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Contagious Runs: Who Initiates?

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Abstract

This paper presents a model of contagious panic between two regions with heterogeneous fragilities. When there is no strategic risk, the spillover is always one-directional; the contagion can only originate from the fundamentally weaker region spilling over to the stronger. When strategic risks due to strategic complementarities cause a self-fulfilling panic, the direction of the contagion could be reversed; panic in the stronger region could generate a contagious panic in the weaker. We show that this depends on the difference in severity of coordination problems between the two regions and the scale of potential spillovers.

The views expressed in this paper are those of the authors and are not necessarily reflective of the views at the Federal Reserve Bank of New York or the Federal Reserve System.

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1. Introduction

The recent financial crisis clearly showed the importance of endogenous risks. In studying the spillover effects across different entities, conventional network analysis takes a "domino-effect" approach in which agents don't ex-ante anticipate potential spillovers and contagion arises in a passive way. This approach is thus not suitable for the analysis of systemic panic where agents could respond preemptively from concerns about the spillovers.

In this note, we present a simple model with potential spillovers between two regions and compare different implications from the domino-effect approach and the panic approach. The two regions are exposed to identical exogenous risk, and importantly, are endowed with heterogenous robustness with respect to this risk. One region's failure generates spillovers to the other, which could lead to a contagious failure. In the domino-effect approach, the weaker region always fails first and the spillover is one-sided; it is always the weaker that causes the contagious failure of the other. We argue that this could be reversed in the panic approach incorporating strategic risk. It is thus different from the previous contagious panic studies, e.g. Dasgupta (2004), and Oh (2013), where agents move sequentially with pre-determined direction of spillovers.

2. Model Setup

We consider a static one-shot game among agents in two different regions, S and W ("Strong" and "Weak"), where spillovers arise to the other when one region fails. Each region is populated with a continuum $[0,1]$ of ex-ante identical risk neutral agents. θ is the "fundamental" of this economy (common to both regions) with an improper prior, whose realization is unobservable to the agents. They instead observe noisy private signals, such that an agent $i(\in[0,1])$ of j-region ($j=S,W$) observes $s_{i,j}=\theta+\epsilon_{i,j}$, where $\epsilon_{i,j}$ is independently uniform over $[-\epsilon,\epsilon]$ and $\epsilon\to 0$ for simplicity. After observing this private signal, the agent chooses either to "stay" ($a_{i,j}=0$) or "exit" ($a_{i,j}=1$) from her region.

A region will either "survive" or "fail" depending on (i) realization of the fundamental, (ii) aggregate action of the agents in that region, and (iii) whether the other region fails. "Failure" occurs if the fundamental turns out to be below a certain threshold, which goes up as more agents exit (i.e., larger l_j where $l_j=\int_0^1 a_{i,j}di$ denotes the total number of the exiting agents in j-region). Moreover, one region's failure also raises the other's failure threshold through "spillover".[1] We introduce two parameters characterizing heterogenous fragilities and scales of the spillovers; f_j denotes j-region's unconditional "fragility" where $f_W>f_S$, and $\delta_{-jtoj}(\geq 0)$ denotes the scale of "spillover" from $-j$-region to j-region such that when $-j$-region fails, the fragility of j-region increases by δ_{-jtoj}. We assume that given the fundamental θ, j-region fails if $\theta\leq f_j+l_j+\delta_j$, where $\delta_j=\delta_{-jtoj}$ if $-j$-region fails (i.e., $\theta\leq f_{-j}+l_{-j}$) and $=0$ otherwise (i.e., $\theta>f_{-j}+l_{-j}$). Ex-ante, a region is more likely to fail if (i) its fragility is greater, (ii) more agents in that region choose to exit, and (iii) the other region fails.

[1] For instance, the spillovers could arise through fire-sale externality, direct exposures, or information contagion.

When choosing to stay, an agent is better off if her region survives but worse off if it fails, compared to when choosing to exit. The payoff function is identical across the agents in the same region, but different from that of the other region's agents. We normalize the upper and lower bounds of the payoffs such that when staying, one receives 1 if her region survives but 0 if it fails. She instead receives $\alpha_j (\in [0,1])$ when exiting.[2] Formally, we define the payoff function $u_{i,j}(a_{i,j}, l_j, l_{-j}, \theta)$ of agent i in j-region given θ as:

$$u_{i,j}(1, l_j, l_{-j}, \theta) = \alpha_j$$

and

$$u_{i,j}(0, l_j, l_{-j}, \theta) = \begin{cases} 0 & \text{if } \theta > f_{-j} + l_{-j} \text{ and } \theta \leq f_j + l_j, \\ & \text{or if } \theta \leq f_{-j} + l_{-j} \text{ and } \theta \leq f_j + l_j + \delta_{-jtoj} \\ 1 & \text{otherwise} \end{cases}$$

The payoff structure is summarized in Table 1. Strategic risk arises due to strategic complementarities, both from her own region (through l_j) and the other region (through δ_j). In the absence of the agents' exit (i.e., $l_S = l_W = 0$) and the spillovers (i.e. $\delta_S = \delta_W = 0$), W-region fails while S-region survives if $f_S < \theta \leq f_W$, thus W-region is fundamentally "weaker".

