Gold–oil prices co-movements and portfolio diversification implications

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Abstract
In this paper we use the bivariate fractionally integrated GARCH (FIGARCH) model to analyze the dynamic relationship between gold and crude oil markets. We also test the role of gold as a hedge or safe haven for crude oil risk. Empirical results show that the dynamic links between the two markets vary over time and decline significantly during major economic and political crisis episodes. This suggests that gold can act as a safe haven during extreme oil market conditions. Finally, Findings indicate that adding gold to crude oil portfolio helps to hedge against the oil risk.
1. Introduction

Modeling the volatility of commodity markets is an essential task for international investors and portfolio managers for several reasons. First, understanding the correct dynamic volatility helps investors to better forecast the volatility evolution of commodity markets and to implement the optimal portfolio diversification and the effective hedging strategies. Second, this task is important to analyze the dynamic relationship between the different commodity prices and to examine the volatility spillovers between markets. Such analysis must help investors and policymakers in monitoring the price of major commodities and in optimizing portfolios.

The co-movement between gold and oil prices has attracted the attention of investors, academics and portfolio managers during the two last decades with the emergence of several geopolitical and economic events such as the Iraq invasion, the Asian financial crisis, the September 11, 2001 terrorist attack and the global financial crisis. These crises have significant impact on the dynamic behavior of commodity markets. Consequently, unveiling the dynamic relationship between gold and oil markets helps investors to optimize the portfolio diversification and maintains optimal hedging strategies against downside risk.

With reference to literature, different mechanisms exist through which oil and gold could be linked. The most obvious channel that explained the link between oil and gold markets is the inflation. As suggested by the traditional macroeconomic models, the raise in the oil prices leads to an increase of the overall price level (Hooker, 2002; Hunt, 2006). Moreover, the demand for gold increases as investors should use this precious metal as a hedge against inflation which leads to the raise in the prices of gold (Jaffe, 1989). Several previous studies investigate the role of gold as a hedge against inflation (see, Worthington and Pahlavani, 2007; Wang et al., 2011; Beckmann and Czudja, 2013). A second channel reflects the link between oil and gold prices was disclosed by Melvin and Sultan (1990). Following the authors, when the oil prices rise, the revenues of oil-exporting countries rise. Consequently, these countries increase their investment in gold in order to preserve a significant share of their diversified portfolios in this precious metal. This pushes up the demand for gold and leads to an increase in its prices. The third channel establishes a relationship between oil and gold markets through economic growth and stock markets. Oil price movements affect economic growth and stock market returns (Holmes and Wang, 2003; Manera and Cologni, 2006, Mohanty et al., 2011, Aloui et al., 2012). Hence, investors turn to gold as a safe-haven asset to protect against stock market fluctuations. The role of gold as a hedge and safe haven for stock markets has been widely studied in the literature (Baur and McDermott, 2010; Baur and Lucey, 2010; Coudert and Raymond-Feingold, 2011; Hood and Malik, 2013; Gurgun and Unalmis, 2014).

In this paper we investigate the dynamic links between gold and oil prices. We contribute to the related literature in three ways. First, we study the dynamic relationship between the gold and crude oil for the period 1997-2014, which characterized by the occurrence of several crises such as the Asian financial crisis in 1997, the Russian and Brazilian crisis between 1998 and 1999, the global financial crisis between 2007-2009 and the European debt crisis of 2011-2012. Such crisis has several effects on the dynamic behavior of the two markets. Second, to investigate the dynamic correlation between the two markets, we employ the bivariate FIGARCH model. This model allows us in one hand to examine the evolution of the correlation between the two commodity markets over time. In the other hand, this model takes into account an important feature of commodity series namely the long memory in volatility. This feature is omitted by previous studies. To the best of our knowledge, ours is the first
study that account for the long memory impacts on the links between the gold and oil markets. Finally, we use the empirical results issue of the estimation of our model to compute the optimal portfolio design and hedging strategies between the two markets.

