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A Rationalization of Selecting Behavior to Release the Cheap Edition

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Abstract

This article examines a specific sales technique employed by certain product suppliers (such as book or gaming software suppliers). Such suppliers release cheap editions (e.g., paperback versions following the initial hardcover edition) of only those products that had high sales volumes and not those that had low sales volumes. This behavior seems counterintuitive because the additional cost of producing such cheap editions is approximately zero (it is a significantly lower cost than releasing a new product) and the cheap editions will generate a profit even if they do not sell well. Hence, it would seem rational for a supplier to release a cheap edition of each of their products following the initial release. This article explains this behavior, showing that it is not necessarily irrational. We construct a differential equation comparable to the replicator dynamics of a strategic form game, and show that if a supplier's decision is biased then their payoff may be better than if it is not biased.

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1 Introduction

This article rationalizes a behavior specific to certain book and software suppliers: releasing cheap editions of only such products (e.g., paperback books and cheap gaming software versions) that sold well on their initial release. The cost of releasing the cheap edition tends to be much lower than of creating a new product and we can consider its cost to be approximately zero: the suppliers thereby achieve a profit even if this cheap edition does not sell well. Releasing cheap versions of only those products that achieved high sales volumes seems to contradict the profit-maximization objective of suppliers; rationally, they should release a cheap edition of each product. It can be argued that there is a lower motivation to release a cheap edition of a product with high sales than of a product with low sales because the latter product has not reached a relatively large potential market. Although somewhat persuasive, this argument contradicts the actual supplier behavior.

This article explains and clarifies such supplier behavior, showing that it is not necessarily irrational. Specifically, if the suppliers release a cheap edition of every product, then most consumers will wait for this version and will tend not to buy the initial release. To avoid such waiting behavior, the supplier does not release a cheap edition when their product has poor sales: the suppliers aim for long run, rather than short run, profits.

We use a dynamic system that is similar to the replicator dynamics of a strategic form game[1] to represent this situation. In this system, the supplier's behavior is biased and they tend not to release a cheap edition of their product when many consumers are waiting for it. We show that if this bias is sufficiently high, then the supplier's payoff may be better than if their bias is zero.

This method is of value, particularly compared with alternative explanations (especially, those that use the Folk theorem). First, it does not require any clever actions by the consumers. Second, the supplier's decision to realize the long run equilibrium emerges specifically and it matches the actual behavior.

In section 2, we present our model and derive its results. The concluding remarks are given in section 3. The proof is in the appendix.

[1]The replicator dynamics was introduced by Taylor and Jonker (1978), and named by Schuster and Sigmund (1983). They consider a symmetric situation. However, the replicator dynamics used in this research is an asymmetric case, which was treated by Selten (1980). See Hofbauer and Sigmund (1998) or Weibull (1997).

2 Model and Result

2.1 Replicator Dynamics

Consider the following strategic form game.

	C	D
A	(a_1, b_1)	(a_3, b_3)
B	(a_2, b_2)	(a_4, b_4)

Let $(x, 1-x)$ (resp. $(y, 1-y)$) be the mixed strategy of the above game in player 1 (resp. player 2), and consider the following dynamic system.

$$\begin{aligned}\dot{x}/x &= (1-x)[y(a_1-a_2)+(1-y)(a_3-a_4)] \\ \dot{y}/y &= (1-y)[x(b_1-b_3)+(1-x)(b_2-b_4)].\end{aligned}$$

This equation is called the **replicator dynamics**, which can be considered as a process representing players' adjustive behavior. Let us explain this dynamics. Consider that x is the proportion of type 1 agents such that they choose A, and that y is the proportion of type 2 agents such that they choose C. At first, all players blindly believe that their current action is the best. As time goes by, however, they may observe that their action is not optimal and they may change it. The change occurs in proportion to the difference between the payoff of the current action and of the social average. Therefore, the growth rates of both proportions x and y are equal to this difference, and the above equation appears.

Selten (1980) showed that, in general, only the pure Nash equilibrium (NE) may become the stable steady state in such an asymmetric replicator dynamics.

2.2 The Model

Consider a market that consists of two players. Player 1 represents a consumer and player 2 represents a supplier. Player 1 considers whether they will immediately purchase a product on its initial release. If they do, then they gain $u > 0$; otherwise they wait for the release of the cheap edition of this product. If player 2 releases a cheap edition, then player 1 can buy it and gain $d > 0$. If player 2 does not release a cheap edition, then player 1 waits until this product becomes dated and its worth tends to zero. The product's original retail price is denoted by p_H and p_L denotes the cheap edition price. We assume $p_H > p_L > 0$.

