

Volume 36, Issue 1

Aggregating trade distortions

Vlad Manole
Rutgers University

Will Martin
The World Bank

Abstract

We analyze the properties of tariff revenue and expenditure aggregators - tariff aggregators that keep expenditure, respectively tariff revenue constant and, in the case of the second aggregator, we correct the definition. Furthermore, under common assumptions in applied trade, we develop closed-form solutions for the expenditure and tariff revenue aggregators, which allows for practical applications of these aggregators in empirical trade. We also explore the relations between the trade-weighted average tariff, the expenditure aggregator and the revenue aggregator. We use these aggregators in a standard general equilibrium model for three countries to analyze the welfare gains from trade liberalization and compare the results.

The views expressed in the paper are those of the authors and should not be attributed to the World Bank or its Executive Directors.

Citation: Vlad Manole and Will Martin, (2016) "Aggregating trade distortions", *Economics Bulletin*, Volume 36, Issue 1, pages 320-328

Contact: Vlad Manole - vlad.manole@rutgers.edu, Will Martin - wmartin1@worldbank.org

Submitted: December 23, 2015. **Published:** March 17, 2016.

1. Introduction

An important challenge in applied international economics is selecting the appropriate method of aggregation of thousands of tariff lines from a country's tariff schedule to match the higher-level aggregation data available for production/consumption. An ad-hoc method of tariff aggregation, like trade-weighted average, may severely underestimate the global benefits of, for example, EU agricultural trade reform - Martin *et al.* (2003) show that these gains are 150 percent higher when appropriate tariff aggregates are used. New approaches, with rigorous theoretical foundations for the aggregation problem have emerged. Anderson and Neary (1993) proposed a Tariff Restrictiveness Index (TRI) uniform tariff that yields the same welfare as the original differentiated tariff structure. Similarly, Bach and Martin (2001) proposed two new tariff aggregators that keep expenditure, respectively tariff revenue constant, as the center of a compensating variation approach to consistent aggregation.

A number of recent papers illustrate the use of these TRI – types of tariff aggregators in the empirical field. Kee *et al.* (2004) calculate tariff aggregators of this type and use them to calculate the welfare losses associated with the existing tariff structures for 88 countries. They conclude that using either simple or weighted average tariffs underestimates the distortion imposed by trade barriers by 30 percent on average. Manole and Spatareanu (2010) computed annual estimates of TRI based solely on tariffs for 131 countries between 1990 and 2004 and found that less trade protection is associated with higher income per capita. Laborde *et al.* (2011) use a modified version of the LINKAGE3 global general equilibrium model to analyze welfare gains from complete liberalization of global trade barriers. The use of tariff revenue and expenditure aggregators instead of trade-weighted averages leads to a significant increase in welfare gains both for aggregate country groups and for most individual countries.

This paper improves upon Bach and Martin (2001) by deriving the properties of the proposed aggregators, which significantly increases the applicability of these aggregators in both empirical economics and policy simulation models. In particular, we show that in the case of their tariff revenue aggregator there may be *multiple* uniform tariffs that yield the same tariff revenue. To deal with this significant challenge we properly define the tariff revenue aggregator so that it leads to a *unique* solution. Furthermore, we derive key theoretical properties of these tariff aggregators, and under common assumptions in applied trade, we develop *closed-form* solutions for the expenditure and tariff revenue aggregators, which allows for practical applications of these aggregators in empirical trade. We also explore the relations between the trade-weighted average tariff, the expenditure aggregator and the revenue aggregator. We use these aggregators in a standard general equilibrium model for three countries to analyze the welfare gains from trade liberalization and compare the results.

2. Model

Bach and Martin (2001) assume that the structure of a competitive, small open economy can be captured by an income-expenditure condition,

$$e(\mathbf{p}, u) - r(\mathbf{p}, \mathbf{v}) - (\mathbf{e}_p - \mathbf{r}_p)'(\mathbf{p} - \mathbf{p}^w) - f = 0 \quad (1)$$

and the vector of behavioral equations¹,

$$e_p(\mathbf{p}, u) - r_p(\mathbf{p}, \mathbf{v}) = \mathbf{m} \quad (2)$$

where $e(\mathbf{p}, u)$ is the expenditure function of the representative household, \mathbf{p} is a given vector of domestic sectoral price aggregates, u is domestic utility, $r(\mathbf{p}, \mathbf{v})$ is domestic revenue from production, and \mathbf{v} is a vector of productive resources; \mathbf{m} is the vector of imports, and f is the exogenously-determined net financial inflow from abroad.