The remainder of the paper is organized as follows. Section 2 offers a short overview of the empirical literature on the links between gold and oil prices. Section 3 presents the empirical methodology. Section 4 describes the data and the preliminary analysis. Empirical results are reported in section 5. Section 6 summarizes the main conclusion.

2. Short overview of the literature

Empirically, few studies have examined the relationship between the two commodity markets and the role of gold as a hedge for the potential movements of crude oil prices. Reberdo (2013) uses the copula approach to analyze the dependence structure between the two commodity markets. Using data from January 2000 to September 2011, he finds a positive and significant relationship between gold and oil suggesting that gold cannot hedge against oil price volatility. Results also reveal a tail independence indicating that gold can act as a good safe haven during the stress periods of crude oil market.

Soytas et al. (2009) use the vector autoregressive model to examine the long-run and short-run relationship between world oil price and domestic gold and silver price in turkey. Empirical evidence reveals significant effects of oil price shocks on precious metal spot price in the short term. Furthermore, this effect is only transitory and dies off pretty quickly.

Zhang and Wei (2010) analyze the long-run and short-run relationship between gold and crude oil market through cointegration test and linear and nonlinear Granger causality tests. They find, in one hand, a significant long-term equilibrium relationship between the two markets. Following the authors, this link is due to the fact that the two markets are affected by some common factors such as the geopolitical events and economic fundamentals. In the other hand, they emphasize the existence of a significant unidirectional linear Granger causality between the crude oil market and the gold market. This causality runs from the former to the latter. More interestingly, oil price movements Granger cause the dynamic of gold prices.

Lee et al. (2012) investigate the asymmetric cointegration and causal relationships between gold and crude oil prices for the period May 1994 to November 2008. They find evidence of asymmetric long-run adjustment between the two commodities. In addition, the gold market responded positively and significantly to oil price movements in the short-run.

Wang and Chueh (2013) suggest that gold and crude oil prices positively influence each other for the period from January 1989 through December 2007. Using a structural vector autoregressive approach, Le and Chang (2012) investigate the impact of oil price movements on gold market returns for the period 1994-2011. Their results reveal that oil price fluctuations are significantly and positively transmitted to real gold returns.

Ewing and Malik (2013) examine the volatility of gold and oil prices through univariate and bivariate GARCH models. Incorporating the structural breaks in the variance, they find strong evidence of volatility spillover between the two markets.

This paper contributes to the existing literature by investigating the dynamic relationship between gold and oil markets using the dynamic conditional correlation (DCC)-FIGARCH model. The aim was to follow the evolution of the co-movement between the two markets...
over time. Thereby, to check whether gold can work as a safe haven for extreme declines in
the oil markets. We also examine the implications of obtained results to portfolio
diversification and hedging effectiveness. It is noteworthy that the GARCH-class models are
heavily used in past studies to explore the co-movement between commodity markets (see
e.g. Mensi et al., 2014; Sensoy, 2013; Chang et al., 2010; Dahl and Iglesias, 2009) and
between commodity and financial markets (see among others: Arouri et al., 2015; Chkili,
Aloui and Nguyen, 2014, Creti et al. 2013, Hammoudeh et al., 2010). Therefore, studies
that examine the links between gold and oil markets using GARCH-type model, are extremely
limited with the exception of Ewing and Malik (2013). The authors employ some univariate
and bivariate GARH model to test the transmission of volatility between the two markets. Our
approach differs in that we extend the DCC model to incorporate the long memory property in
the conditional volatility and explore this feature in portfolio management and hedging
strategies.

3. Methodology

Our main objective is to examine the dynamic relationship between gold and oil markets. As
mentioned above, we use the DCC-FIGARCH which combines the FIGARCH specification
and the DCC process of Tse and Tsui (2002). This model allows us to model jointly the long
memory feature for the two series and to follow the evolution of the dynamic correlation over
time. Assume that the conditional mean equation is defined as:

\[ y_t = \mu(\theta) + \varepsilon_t \]  

Where \( \mu(\theta) = \{\mu_1, \mu_2, \ldots, \mu_N\} \) means the conditional vector of \( y_t \).