If $u - p_H \geq d - p_L$, then the consumer should immediately purchase the product. This consumer must not consider a cheap edition release in their purchase decision, and thus we restrict our consideration to the case in which $d - p_L > u - p_H$. Moreover, if $u - p_H \leq 0$, then the consumer should wait for the cheap edition. This consumer behavior is simple and we treat it implicitly. We assume that if player 2 does not release the cheap edition, then they automatically lose $D > 0$, (the sales for such consumers). Therefore, we consider only the case where $d - p_L > u - p_H > 0$.

The following strategic form game represents the above situation. Strategy B represents "Buy this product immediately" and strategy NB represents "Not buy until the supplier releases the cheap edition". Similarly, strategy R represents "Release the cheap edition" and strategy NR represents "Not release the cheap edition".

	R	NR
B	$(u - p_H, p_H)$	$(u - p_H, p_H - D)$
NB	$(d - p_L, p_L)$	$(0, -D)$

Clearly, the unique NE of this game is (NB, R). Actually, we see that R is the dominant strategy for player 2. We call the payoff p_L of player 2 in this NE a **short run profit** for the supplier. In NE, the supplier must release the cheap edition and the consumer must wait for this cheap edition: as in the introduction, there is a mismatch between this NE and the actual behavior of suppliers.

Next, let $(x, 1 - x)$ (resp. $(y, 1 - y)$) be the mixed strategy for the above game of player 1 (resp. player 2). Consider the following dynamic system:

$$\begin{aligned} \dot{x}/x &= (1 - x)[(u - p_H) - y(d - p_L)], \\ \dot{y}/y &= (1 - y)[(1 - x)p_L + D] - b(1 - x). \end{aligned} \tag{1}$$

If a parameter b (indicating supplier's bias) is zero, then this dynamic system is the same as the replicator dynamics of this game. However, in such a case, the unique NE of the game is globally stable. Therefore, a mismatch between theory and the actual behavior of suppliers still remains.

Therefore, we add the term $-b(1 - x)$ to represent the supplier's decision bias. This term represents the supplier behavior of "not releasing the cheap edition if the initial product sales x are not sufficiently high". Note that **the existence of a positive b is the observed fact**: in the real world, many suppliers do not release a cheap edition unless the initial product sales are sufficiently high. However, if $b > 0$, then the unique NE is not the steady state of equation (1), and it may seem that the existence of positive b indicates that the supplier is irrational.

We define the **long run profit** as the supplier's payoff for the mixed strategy profile $((x^*, 1-x^*), (y^*, 1-y^*))$, where (x^*, y^*) is the asymptotically stable steady state of equation (1).[2] Our purpose is to show that there exists $b > 0$ such that the long run profit is greater than the short run profit p_L, which indicates that the existence of a positive b does not mean the irrationality of the supplier.[3]

In fact, the following proposition shows the existence of such a b.

Proposition: If $b > \frac{(p_H+D)[(d-p_L)-(u-p_H)]}{d-p_L}$, then this dynamic system has an asymptotically stable steady state such that,

$$(x^*, y^*) = (\frac{(d-p_L)(p_L+D-b)-(u-p_H)(p_H+D)}{(d-p_L)(p_L-b)-(u-p_H)p_H}, \frac{u-p_H}{d-p_L}).$$

Moreover, if $D < \frac{(p_H-p_L)(d-p_L)}{(d-p_L)-(u-p_H)}$ and $b > 0$ is sufficiently high, then the long run payoff exceeds p_L.

Proof: See appendix.

3 Concluding Remarks

This article explained and clarified the specific supplier (of books or gaming software) behavior of releasing a cheap edition of only such products that sell well. We showed that if the supplier's short run decision is biased, then their long run payoff may improve. Therefore, despite its appearance such supplier behavior is not necessarily irrational and may actually improve their payoff.

We used a dynamic system similar to the replicator dynamics of a strategic form game to represent this situation, including a biased supplier behav-

[2]The terms *long run* and *short run* reflect our interpretation of the model. We think that if suppliers only consider today's profit, then they must choose the dominant strategy R. Therefore, if their behavior against R can be justified, then they must consider the profit in the future.

[3]The left-hand side of (1) is $\dot{y}/y$ for technical reasons only. If we rewrite (1) as

$$\dot{y} = y(1-y)[(1-x)p_L + D] - b(1-x),$$

then y may become to be negative because the effect of the term $-b(1-x)$ is not restricted, which contradicts the interpretation of y. Later our lemma 1 will show that such a problematic case does not occur for the solution of (1).

ior. This method is of value, particularly compared with alternative explanations, especially using the Folk theorem. First, it does not require any clever actions by the consumers. Second, the supplier's decision to realize the long run equilibrium emerges specifically and matches the actual behavior.

A Appendix: Proof of Proposition

First, we will prove the following lemma:

Lemma 1. Consider equation (1) with an initial value condition $(x(t^*), y(t^*)) = (c_1, c_2) \in]0,1[^2$, where $b \geq 0$. Then, the solution $(x(t), y(t))$ can be extended on $[t^*, +\infty[$ and $(x(t), y(t)) \in]0,1[^2$ for every $t \geq t^*$.