Based on equation (1) and considering prices \mathbf{p} and the level of utility u^0 as exogenous, the balance-of-trade function \mathbf{B} can be written as:

$$\mathbf{B}(\mathbf{p}, u^0) = e(\mathbf{p}, u^0) - r(\mathbf{p}, \mathbf{v}) - (\mathbf{e}_p - \mathbf{r}_p)(\mathbf{p} - \mathbf{p}^w) - f \quad (3)$$

In the rest of the paper we base our analysis on the assumption that the conditions proposed by Bach and Martin (2001) hold. In this framework, let e be the expenditure function, $e(\mathbf{p}_j, u_j^0)$, where \mathbf{p} is the vector of domestic price for goods and u^0 is the utility level associated with consumption of goods. The goods can be divided into domestically produced goods - with price \mathbf{p}^d , and imported goods - with domestic price \mathbf{p}^t , so the complete domestic vector price may be written as $\mathbf{p} = (\mathbf{p}^d, \mathbf{p}^t)$, where the disaggregated tariffs enter the definition via domestic prices of imported goods - \mathbf{p}^t .

Bach and Martin (2001) define the tariff aggregator for expenditure as the uniform tariff, τ^e , which requires the same level of expenditure on imported commodities as the observed vector of tariffs to maintain utility level u^0 :

$$\tau^e = [\tau^e \mid e(\mathbf{p}^d, \mathbf{p}^w(1+\tau^e), u^0) = e(\mathbf{p}^d, \mathbf{p}^t, u^0)] \quad (4)$$

A tariff revenue aggregator may be defined as the uniform tariff that yields the same tariff revenue as the observed vector of disaggregated tariffs, conditional on the utility level underlying the expenditure function and the resource endowments underlying the domestic revenue function (Bach and Martin, 2001):

$$\tau^R = [\tau^R \mid tr(\mathbf{p}^w(1+\tau^R), \mathbf{p}^w, u^0, v^0) = tr(\mathbf{p}^t, \mathbf{p}^w, u^0, v^0)] \quad (5)$$

3. Properties of the aggregators

The definitions given in equations (4) and (5) do not guarantee existence, uniqueness or economic meaning for the proposed tariff aggregators. In the rest of this paper we consider a Constant-Elasticity of Substitution (CES) functional form for the expenditure function and for the import demand functions. With this functional form, the expenditure function and the tariff revenue function are:

$$e = \left(\beta^d p^d + \sum_i \beta_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} u^0$$

$$tr = \sum_i \beta_i \left(\frac{p}{p_i} \right)^\sigma (p_i - p_i^w) u^0$$

¹ We use bold letters for vectors.

where $p = \left(\beta^d p^d + \sum_i \beta_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ is a price index, p^d is the price of the domestic good and p_i and p_i^w are the domestic and world prices of the import good i , the parameters $\beta^d, \beta_1, \beta_2, \dots$ are the expenditure shares (domestic and import). By appropriate selection of the units of measurement, all domestic prices are set equal to 1 in the base equilibrium and, in consequence, $p_i^w = 1/(1 + \tau_i)$, where τ_i is the *ad-valorem* tariff for good i . In this context, Bach and Martin (2001) showed that the expenditure tariff aggregator has the closed-form form:

$$\tau^e = \left(\frac{1 - \beta^d}{\sum_{i=1}^n \beta_i (1 + \tau_i)^{-(1-\sigma)}} \right)^{\frac{1}{1-\sigma}} - 1 \quad (6)$$

This solution guarantees the existence and uniqueness of the expenditure tariff aggregator. We prove that the expenditure aggregator is always positive when $\sigma > 1$, meeting this basic criterion for economic relevance.

Proposition 1. For $\sigma > 1$, τ^e is positive.