The DCC model of Tse and Tsui (2002) can be specified as follows:

\[ H_t = D_t R \Sigma D_t \]  

Where \( D_t = \text{diag}(\sqrt{h_{11}}, \ldots, \sqrt{h_{NN}}) \)

\( h_t \) is defined as an univariate FIGARCH model as follows:

\[ h_t = \omega + \beta(L)\varepsilon_t + \left(1 - \beta(L)\varepsilon_t^2 - \phi(L)(1 - L)^d\varepsilon_t^2 \right) \]  

Where the long memory coefficient \( d \) satisfies the condition \( 0 \leq d \leq 1 \), \( \phi(L) \) and \( \beta(L) \) are
finite order lag polynomials that assumed to have all their roots lying outside the unite circle,
and \( (1 - L)^d \) is the fractional differencing operator.

\[ R_t = (1 - \theta_1 - \theta_2) + \theta_1 \Psi_{t-1} + \theta_2 R_{t-1} \]

\( R \) is a symmetric \( N \times N \) positive definite parameter matrix with \( \rho_{ij} = 1 \), \( \psi_{t-1} \) is the \( N \times N \) correlation matrix of \( \varepsilon_t \) while \( \theta_1 \) and \( \theta_2 \) are non negative parameters that satisfy the condition
\( \theta_1 + \theta_2 < 1 \).

The dynamic correlation coefficient for the bivariate framework is given by:

\[ \rho_{12,t} = (1 - \theta_1 - \theta_2)\rho_{12} + \theta_2 \rho_{12,t-1} + \theta_1 \frac{\sum_{m=1}^{M} \mu_{1,t-m} \mu_{2,t-m}}{\sqrt{\sum_{m=1}^{M} \mu_{1,t-m}^2} \sqrt{\sum_{m=1}^{M} \mu_{2,t-m}^2}} \]  

(4)
Where \( m = 1,2, \ldots, M \) mean the time lag.

4. Data and preliminary analysis

The data set consist of daily spot prices of two major commodity markets namely gold and West Texas Intermediate (WTI) crude oil. The period spans from January 1997 to December 2014 yielding a total of 4578 daily observations. Our choice of the two commodities is motivated by several factors. First, these two commodities are commonly used in previous studies but none has analyzed the transmission of volatility through long memory model. Second, the movements of their prices have substantial implications for real economy and commodity and financial markets\(^1\). Therefore, understanding their co-movement prices has great significance for investors, policy makers and portfolio managers. Finally, the WTI is often used as a reference for deciding the price of other light crudes in the USA and has the same trend with the Brent oil prices (see Reboredo, 2013). The WTI crude oil prices ($/barrel) are obtained from the US Energy Information Agency\(^2\) while the gold prices ($/Troy ounce) are collected from the Bloomberg database.

Table 1 reports the descriptive statistics as well as the unit root tests for the commodity return series. As shown the daily average returns are positive for the two markets and range from 0.0265 % for gold to 0.0155% for oil. The volatility of the WTI market, measured by the standard deviations, is about twice times higher than volatility of the gold market. Both Skewness and Kurtosis statistics indicate that return series are away from the normal distribution. The rejection of normality is confirmed by the Jarque-Bera test results. The results of the Ljung-Box test applied to squared returns and the Engle (1982) test reject the null hypothesis of no serial correlation and suggest the presence of ARCH effects, respectively. Finally, to examine the stationarity of the commodity time series, we report in the two last lines the results of the Augmented Dickey-Fuller (ADF) unit root test and the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) stationarity test. The results of these tests show that all the considered series are stationary and thus appropriate for further analysis.

