *more* goods than bundles;

2. Proofs that when $K \geq N$ linearly independent bundles are offered, truthful revelation of each goods' valuation can be achieved:

    (a) as a strictly dominant strategy in a generalized BDM; and

    (b) as the unique Nash equilibrium in weakly dominant strategies in a Vickrey auction.

We have considered here the two leading mechanisms for truthful revelation: BDM in the non-competitive case where there is a single bidder and Vickrey auction when multiple bidders compete. The randomization approach employed to obtain truthful bids in these mechanisms could easily be extended elsewhere. For instance, The n^{th} price auctions (Shogren et al. 2001) could be extended to the sale of multiple identical bundles. Incentive compatible mechanisms that involve a mix of Vickrey auction and BDM could be similarly constructed. For instance, multiple bidders could submit bids, with the highest bid value on a bundle gaining the right to participate in the GBDM described above. It should also be possible to extend the principles described in this paper to the provision of bundles of public goods by appropriately modifying the Random Price Voting Mechanism developed by Messer et al. (2010).

[1] There may exist other equilibria that do not involve undominated strategies (Blume and Heidhues 2004).

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