	$\theta > l_j + f_j + \delta_j$	$\theta \leq l_j + f_j + \delta_j$
Stay	1	0
Exit	α_j	α_j

Table 1: Payoff Structure for j-region agents

Timeline is as follows. The fundamental θ realizes and the agents receive $s_{i,j}$. Based on $s_{i,j}$, they choose either stay or exit to maximize expected payoffs. Each region either fails or survives depending on θ and the agents' collective action, then the agents receive their payoffs.

3. Results

We solve for a Bayesian equilibrium within a global games setup, which is the unique equilibrium (see, e.g, Morris and Shin 2003). We focus on the threshold strategies characterized by "switching thresholds" s_j^* such that agent i in j-region stays if $s_{i,j} > s_j^*$ and exits otherwise, which give "failure thresholds" θ_j^* such that j-region fails if and only if $\theta \leq \theta_j^*$. If $|\theta_S^* - \theta_W^*| \to 0$ (i.e., $|s_S^* - s_W^*| \to 0$ since $\theta_j^* \to s_j^*$) as $\epsilon \to 0$, we refer to this case as an occurrence of "contagious failures".[3]

We compare two approaches: (i) domino-effect approach ignoring the strategic risk; and (ii) panic approach incorporating the strategic risk. In the domino-effect approach, we assume agents are "passive", i.e. no one exits preemptively to avoid the failure. There is no coordination problem and without preemptive runs, a region fails only when the fundamental

[2]This normalization is without loss of generality. See discussion following Lemma 1.
[3]This corresponds to "strong interdependence" in Goldstein (2004).

is below its "fragility".

Proposition 1. Under the domino-effect approach,

(i) Suppose $\delta_{WtoS} < f_W - f_S$. W-region fails when $\theta \leq f_W (= \theta_W^*)$ and S-region fails when $\theta \leq f_S + \delta_{WtoS} (= \theta_S^*)$. There is no contagious failure since $\theta_W^* > \theta_S^*$.

(ii) Suppose $\delta_{WtoS} > f_W - f_S$. Both regions fail when $\theta \leq f_W$. Contagious failures arise with $\theta_S^* = \theta_W^* = f_W$.

Here, the weaker W-region always fails with the (weakly) higher fundamental. If the spillover from W-region's failure is large, it causes a contagious failure of S-region when $f_S < \theta \leq f_W$. Note that the failure threshold for the contagious failures in case (ii) only depends on W-region's fragility f_W.

The panic approach incorporates strategic risk; agents anticipate the potential impact of others' choices. This could reverse our previous result—the stronger region could trigger contagious failure of the weaker. When all agents follow equilibrium strategies (s_S^*, s_W^*), one should be indifferent between the two actions on one's switching threshold. For agent i in j, defining the payoff advantage of exiting over staying as $\triangle u_{i,j}(l_j, l_{-j}, \theta) \equiv u_{i,j}(1, l_j, l_{-j}, \theta) - u_{i,j}(0, l_j, l_{-j}, \theta)$, the following thus needs to hold given $s_{i,j} = s_j^*$:

$$\int_{s_j^*-\epsilon}^{s_j^*+\epsilon} \triangle u_{i,j}(l_j, l_{-j}, \theta) \cdot \frac{1}{2\epsilon} d\theta = 0, \tag{1}$$

since $\theta \sim U[s_j^* - \epsilon, s_j^* + \epsilon]$ given $s_{i,j} = s_j^*$.

Given θ, agents with $s_{i,j} < s_j^*$ exit where $s_{i,j} = \theta + \epsilon_{i,j}$ with $\epsilon_{i,j} \sim U[-\epsilon, \epsilon]$. Thus, (1) becomes

$$\int_{s_j^*-\epsilon}^{s_j^*+\epsilon} \triangle u_{i,j}\Bigg(\frac{s_j^* - (\theta - \epsilon)}{2\epsilon}, \frac{s_{-j}^* - (\theta - \epsilon)}{2\epsilon} \times \mathbb{1}_{[0 < \frac{s_{-j}^* - (\theta-\epsilon)}{2\epsilon} < 1]} + 1 \times \mathbb{1}_{[\frac{s_{-j}^* - (\theta-\epsilon)}{2\epsilon} \geq 1]} + 0 \times \mathbb{1}_{[\frac{s_{-j}^* - (\theta-\epsilon)}{2\epsilon} \leq 0]}, \theta\Bigg) \cdot \frac{1}{2\epsilon} d\theta = 0$$

This gives us two equations (for $j = S, W$) and solving jointly we get (s_S^*, s_W^*).