Fig. 1 plots the prices and returns of the two commodity markets for the period 1997-2014. For the upper panel, we notice that the gold prices experienced an increasing trend over the period under investigation. This phase continues until mid-2013. Henceforth, the prices skilled a slight decline during the last two years of study. The crude oil market also experienced increasing prices during the first phase of our period study. More interestingly, the market reaches their highest values in mid-2008. This period is followed by a sharp falls during the global financial crisis that occurred between 2008 and 2009. From 2010, the price resumes the increasing phase. The lower panel plots the commodity return series. The two series indicate the presence of volatility clustering: large (small) changes in the commodity prices tend to be followed by large (small) changes by either sign. This feature confirms the presence of ARCH effects and justifies our choice of GARCH type models to model the volatility of the two series.

\(^1\) Several studies examine the relationship between these two commodity prices and financial markets (see among others Aloui and Jammazi, 2009; Sari et al., 2010; Ciner et al., 2013; Chkili, Aloui and Nguyen, 2014). Other studies investigate the link between commodity price fluctuations and real economy (Hooker, 1999; Barsky and Kilian, 2004; Hamilton, 2008).

\(^2\) [http://www.eia.doe.gov](http://www.eia.doe.gov)
Table 1
descriptive statistics and unit root tests

<table>
<thead>
<tr>
<th></th>
<th>Gold</th>
<th>WTI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0265</td>
<td>0.0155</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.1115</td>
<td>2.4324</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.2038</td>
<td>-0.1738</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>5.975</td>
<td>5.2153</td>
</tr>
<tr>
<td>J.B</td>
<td>6840.1 [0.000]</td>
<td>5210.3 [0.000]</td>
</tr>
<tr>
<td>Q²(10)</td>
<td>572.53 [0.000]</td>
<td>990.88 [0.000]</td>
</tr>
<tr>
<td>ARCH(5)</td>
<td>51.576 [0.000]</td>
<td>92.834 [0.000]</td>
</tr>
<tr>
<td>ADF t-tests</td>
<td>-38.357**</td>
<td>-39.544**</td>
</tr>
<tr>
<td>KPSS t-tests</td>
<td>0.2337</td>
<td>0.1051</td>
</tr>
</tbody>
</table>

Notes: JB designs the Jarque-Bera test for normality, Q²(10) and ARCH(5) refer to the statistics of Ljung-Box test for 10-order serial correlation applied to squared returns and Engle (1982)’s test for conditional heteroscedasticity, respectively. ADF and KPSS are the Augmented Dickey-Fuller and Kwiatkowski-Phillips-Schmidt-Shin tests for the unit root and stationarity. * denotes the rejection of the null hypotheses at the 5% level. ** denotes the rejection of the null hypotheses at the 1% level.

![Gold prices](image1)
![WTI prices](image2)
![Gold returns](image3)
![WTI returns](image4)

**Fig.1.** prices and returns for gold and oil markets
5. Empirical results

5.1. Long memory in the commodity volatility

The aim of the paper is to determine the appropriate model to describe the volatility dynamics of two major commodities markets: gold and WTI. We start our analysis by checking the long memory property in the volatility of the two considered markets. In this vein, we employ three long memory tests namely the rescaled variance ($V/S$) test of Giraitis et al. (2003), the log periodogram regression (GPH) test of Geweke and Porter-Hudak (1983) and the Gaussian semi-parametric (GSP) test of Robinson (1995). These three tests have been, often used by several previous studies examining the dynamic of commodity markets (Choi and Hammoudeh, 2009; Aloui and Mabrouk, 2010; Chkili, Aloui and Nguyen, 2014 and Chkili, Hammoudeh and Nguyen, 2014).