This lemma is needed for assuring the following two properties. First, the stability of the equation has a meaning, because the solution $(x(t), y(t))$ can be defined for an infinitely long period. Second, the value of $(x(t), y(t))$ is consistent with the interpretation of these: if $(x(t), y(t)) \notin [0,1]^2$, then this cannot be treated as a mixed strategy.

Proof. Consider the following modified dynamic system of (1):

$$\begin{aligned} \dot{x} &= x(1-x)[(u-p_H) - y(d-p_L)], \\ \dot{y} &= y[(1-y)[(1-x)p_L + D] - b(1-x)], \end{aligned} \tag{2}$$

where the right-hand side is defined on $\mathbb{R}^2$. Clearly, any solution $(x(t), y(t))$ of (2) with $x(t) > 0, y(t) > 0$ for any t is also a solution of (1). Therefore, it suffices to show that the solution $(x(t), y(t))$ of (2) can be extended on $[t^*, +\infty[$ and $(x(t), y(t)) \in]0,1[^2$ for every $t \geq t^*$. If $(x(t), y(t))$ cannot be extended on $[t^*, +\infty[$, then there exists $\hat{t}$ such that $(x(t), y(t)) \notin [0,1]^2$ for $t > \hat{t}$.[4] Therefore, it suffices to show that $(x(t), y(t)) \in]0,1[^2$ for all t in the domain of $(x(t), y(t))$ with $t \geq t^*$.

Suppose that $x(t) \leq 0$ for some $t > t^*$. Let $t^+ = \min\{t' \in [t^*, t] | x(t') = 0\}$, and consider the following equation

$$\dot{z} = z[(1-z)[p_L + D] - b], z(t^+) = y(t^+).$$

Choose any solution $z(t')$. Then, $(0, z(t'))$ is a solution of (2) defined on a neighborhood of t^+. Because of the local uniqueness of the solution, we

[4]It is well known that for any nonextendable solution $z(t)$ of an autonomous ordinary differential equation $\dot{z} = f(z)$ defined on $]a, b[$, either $b = +\infty$ or for any compact set C included in the domain of f, there exists $t \in]a, b[$ such that $z(t') \notin C$ for any $t' \in [t, b[$. See Pontryagin (1962) for more detailed arguments.

have $x \equiv 0$ on a neighborhood of t^+, which contradicts the definition of t^+. Therefore, we have $x(t) > 0$ for all $t > t^*$. By the same arguments, we can verify that $x(t) < 1$ and $y(t) > 0$ for all $t > t^*$, and if $b = 0$, then $y(t) < 1$ for all $t > t^*$.

Next, suppose that $b > 0$ and $y(t) \geq 1$ for some $t > t^*$. Let $t^+ = \min\{t' \in [t^*, t] | y(t) = 1\}$. Because $y(t^*) < 1$, we have $y(t') < 1$ for any $t' \in [t^*, t^+[$, and thus

$$\dot{y}(t^+) = \lim_{t' \uparrow t^+} \frac{y(t') - y(t^+)}{t' - t^+} \geq 0.$$

Meanwhile, because $y(t^+) = 1$, we have $\dot{y}(t^+) = -b(1 - x(t^+))$. This implies that $x(t^+) \geq 1$, which contradicts what we proved in above arguments. This completes the proof of lemma 1. ■