Proof: From (6) and with domestic prices set to 1 in the base equilibrium, world prices are $p_i^w = 1/(1 + \tau_i)$ and $\sum_{i=1}^n \beta_i (p_i^w)^{1-\sigma} = \sum_{i=1}^n \beta_i (1 + \tau_i)^{\sigma-1} > \sum_{i=1}^n \beta_i$ as long as there is at least one positive tariff. As the denominator is $1 - \beta^d = \sum_{i=1}^n \beta_i$, the ratio in parentheses in (6) is less than 1.

The ratio to a negative power is greater than one, so τ^e is positive.

The tariff revenue index, τ^R can be obtained by setting the tariff revenue function (5) equal to the corresponding expression with a uniform tariff, and solving for τ^R . This is similar to solving:

$$c = h(\tau^R) \quad (7)$$

where $h(\cdot; \mathbf{p}_j^w, u_j^0, v_j^0)$ is a real function of τ^R and c depends on disaggregated tariffs.

Proposition 2. For $\sigma > 1$, there are certain values of the parameter c for which equation (7) has at least two solutions.

Proof: We write an explicit form for equation (7):

$$\frac{\sum_{i=1}^n \beta_i p_i^w \tau_i}{\sum_{i=1}^n \beta_i (p_i^w)^{1-\sigma}} = \tau^R (1 + \tau^R)^{-\sigma} \left[\beta^d + (1 + \tau^R)^{1-\sigma} \sum_{i=1}^n \beta_i (p_i^w)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}} \quad (8)$$

We define function $h: R^+ \rightarrow R^+$, $h(t) = t(1+t)^{-\sigma} \left[\beta^d + (1+t)^{1-\sigma} \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}}$. The function h can be written as $h(t) = k(t) * m(t)$, with $k(t) = t(1+t)^{-\sigma}$ and

$$m(t) = \left[\beta^d + (1+t)^{1-\sigma} \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}}.$$

The function $k(t)$ starts from zero, increases until it reaches the maximum in $t = 1/(\sigma - 1)$ after which it decreases, converging asymptotically to zero.

The function $m(t)$ has the following properties:

1. $m(0) = \left[\beta^d + \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}}$ with $m(0) > 0$. Similar with the proof of Proposition 2, $\beta^d + \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} > 1$ so $m(0) < 1$.
2. $\lim_{t \rightarrow \infty} m(t) = (\beta^d)^{\frac{\sigma}{1-\sigma}}$. As $\beta^d < 1$, $\lim_{t \rightarrow \infty} m(t) > 1$.
3. $m'(t) = \frac{\sigma}{1-\sigma} \left[\beta^d + (1+t)^{1-\sigma} \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right]^{\frac{\sigma}{1-\sigma}-1} \sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} (1-\sigma)(1+t)^{-\sigma} > 0$, so $m(t)$ is an increasing function.

Note that the function $h(t)$ is a continuous and positive function with $h(0) = 0$, $\lim_{t \rightarrow \infty} h(t) = 0$, has a maximum on its domain and it reaches this maximum M . For any tariffs, imports and domestic consumption such that $c = \left(\sum_{i=1}^n \beta_i p_i^W \tau_i \right) / \left(\sum_{i=1}^n \beta_i (p_i^W)^{1-\sigma} \right) < M$ we may apply the intermediate value property and find at least two solutions.

Observation. The definition of the tariff revenue aggregator may be amended as follows: In any case where there are two feasible solutions for this aggregator, the tariff revenue aggregator may be defined as **the lowest valued** uniform tariff that will yield the same tariff revenue as the observed vector of disaggregated tariffs.

4. Separability between domestic and imported goods

With the additional assumption of separability between domestic and imported products – an assumption that is made routinely in computational general equilibrium (CGE) models – the aggregators can be applied only over foreign products. This specification requires the usual assumptions for such two-stage budgeting, such as weak separability in demand and homotheticity of the sub-utility functions at the lower level of nesting, but these assumptions are virtually universal in trade applications. Therefore, we can rewrite the expenditure function and the tariff revenue function:

$$e = \left(\sum_i \beta_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} u^0$$

$$tr = \sum_i \beta_i \left(\frac{p}{p_i} \right)^\sigma (p_i - p_i^w) u^0$$

where $p = \left(\sum_i \beta_i p_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ is a price index and the variables were described in section 3.