The results of the three long memory tests are reported in Table 2. The statistics of the GPH and GSP tests applied to absolute and squared returns are significant at 1% significance level. Thus, we can reject the null hypothesis of no long-range memory for all commodity series. More interestingly the parameter $d$ varies between 0.261 and 0.745 for gold market and between 0.289 and 0.702 for crude oil market, depending on the bandwidth $m$. Likewise, the result of the rescaled variance test confirms those provided by the GPH and GSP tests. The statistics of this test applied to absolute and squared returns are significant at 1% level suggesting the presence of long memory effect for both gold and crude oil markets. Given these results, we choose to model their conditional volatility through the FIGARCH model that take into account the long memory feature in conditional volatility. These results are consistent with the finding of Chkili, Hammoudeh and Nguyen (2004). Using data for four major commodities (WTI crude oil, natural gas, gold and silver), they find strong evidence of the presence of long memory propriety in the volatility of all considered markets.

Table 2
Long memory test results

<table>
<thead>
<tr>
<th></th>
<th>Gold</th>
<th>WTI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Absolute return</td>
<td>Squared return</td>
</tr>
<tr>
<td><strong>Panel A: rescaled variance test</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m = 5$</td>
<td>1.5883 [0.000]</td>
<td>1.5038 [0.000]</td>
</tr>
<tr>
<td>$m = 10$</td>
<td>2.3512 [0.000]</td>
<td>2.0961 [0.000]</td>
</tr>
<tr>
<td><strong>Panel B: GPH test</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m = T^{0.5}$</td>
<td>0.745 [0.000]</td>
<td>0.607 [0.000]</td>
</tr>
<tr>
<td>$m = T^{0.6}$</td>
<td>0.624 [0.000]</td>
<td>0.528 [0.000]</td>
</tr>
<tr>
<td>$m = T^{0.8}$</td>
<td>0.315 [0.000]</td>
<td>0.261 [0.000]</td>
</tr>
<tr>
<td><strong>Panel C: GSP test</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m = T/4$</td>
<td>0.332 [0.000]</td>
<td>0.257 [0.000]</td>
</tr>
<tr>
<td>$m = T/8$</td>
<td>0.410 [0.000]</td>
<td>0.305 [0.000]</td>
</tr>
<tr>
<td>$m = T/16$</td>
<td>0.498 [0.000]</td>
<td>0.402 [0.000]</td>
</tr>
</tbody>
</table>

Notes: $m$ is the bandwidth for the GPH, GSP and rescaled variance tests. $T$ is the sample size. $p$-values are in brackets.

5.2. Volatility spillovers between gold and oil markets

The first setup of our analysis consists to examine the dynamic correlation between the two commodity markets. More interestingly, we verify the evolution of this correlation over the period under study characterizing by the emergence of several crises namely the Asian financial crisis of 1997, the 2007-2008 global financial crisis and the European debt crisis.
2011-2012. In the second setup, we seek to determine the optimal portfolio diversification and hedging strategy between gold and crude oil that minimize the risk for international investors.

The results of the estimation of the bivariate FIGARCH\((p,d,q)\) model under student-\(t\) distribution are displayed in Table 3. Before estimating the considered model, we specify the order of \((p,q)\). Following the Akaike and Schwartz information criteria the FIGARCH\((1,d,1)\) specification is the appropriate model to successfully capture the volatility clustering and long-range memory in commodity market returns.

As shown in Table 3, the estimated parameters of the ARCH and GARCH coefficients are all significant at the conventional levels for all cases and are more important for crude oil market. The fractional differencing parameters \((d)\) are significant at 1% significance and range between 0.336 and 0.374 for gold and oil, respectively. This result confirms those found above suggesting the presence of long memory in the volatility of commodity markets. The student-\(t\) degrees of freedom parameter is also significant which justify the goodness of the student-\(t\) distribution in volatility modeling. These results corroborate with those of some previous studies focusing on the volatility of oil and gold markets (Chkili, Hammoudeh and Nguyen, 2014; Arouri et al., 2012; Aloui and Mabrouk, 2010).

Panel C reports some diagnostic tests. As shown the results of Ljung-Box test applied to standardized residuals and standardized squared residuals cannot reject the null hypothesis of no serial autocorrelation in all cases. Thus our DCC-FIGARCH model is correctly specified and sufficiently depicts the links between gold and oil markets in the presence of long memory property.