Before fully solving the model, we begin by analyzing each region individually, ignoring the inter-regional spillover effect (i.e., assuming $l_{-j} = 0$ and thus $\delta_{StoW} = \delta_{WtoS} = 0$).[4]

Lemma 1.

Suppose $\delta_{StoW} = \delta_{WtoS} = 0$. S-region fails if $\theta \leq \theta_S^I \equiv f_S + \alpha_S$, and W-region fails if $\theta \leq \theta_W^I \equiv f_W + \alpha_W$, where θ_j^I is j-region's failure threshold when analyzed individually.

Lemma 1 implies that self-fulfilling panic raises the failure threshold of each region by α_j. Note that in our normalized payoff structure, α_j is the ratio of (payoff of exit – payoff of

[4]This approach is similar to a "microprudential" approach that analyzes an individual entity in isolation, ignoring any externalities across different entities.

failure) to (payoff of stay and survival – payoff of failure), which reflects the attractiveness of a preemptive exit relative to staying. If this ratio is lower, the coordination problem itself becomes less critical since the added-benefit of the preemptive exit becomes smaller compared to the risks from strategic uncertainty when staying. Thus, α_j reflects severity of the coordination problem in j-region, and the self-fulfilling run becomes more likely with larger α_j.[5]

Lemma 1 also implies $\theta_W^I > \theta_S^I$ if $f_W - f_S > \alpha_S - \alpha_W$, but $\theta_W^I < \theta_S^I$ if $f_W - f_S < \alpha_S - \alpha_W$. When the coordination problem is very critical in S-region compared to W-region while heterogeneity in the fragilities is not large, the self-fulfilling panic could happen in the stronger region even if no panic would arise in the weaker.

Finally, incorporating concern about the spillovers from the other region (i.e., non-zero δ_j), we get:

Proposition 2.

(i) Suppose $\alpha_S - \alpha_W > f_W - f_S > \alpha_S - \alpha_W - \delta_{StoW}$. Both regions fail if and only if $\theta \leq \theta_S^I$ (i.e., $\theta_S^* = \theta_W^* = f_S + \alpha_S$).

(ii) Suppose $f_W - f_S < \alpha_S - \alpha_W - \delta_{StoW}$. There is no contagious failure and S-region fails if and only if $\theta \leq \theta_S^I$ and W-region fails if and only if $\theta \leq \theta_W^I + \delta_{StoW}$ (i.e., $\theta_S^* > \theta_W^*$).

(iii) Suppose $f_W - f_S > \alpha_S - \alpha_W > f_W - f_S - \delta_{WtoS}$. Both regions fail if and only if $\theta \leq \theta_W^I$ (i.e., $\theta_S^* = \theta_W^* = f_W + \alpha_W$).

(iv) Suppose $\alpha_S - \alpha_W < f_W - f_S - \delta_{WtoS}$. There is no contagious failure and S-region fails if and only if $\theta \leq \theta_S^I + \delta_{WtoS}$ and W-region fails if and only if $\theta \leq \theta_W^I$ (i.e., $\theta_S^* < \theta_W^*$).

Contagious failures arise in case (i) and (iii) with sufficiently strong spillovers (δ_{StoW}, δ_{WtoS} respectively). In case (ii) and (iv), the spillover is not strong enough to trigger contagious failure. An interesting result is case (i), where self-fulling panic causes contagious failures due to the potential spillover from S-region to the W-region. That is, a contagious panic spills over from the stronger region to the weaker when $\theta_W^I < \theta \leq \theta_S^I$; a panic arises in W-region which would not have been the case were it not for the potential spillover from S-region. This is when (i) the coordination problem is severer in S-region, (ii) the heterogeneity of fragilities between the two regions is relatively small, and (iii) the stronger region could impose significant spillover.

4. Discussion

Our results suggest several novel implications that the conventional domino-effect approach misses. Suppose that one simulates exogenous shocks (i.e., θ) to test financial system stability incorporating potential spillovers through interconnection (i.e., δ_j), e.g. interbank exposures. If δ_{WtoS} is small while δ_{StoW} is large, simulations adopting the domino-effect

[5]See Choi (2014) for more discussion. Sákovics and Steiner (2012) also study a global games model with heterogeneous payoff structure, but all agents in their model belong to the same "region" within our setup.

approach would not capture the possibility of contagious failures (Proposition 1, case (i)) while contagious failures could arise if the strategic risk leads to contagious panic (Proposition 2, case (i)). Conventional network analysis could underestimate systemic risk and the criticality of "systemically important" institutions.