These assumptions (and functional forms) hold for the rest of the paper.

4.1 The Expenditure Tariff Aggregator

Based on the above assumptions, (6) leads to:

$$\tau^e = \left(\sum_{i=1}^n \beta_i (1 + \tau_i)^{\sigma-1} \right)^{\frac{1}{\sigma-1}} - 1 \quad (9)$$

4.2 The Tariff Revenue Aggregator

The tariff revenue aggregator, τ^R , is obtained by setting the left-hand side in (5) equal to the corresponding expression with a uniform tariff, and solving the equation for τ^R . Therefore, a tariff aggregator ranging over the imported goods alone (with $p_i = (1 + \tau^R)/(1 + \tau_i)$ and $p_i^w = 1/(1 + \tau_i)$) can be defined and the left-hand side of the equation (5) becomes:

$$\sum_i \beta_i \left(\frac{\left(\frac{\sum_j \beta_j}{(1+\tau_j)^{1-\sigma}} \right)^{\frac{1}{1-\sigma}} (1 + \tau^R)}{\frac{1 + \tau^R}{1 + \tau_i}} \right)^\sigma \left(\frac{1 + \tau^R}{1 + \tau_i} - \frac{1}{1 + \tau_i} \right) u^0 = \tau^R \left(\sum_i \beta_i (1 + \tau_i)^{\sigma-1} \right)^{\frac{1}{1-\sigma}} u^0 \quad (10)$$

Consequently, for the right-hand side of the equation (5) we have $p_i = 1$ and $p_i^w = 1/(1 + \tau_i)$, therefore the price index $p = 1$ and the right-hand side of the equation (5) is $\sum_{i=1}^n \beta_i \tau_i / (1 + \tau_i) u^0$.

From (10) and the above expression, we obtain a closed-form solution for τ^R :

$$\tau^R = \frac{\sum_{i=1}^n \beta_i \tau_i / (1 + \tau_i)}{\left(\sum_{j=1}^n \beta_j (1 + \tau_j)^{\sigma-1} \right)^{\frac{1}{1-\sigma}}} \quad (11)$$

where the β_j 's are value shares of imports at domestic prices ($\beta_j = M_j (1 + \tau_j) / \sum_k M_k (1 + \tau_k)$), where M_k is the value of imports of product k at border prices, and τ_k is the tariff on product k),

p_j^w is the world price for product j , where ($p_j^w = 1/(1 + \tau_j)$) because the domestic price is unity, and σ is the elasticity of substitution.

Lemma. Consider x_1, x_2, \dots, x_n n positive real numbers not all equal and w_1, w_2, \dots, w_n n positive real numbers such that $\sum_{i=1}^n w_i = 1$. For r a positive real number, we define the r -weighted

mean of the x_1, x_2, \dots, x_n numbers as $A_r = \left(\sum_{i=1}^n w_i x_i^r \right)^{\frac{1}{r}}$. For the given weights w_1, w_2, \dots, w_n the r -weighted mean of the x_1, x_2, \dots, x_n numbers has the following properties:

1. If $r > s$ then $A_r > A_s$.
2. $\lim_{r \rightarrow 0} A_r = x_1^{w_1} x_2^{w_2} \cdot \dots \cdot x_n^{w_n}$.

Proof: see Qi *et al.* (2000).

We use the notation t_{wa} for the weighted average tariff ($t_{wa} = \sum_{i=1}^n w_i \tau_i$), where $w_i = M_i / \sum_{k=1}^n M_k$, $i = 1, \dots, n$ are the weights.

Proposition 3. For $\sigma > 0$, $\tau^e \geq t_{wa}$.