**Table 3**

<table>
<thead>
<tr>
<th></th>
<th>Gold</th>
<th>WTI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: estimation results of DCC-FIGARCH model</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(C(m))</td>
<td>0.020 (0.015)</td>
<td>0.056* (0.031)</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.001 (0.016)</td>
<td>-0.023 (0.017)</td>
</tr>
<tr>
<td>(C(v))</td>
<td>0.068* (0.035)</td>
<td>0.174 (0.131)</td>
</tr>
<tr>
<td>(d)</td>
<td>0.336*** (0.065)</td>
<td>0.374*** (0.081)</td>
</tr>
<tr>
<td>ARCH</td>
<td>0.287* (0.147)</td>
<td>0.392* (0.162)</td>
</tr>
<tr>
<td>GARCH</td>
<td>0.515*** (0.153)</td>
<td>0.635*** (0.188)</td>
</tr>
<tr>
<td><strong>Panel B: Dynamic correlation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average correlation</td>
<td>0.1667*** (0.053)</td>
<td></td>
</tr>
<tr>
<td>alpha</td>
<td>0.008 (0.006)</td>
<td></td>
</tr>
<tr>
<td>beta</td>
<td>0.988*** (0.011)</td>
<td></td>
</tr>
<tr>
<td>LL</td>
<td>-16186.2</td>
<td></td>
</tr>
<tr>
<td>student-df</td>
<td>5.998***</td>
<td></td>
</tr>
<tr>
<td><strong>Panel C: Diagnostic tests</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Q(10))</td>
<td>5.302 [0.870]</td>
<td>3.157 [0.977]</td>
</tr>
<tr>
<td>(Q^2(10))</td>
<td>4.352 [0.930]</td>
<td>8.053 [0.624]</td>
</tr>
<tr>
<td>AIC</td>
<td>7.0798</td>
<td></td>
</tr>
<tr>
<td>SIC</td>
<td>7.1023</td>
<td></td>
</tr>
<tr>
<td>HQIC</td>
<td>7.0877</td>
<td></td>
</tr>
</tbody>
</table>

*Notes: \(C(m)\) and \(C(v)\) are the constants of the mean and variance equations, respectively. \(d\) is the long memory parameter. \(Q(10)\) and \(Q^2(10)\) refer to the statistics of Ljung-Box test applied to residuals and squared residuals. \(p\)-values are in brackets. Standard deviations are reported in parentheses. *, ** and *** denote the significance at the 10%, 5% and 1%, respectively.*
The dynamic conditional correlation between gold and crude oil for the period under investigation is reported in Fig. 2. We can observe that the correlation is time varying and varies between 0.35 and -0.05 with an average value of 0.166. This low value emphasizes the role of gold as a hedge for crude oil volatility. More interestingly, the correlation between the two markets declines significantly during the major crises that occur during the last three decades. The important drop in the correlation can be seen during the global financial crisis and less important over the European debt crisis. However, the oil market has reacted significantly to these two economic events while the gold prices continue its upward phase. This indicates the role of gold as a safe haven against losses in oil market prices and its ability to provide protection during extreme declines. The drop of the conditional correlation between the two markets can also be detected during the Asian financial crisis of 1997. This confirms the role of gold as a safe haven during extreme market conditions. This result is consistent with the evidence provided in Reboredo (2013). Using a copula approach, he finds that gold plays a role as an effective safe haven against extreme oil market movements.

Fig. 2. The Dynamic conditional correlation between WTI and gold markets

5.3. The DCC behavior over time

Fig. 2 shows that the DCC is time varying over time and declines significantly during major economic and geopolitical events. To more investigate the potential shift behavior of the DCCs between the two markets, we specify the following model including dummy variables which represent the economic and geopolitical crisis periods:

\[
\rho_t = c_0 + c_1 \rho_{t-1} + \sum_{k=1}^{K} \delta_k dummy_{k,t} + \nu_t \\
\eta_t = \alpha_0 + \alpha_1 \eta_{t-1} + \alpha_2 \nu_{t-1}^2
\]

Following Baur and Lucey (2010) and Baur and McDermott (2010): an asset is considered as a safe haven if it is uncorrelated or negatively correlated with another asset in extreme market conditions.