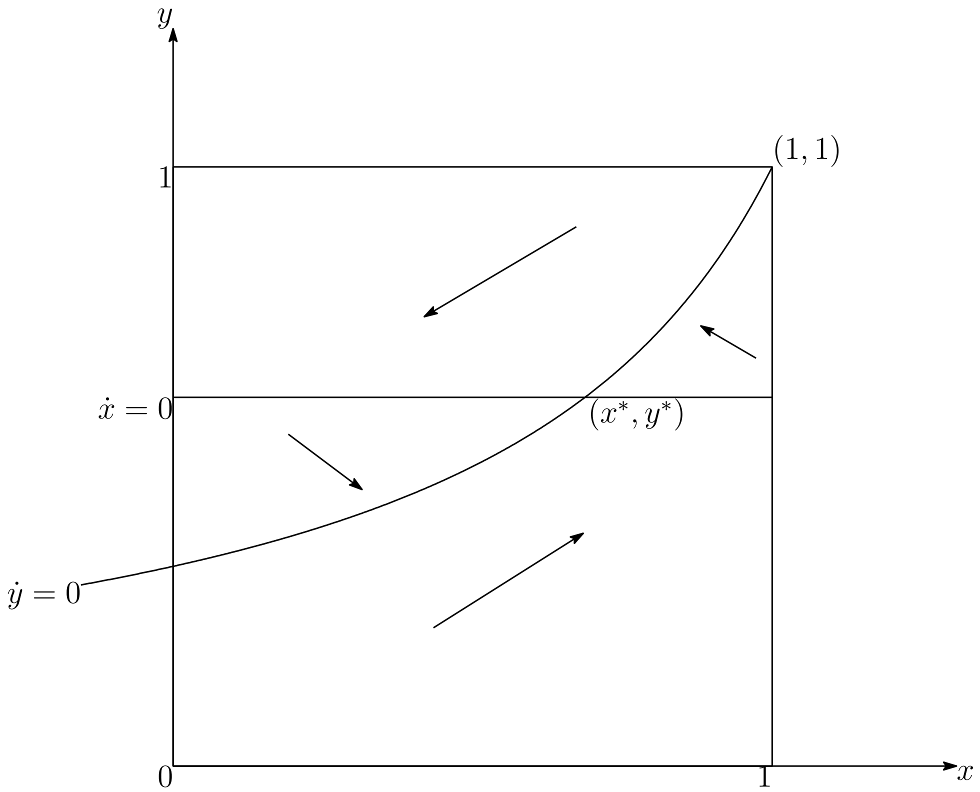

Figure 1: Phase diagram.

Hereafter, we assume that $b > \frac{(p_H + D)[(d - p_L) - (u - p_H)]}{d - p_L}$. Figure 1 exhibits the phase diagram for equation (1). It is easy to show that the $\dot{x} = 0$ curve intersects the $\dot{y} = 0$ curve at only one point in $]0, 1[^2$, say, (x^*, y^*).

Lemma 2. The steady state (x^*, y^*) is asymptotically stable.

Proof. It suffices to show that for every eigenvalue λ of the Jacobian matrix of the right-hand side of (2), its real part is negative.[5] Since $\dot{x} = 0$ and $\dot{y} = 0$ at (x^*, y^*), its Jacobian matrix at (x^*, y^*) is

$$\begin{pmatrix} 0 & -x^*(1-x^*)(d-p_L) \\ y^*[b-(1-y^*)p_L] & -y^*((1-x^*)p_L+D) \end{pmatrix}$$

and thus, its characteristic equation is

$$\lambda^2 + y^*((1-x^*)p_L + D)\lambda + x^*(1-x^*)y^*(d-p_L)[b-(1-y^*)p_L] = 0.$$

Let λ_1, λ_2 be the solution of this equation. Then,

$$\lambda_1 + \lambda_2 = -y^*((1-x^*)p_L + D) < 0,$$

$$\lambda_1\lambda_2 = x^*(1-x^*)y^*(d-p_L)[b-(1-y^*)p_L].$$

Note that, if λ_1 is not a real number, then $\bar{\lambda}_1 = \lambda_2$ and $\lambda_1 + \lambda_2 < 0$, and thus the real part of λ_1, λ_2 must be less than 0. Therefore, we can assume that $\lambda_1, \lambda_2 \in \mathbb{R}$ without loss of generality. Then,

$$b - (1-y^*)p_L > \frac{D[(d-p_L)-(u-p_H)]}{d-p_L} > 0,$$

implying $\lambda_1, \lambda_2 < 0$. This completes the proof of lemma 2. ■

Finally, we should check that the payoff of player 2 in (x^*, y^*) exceeds p_L if $D < \frac{(p_H-p_L)(d-p_L)}{(d-p_L)-(u-p_H)}$ and b is sufficiently high. To show this, it suffices to show that if $b \to \infty$, then the payoff of player 2 in (x^*, y^*) exceeds p_L. Actually, if $b \to \infty$, then $(x^*, y^*) \to (1, \frac{u-p_H}{d-p_L})$.[6] At this point, the payoff of player 2 is

$$p_H - \frac{(d-p_L)-(u-p_H)}{d-p_L}D > p_L.$$

Therefore, for any sufficiently high b, the long run profit of player 2 exceeds the short run profit p_L. This completes the proof. ■

Reference

[1] Hofbauer, J. and Sigmund, K. (1998) *Evolutionary Games and Populational Dynamics*, Cambridge University Press, Cambridge.

[5] See Pontryagin (1962) for more detailed arguments.

[6] Use l'Hospital's rule.

[2] Pontryagin, L. S. (1962) *Ordinary Differential Equations*, Addison-Wesley, Massachusetts.

[3] Schuster, P. and Sigmund, K. (1983) “Replicator Dynamics”, *Journal of Theoretical Biology* **100**, 533-538.

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[5] Taylor, P. D. and Jonker, L. B. (1978) “Evolutionarily Stable Strategies and Game Dynamics”, *Mathematical Biosciences* **40**, 145-156.

[6] Weibull, J. (1997) *Evolutionary Game Theory*, MIT Press, Massachusetts.