Proof. From Lemma, we define $A_{1-\sigma} = \left(\sum_{i=1}^n \beta_i (p_i^w)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$ as the $1-\sigma$ weighted mean of world prices $p_1^w, p_2^w, \dots, p_n^w$ with the weights $\beta_1, \beta_2, \dots, \beta_n$. For the same tariff for all goods, $\tau^e = t_{wa}$. Otherwise, we use Lemma and, as $1-\sigma < 1$, then $A_{1-\sigma} < A_1$:

$$\left(\sum_{j=1}^n \beta_j (p_j^w)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} < \sum_{i=1}^n \beta_i p_i^w \quad (12)$$

We can write:

$$\sum_{i=1}^n \beta_i p_i^w = \sum_{i=1}^n \frac{M_i (1 + \tau_i)}{\sum_{k=1}^n M_k (1 + \tau_k)} \frac{1}{1 + \tau_i} = \sum_{i=1}^n \frac{M_i}{\sum_{k=1}^n M_k (1 + \tau_k)} = \frac{\sum_{i=1}^n M_i}{\sum_{k=1}^n M_k + \sum_{k=1}^n M_k \tau_k} = \frac{1}{1 + t_{wa}}$$

and (12) becomes:

$$\left(\sum_{j=1}^n \beta_j (p_j^w)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} < \frac{1}{1 + t_{wa}}$$

and further:

$$\tau^e = \left(\sum_{j=1}^n \beta_j (1 + \tau_j)^{\sigma-1} \right)^{\frac{1}{\sigma-1}} - 1 > t_{wa} \quad (13)$$

Proposition 4. For $\sigma > 0$, $\tau^e \geq \tau^R \geq t_{wa}$.

Proof. Using (9) and (11):

$$\tau^R = (\tau^e + 1) \sum_{i=1}^n \beta_i p_i^w \tau_i \quad (14)$$

Further:

$$\sum_{i=1}^n \beta_i p_i^w \tau_i = \sum_{i=1}^n \frac{M_i (1 + \tau_i)}{\sum_{k=1}^n M_k (1 + \tau_k)} \frac{\tau_i}{1 + \tau_i} = \sum_{i=1}^n \frac{M_i \tau_i}{\sum_{k=1}^n M_k (1 + \tau_k)} = \frac{\sum_{i=1}^n M_i \tau_i}{\sum_{k=1}^n M_k + \sum_{k=1}^n M_k \tau_k} = \frac{t_{wa}}{1 + t_{wa}}$$

Therefore:

$$\tau^R = (\tau^e + 1) \frac{t_{wa}}{1 + t_{wa}} \quad (15)$$

From Proposition 3:

$$\tau^e + 1 \geq 1 + t_{wa} \Rightarrow \tau^R = (\tau^e + 1) \frac{t_{wa}}{1 + t_{wa}} \geq t_{wa} \quad (16)$$

Again Proposition 3:

$$\tau^e \geq t_{wa} \Leftrightarrow 1 - \frac{1}{1 + \tau^e} \geq 1 - \frac{1}{1 + t_{wa}} \Leftrightarrow \frac{\tau^e}{1 + \tau^e} \geq \frac{t_{wa}}{1 + t_{wa}} \Leftrightarrow \tau^e \geq (\tau^e + 1) \frac{t_{wa}}{1 + t_{wa}} = \tau^R$$

From (16) and above:

$$\tau^e \geq \tau^R \geq t_{wa}$$

5. Application

To illustrate the differences between various aggregators for some real-world economies, we use a dual version of the model presented in de Melo and Tarr (1992). The model is known as the “1-2-3 model”, one country, two sectors and three goods. The two sectors produce an export good E , that is not consumed in the country, and a domestic good D (E and D are different), which is consumed only domestically. The third good in the model is the imported good M . The model uses the Armington assumption that domestically produced goods and imported goods are imperfect substitutes, and that weak separability and homotheticity of preferences at the lower level allow decision-making at the aggregate level to be based on prices and quantities of broad product groups—in our case, all domestic and imported goods. The aggregate production \bar{X} is fixed – an implicit assumption of full employment of all inputs. In the model the country is small, so world prices ($\bar{\pi}^m$ for imports and $\bar{\pi}^e$ for exports) are constant.