4 This model is used by Miyazaki et al. (2012) to highlight the role of crisis in the dynamic links between gold and other financial markets.
Where $\rho_t$ is the estimate of the dynamic conditional correlation between gold and oil markets. The dummy variables take the value of 1 during the crisis period and 0 otherwise. We identify four important geopolitical and economic events that occur during the period under study$^5$. These events are defined in Table 4.

### Table 4
Impact of major events on the DCC between gold and oil markets

We consider four economic and geopolitical events:

<table>
<thead>
<tr>
<th>Event</th>
<th>Dummy Variable</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asian financial crisis</td>
<td>$\delta_1$</td>
<td>July 1997-December 1997</td>
</tr>
<tr>
<td>September 11, 2001 terrorist attack</td>
<td>$\delta_2$</td>
<td>September 2001-December 2001</td>
</tr>
<tr>
<td>Global financial crisis</td>
<td>$\delta_3$</td>
<td>August 2007-December 2008</td>
</tr>
<tr>
<td>European debt crisis</td>
<td>$\delta_4$</td>
<td>January 2010-December 2011</td>
</tr>
</tbody>
</table>

Panel A: mean equation

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0$</td>
<td>0.004**</td>
<td>0.0019</td>
</tr>
<tr>
<td>$c_1$</td>
<td>0.998***</td>
<td>0.0013</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>-0.0015***</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>0.0016**</td>
<td>0.0007</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>-0.0053</td>
<td>0.0032</td>
</tr>
<tr>
<td>$\delta_4$</td>
<td>0.0030</td>
<td>0.0031</td>
</tr>
</tbody>
</table>

Panel B: variance equation

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Estimate</th>
<th>S.E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_0$</td>
<td>4.01 E-05***</td>
<td>7.37 E-6</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.214***</td>
<td>0.054</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>-0.332*</td>
<td>0.177</td>
</tr>
</tbody>
</table>

Notes: ***, **, and * represent statistical significance at 1%, 5%, and 10% levels, respectively. S.E. is the standard error.

The empirical estimation results of Eq. (8) are reported in Table 4. As shown, the coefficient of the first dummy variable ($\delta_1$) in the regression of the gold-oil correlation is negative and statistically significant. This implies that the correlation between the two markets has decreased significantly during the Asian financial crisis. Furthermore, gold can be regarded as a strong safe haven against the oil market volatility during this crisis period. Regarding the global financial crisis estimation, the coefficient associated with the dummy variable is negative and insignificant, suggesting that gold can act as a weak safe haven against the extreme declines of oil prices. This result is not surprising as the oil price has experienced a sharp fall during the subprime crisis.

Finally, it is worth noting that the oil and gold markets are significantly and positively correlated during the September 11 terrorist attack. A possible explanation for this result is that both gold and oil prices do not affect by this event and exhibited a continuously increasing trend during this geopolitical event.

### 5.4. Optimal portfolio diversification and hedging strategy

Given the low dynamic relationship between gold and crude oil markets, we compute in this subsection the optimal portfolio design that investor should possess in order to reduce the risk without lowering the expected portfolio returns. Following Kroner and Ng (1998), the optimal weight of gold in one dollar portfolio of gold/oil at time $t$ is given by:

$\omega_h(t) = \frac{(1 - \rho_t) \delta_1}{(1 - \rho_t) \delta_1 + \delta_2}$

$^5$ The choice of these geopolitical events and crisis periods is based on several previous studies (see eg. Chkili, Aloui and Nguyen, 2014; Miyazaki et al., 2012; Morales and Andreosso-O’Callaghan, 2011 ; Filis et al., 2011).
\[ w_t^{OG} = \frac{h_t^{OG} - h_t^G}{2h_t^{OG} + h_t^O} \] (5)

and

\[ w_t^{OG} = \begin{cases} 
0 & \text{if } w_t^{OG} < 0 \\
1 & \text{if } w_t^{OG} > 1 \\
0 < w_t^{OG} < 1 & \text{if } \end{cases} \]