Even when δ_{WtoS} is large, the two approaches suggest different implications on who could be more critical within the network. Suppose the parameters satisfy conditions for Proposition 1 (ii) and Proposition 2 (i). Contagious failures could arise in both approaches, but the failure threshold depends only on W-region's fragility in the first case, while only on S-region's in the second; Policy makers could enhance systemic stability more effectively (i.e. lowering the failure thresholds) by bolstering the weaker region (reducing f_W) in the first, but the stronger (reducing f_S) in the second.

We finally provide an application of these results. Consider two banks with the same illiquid risky assets and size. f_j could be interpreted as different leverage levels, i.e., fragilities to insolvency risk θ. Suppose that a bank liquidiates its assets upon failure, which causes capital loss of the other. All else equal, the less-capitalized bank would always fail first and the spillover should always arise from the weaker to the stronger. Now suppose the banks face maturity mismatch but the weaker is mostly funded by insured deposits[6] and its creditors care less about the consequences of its failure, while the stronger is funded by uninsured wholesale funding. This implies that α_S is large while α_W is small, thus S-bank is more exposed to the strategic (i.e., creditor run) risk. If the creditor panic in the stronger could cause the contagious panic in the weaker, as in Proposition 2 (i), it would be the better capitalized S-bank that would be systemically important and need to be bolstered to enhance systemic stability, when concerns about the panic are critical.

Appendix

Proof of Lemma 1

This could be solved from (1) by assuming $l_{-j} = 0$, which becomes

$$\int_{s_j^*-\epsilon}^{s_j^*+\epsilon} \triangle u_{i,j}\left(\frac{s_j^* - (\theta - \epsilon)}{2\epsilon}, 0, \theta\right) \cdot \frac{1}{2\epsilon} d\theta = 0, \tag{2}$$

we get $s_j^* = \alpha_j + f_j$. ■

Proof of Proposition 2

We prove case (i) with $\alpha_S - \alpha_W > f_W - f_S > \alpha_S - \alpha_W - \delta_{StoW}$. Proof of (ii) is straightforward since $\theta_S^I > \theta_W^I + \delta_{StoW}$ in this case and we could solve for θ_S^* and θ_W^* separately by assuming $\delta_S = 0$ and $\delta_W = \delta_{StoW}$.

[6] We could alternatively consider secured versus unsecured funding, or short-term versus long-term funding.

Denote the switching threshold for Lemma 1 as $\underline{s}_j^*(= \alpha_j + f_j)$, suppose agents adopt $s_S^* = \underline{s}_S^*$, and $s_W^* = \underline{s}_S^* + 2(\alpha_W - \alpha_S)\epsilon$. We verify these (s_S^*, s_W^*) indeed satisfy equation (1) for both $j = S, W$. Since there only exists a unique equilibrium (see Morris and Shin 2003), this would indeed be the only solution.

Note that given l_S, as long as $0 < l_W < 1$, l_W can be written as

$$l_W = l_S + \frac{s_W^* - s_S^*}{2\epsilon} = l_S + (\alpha_W - \alpha_S).$$

When $\alpha_S - \alpha_W > f_W - f_S > \alpha_S - \alpha_W - \delta_{StoW}$, the above equation implies

$$l_W + f_W + \delta_{StoW} > l_S + f_S > l_W + f_W.$$

Therefore, W-region doesn't fail unless S-region fails ($\because \theta > l_S + f_S$ implies $\theta > l_W + f_W$), but W-region always fails if S-region fails ($\because \theta < l_S + f_S$ implies $\theta < l_W + f_W + \delta_{StoW}$).

Thus, for W-region agents, equation (1) can be written as

$$1 \times Pr[l_S < \theta - f_S | s_{i,W} = s_W^*] + 0 \times Pr[l_S > \theta - f_S | s_{i,W} = s_W^*] = \alpha_W.$$

This indeed holds for our (s_S^*, s_W^*) because

$$\begin{aligned} Pr[l_S < \theta - f_S | s_{i,W} = s_W^*] &= Pr[l_S < \alpha_S | s_{i,W} = s_W^*] \\ &= Pr[l_W - (\alpha_W - \alpha_S) < \alpha_S | s_{i,W} = s_W^*] \\ &= Pr[l_W < \alpha_W | s_{i,W} = s_W^*] \\ &= \alpha_W \end{aligned}$$

where the first equality is from $s_W^* = s_S^* + 2(\alpha_W - \alpha_S)\epsilon \to \alpha_S + f_S$ with $\epsilon \to 0$, and the last equality is from $l_W \sim U[0,1]$ on $s_{i,W} = s_W^*$ (see Morris and Shin 2003). It is easy to verify (s_S^*, s_W^*) also satisfy S-region agent's problem since W doesn't fail unless W fails, so Lemma 1 applies in this case.

We can prove (iii) and (iv) analogously. ∎

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