One of the advantages of the “1-2-3 model” is that the necessary data to run the model are readily available. While it is simple, lacking information on production structures and returns to individual factors present in conventional CGE models, this simplicity provides much greater transparency than is possible with a more complex model. We obtained macro-economic data

from the World Bank's World Development Indicators 2004. The three elasticity parameters needed in the model were:

- (i) The elasticity of substitution between domestic and imported goods, σ_m
 - (ii) The elasticity of transformation between domestic and exported goods in production, σ_T , and
 - (iii) The elasticity of substitution between different six-digit imported goods, σ_i
- For our analysis, we used $\sigma_m=1.5$, $\sigma_T= 1.5$, and $\sigma_i =6.0$.

The trade data were obtained from UNCTAD's TRAINS database and from the UN Statistical Division's COMTRADE database. We used WITS (World Bank data portal and software) to extract the trade data for Brazil, India and Venezuela.

The goal of this exercise is to compute and to compare the welfare gains from trade liberalization (complete elimination of all tariffs), considering three different approaches: (1) tariffs aggregated using the import-weighted average tariff; (2) tariffs aggregated using the expenditure tariff aggregator; (3) tariffs aggregated using the expenditure tariff aggregator but the balance of trade calculated using the expenditure aggregator for the expenditure function and a tariff revenue aggregator factor for tariff revenues, as suggested by Bach and Martin (2001).

Table I. Welfare effects after trade liberalization (% of GDP)

Countries	Tariff aggregators (%)			Welfare Effects after Trade Liberalization % of GDP		
	Trade Weighted Average	Tariff Revenue Aggregator	Expenditure Aggregator	Trade Weighted Aggregator	Expenditure Aggregator	Expenditure and Revenue Aggregator
Brazil	10	10.1	11.7	0.04	0.05	0.20
India	26.6	28.3	34.6	0.23	0.35	0.79
Venezuela	13.7	13.9	15.8	0.09	0.11	0.31

Table I presents the results. The empirical relation between the tariff aggregators is the one demonstrated in the theoretical section of this paper – that is, the trade weighted average is lower than tariff revenue aggregator and the latter is lower than the expenditure aggregator. For all countries, there is a difference of 2% or less between the trade-weighted average and the tariff-revenue aggregator. In contrast, the difference between the tariff-revenue and the expenditure aggregators is much bigger, from 2% in the case of Brazil, up to 8% in the case of India. The welfare effects vary considerably, depending on the type of aggregator used. Comparative with the trade-weighted average aggregator, the welfare gains are at least three times larger when we model trade liberalization using the third scenario (expenditure tariff aggregator corrected by a tariff revenue aggregator factor).

The results presented strongly suggest that aggregating correctly is very important and that the welfare gains from trade liberalization are severely underestimated when using the traditional tariff aggregator. This has very important policy implications, especially for developing countries, which frequently have high levels of protection

References

- Anderson, J. E., and J. P. Neary (1993) "A New Approach to Evaluating Trade Policy" Mimeo, Boston College
- Bach, C. F., and W. Martin (2001) "Would the Right Tariff Aggregator for Policy Analysis Please Stand Up?" *Journal of Policy Modeling*, **23**, 621- 635
- De Melo, J., and D. G. Tarr (1992) *A General Equilibrium Analysis of US Foreign Trade Policy*, Cambridge, Mass: MIT Press
- Kee, H.L., A. Nicita and M. Olarreaga (2004) "Import Demand Elasticities and Trade Distortions" World Bank Paper No. 3452
- Laborde, D., W. Martin, and D. van der Mensbrugge (2011) "Measuring the Impacts of Global Trade Reform with Optimal Aggregators of Distortions" World Bank Policy Research Working Paper 5665
- Manole, V. and M. Spatareanu (2010) "Trade Openness and Income—A Re-examination," *Economics Letters*, **106**, 1-3
- Martin, W., D. van der Mensbrugge, and V. Manole (2003) "Is the Devil in the Details?: Assessing the Welfare Implications of Agricultural and Non Agricultural Trade Reforms" presented at The 6th Annual Conference on Global Economic Analysis, June 12 - 14, Scheveningen, The Hague, The Netherlands
- Qi, F., Mei, J. Q., Xia, D. F., and S. L. Xu (2000) "New proofs of weighted power mean inequalities and monotonicity for generalized weighted mean values" *Mathematical Inequalities & Applications*, **3(3)**, 377-383