Where in this expression, \( h_t^O \) and \( h_t^G \) are the conditional variance of oil and gold markets, respectively and \( h_t^{OG} \) is the conditional covariance between the two considered series returns at time \( t \). In the same way, the weight of crude oil in the one dollar gold/oil portfolio at time \( t \) is computed as \( 1 - w_t^{OG} \).

Given our main objective to investigate the role of gold as a hedge asset for crude oil, we also compute the optimal hedge ratio. The hedge ratio allows us to verify whether the addition of gold in the portfolio can reduce the oil market risk. Following Kroner and Sultan (1993), in order to minimize risk, a long position of one dollar taken in crude oil should be hedged by a short position of \( \beta_t \) dollar invested in gold at time \( t \). The optimal hedge ratio is given as:

\[ \beta_t^* = \frac{h_t^{OG}}{h_t^G} \] (6)

Finally, we can evaluate the percentage reduction in the risk of a hedge portfolio compared to the oil portfolio through the hedging effectiveness (HE) index. For our instance, the HE is computed as follows:

\[ HE = \frac{Var_{\text{unhedged}} - Var_{\text{hedged}}}{Var_{\text{unhedged}}} \] (7)

Where \( Var_{\text{hedged}} \) denotes the variance of hedged portfolio and \( Var_{\text{unhedged}} \) refers to the variance of crude oil portfolio.

We report in Table 5 the average values of the optimal weight of gold, the hedge ratio and the HE index calculated from the estimation results of our DCC-FIGARCH benchmark model. The optimal weight is equal to 0.8279. This suggests that investors should hold 82.79% of their wealth in gold while the remaining should be invested in the crude oil markets. This result is not surprising given that the gold price returns are less volatile than the WTI price return. Therefore, investors should hold more gold than oil in their portfolio in order to benefit of the diversification and reduce the risk without lowering the expected return of their portfolio. In the same vein, Hammoudeh, Malik, and McAleer (2011) emphasize that an optimal portfolio of precious metals that minimizes risk should be dominated by gold. Arouri et al. (2015) suggest that the emergence of successive financial turbulences and crises have prompted investors to account alternative investment instruments by adding gold in their portfolio as a refugee asset.
The hedge ratios, obtained from the estimation of the model, is time varying\(^6\). The average value is low and is around 0.2916 for the period of study. This indicates that one dollar long position taken in the crude oil market should be hedged by a short position of about 30 cents in the gold market. Finally, the average value of the HE index for the pair gold/oil reported in the last column of Table 5 shows that the hedging strategy implicating crude oil and gold assets reduce significantly the portfolio’s risk. More interestingly, the risk is reduced in average by 18%.

<table>
<thead>
<tr>
<th>portfolio</th>
<th>( \omega_{12} )</th>
<th>( \beta_{12} )</th>
<th>HE (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold/oil</td>
<td>0.8279</td>
<td>0.2916</td>
<td>17.92</td>
</tr>
</tbody>
</table>

6. Conclusion

In this paper, we use the DCC-FIGARCH model that take into account the long memory feature to examine the dynamic relationship between two major commodity markets namely gold and crude oil. Empirical results show that the dynamic correlation between the two markets is time varying and has decline significantly during the main economic and geopolitical events that occurred during the period under examination. Thereby, gold can work as a safe haven against extreme downward market movements. Empirical findings also reveal that adding gold into crude oil portfolios enhances their risk-adjusted performance. In addition, investors should hold more gold than crude oil in order to reduce the risk of their investment.

References


\(^6\) We report here only the average value, the evolution of the hedging ratio over time is available upon request.


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