Economics Bulletin

Volume 36, Issue 1

Market Foreclosure and the Welfare Impacts of Price Discrimination

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Abstract

There is an extensive literature studying the welfare comparison of third-degree price discrimination vs. uniform pricing, typically under the assumption that all markets are served under uniform pricing. In this study, we allow market foreclosure and show that the welfare comparison of price discrimination vs. uniform pricing depends on whether market foreclosure is allowed. We also analyze how firms' foreclosure incentives vary with competition intensity. Our results show that an increase in competition intensity makes complete foreclosure less likely to be an equilibrium. On the other hand, the impact of competition intensity on partial foreclosure is non-monotonic. We also show that equilibrium under uniform pricing may feature strategic market foreclosure, defined as committing not to serve a market when demand there is positive.

I would like to thank the Associate Editor Parimal Bag, an anonymous referee, Yuk-fai Fong, Thomas Jeitschko and Catherine Mooney for helpful comments and suggestions. I would also like to thank Michael Salinger for his comments on another paper which inspired this study.
Citation: Qihong Liu, (2016) "Market Foreclosure and the Welfare Impacts of Price Discrimination", *Economics Bulletin*, Volume 36, Issue 1, pages 337-348
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Submitted: December 04, 2015. **Published:** March 17, 2016.

1 Introduction

Welfare impact of third-degree price discrimination is an extensively studied topic, whether under monopoly or competition.[1] In the case of monopoly, it is well understood that the monopolist may find it optimal to foreclose a weak market under uniform pricing. Consequently, price discrimination would expand market coverage and benefit the monopolist as well as the consumers. Similar logic has been extended to oligopolistic price discrimination. But little has been said on exactly when firms would have an incentive to foreclose a market and how this incentive varies with market conditions.[2] Instead, studies on third-degree price discrimination usually brush this problem aside by assuming that all markets are served under uniform pricing, and then derive conditions governing the welfare impacts of price discrimination.[3] Intuitively, these conditions do not necessarily hold if the assumption of complete market coverage is violated (See Section 4 for more details).

In this paper, we analyze firms' incentive for market foreclosure under uniform pricing. Two firms serve two markets (one strong and the other weak).[4] If a firm serves both markets, it has to charge a uniform price for the two markets (uniform pricing). When both markets are served by both firms (*complete coverage*), there is best-response symmetry in the sense that the two firms' strong markets coincide (Corts (1998)). However, a market may be served by one firm (*partial foreclosure*) or none (*complete foreclosure*), depending on firms' foreclosure decisions.[5]

We are interested in firms' foreclosure incentives and how they vary with competition intensity. We find that market foreclosure has two effects on the foreclosing firm's profit. The direct effect is the loss of sales from the foreclosed market. The indirect effect is, foreclosing a market gives firms an incentive to adjust their prices which in turn affects their profits from the other market. If the overall effect improves the foreclosing firm's profit, then the firm will have an incentive for market foreclosure. We find that *complete foreclosure* is less likely to be an equilibrium when competition intensity increases. Correspondingly, more intense competition raises consumer welfare through not only lower prices, but also potential market expansion. On the other hand, an increase in competition intensity makes *complete coverage* more likely to be an equilibrium when competition intensity is low but the result is opposite when competition intensity is high.

We then analyze strategic market foreclosure, defined as foreclosing a market when de-

[1]See, for example, Armstrong (2006) and Stole (2007) for surveys of this literature.

[2]There is an extensive literature studying market foreclosure in the presence of vertical integration.

[3]This is pointed out in Schmalensee (1981) footnote 1 where a few exceptions are also listed. In particular, Battalio and Ekelund (1972) graphically illustrates the possibility that a monopolist may choose not to serve one of two markets with linear demand. In this paper we consider duopoly and we show that a market may be foreclosed even when demand there is positive under the market price. That is, a firm has to commit not to serve that market.

[4]Robinson (1933) characterizes the two markets served by a monopolist as "strong" and "weak" markets in the sense that its discriminatory price is higher (lower) in the strong (weak) market. This characterization has been extended to imperfectly competitive markets.

[5]Consider the following example. A firm produces a drug which can cure disease A. It can also add certain ingredients (at little additional cost) to the drug to cure disease B. One can view the patients of disease A and B as market A and B respectively. Due to fixed offering costs (Evans and Salinger (2005)), the firm will offer only one drug. Without loss of generality, assume that this drug will always cure A and the question is whether it will cure B as well. If it won't, then the firm is committing to foreclose market B.

mand there is positive in the equilibrium.[6] This occurs when the weak market is neither too weak (otherwise it will have no positive demand) nor too strong (otherwise it will not be foreclosed). The intuition is the following. Foreclosing the weak market reduces competition and raises prices and profits for both firms in the strong market. If the profit increase from the strong market (reduced competition effect) more than offsets the loss of profit from the weak market (loss of sales effect), then a firm would have an incentive to foreclose the weak market even though demand there is positive. Note that strategic market foreclosure cannot occur in equilibrium. Without competition, there is reduced competition effect, leaving only the loss of sales effect.

2 The model

There are two markets: one strong (denoted by subscript s) and the other weak (w). A firm's demand depends on whether the market is strong/weak and on how many firms serve that market. The flexibility that a market can be served by 1 or 2 firms complicates our analysis. For tractability we consider linear demand functions with variable competition intensity. For the strong market, if served by both firms, then firm i's demand is given by[7]

$$p_i = 1 - q_{is} - bq_{js}, \quad i \neq j = 1, 2.$$

The parameter $b \in [0, 1]$ measures the intensity of competition.[8] In particular, when $b = 0$, the two products are not substitutes and there is no competition. The other extreme of $b = 1$ captures the case where the two products are perfect substitutes (homogeneous). We assume that the firms' products are imperfect substitutes so $b \in (0, 1)$.

Combining both firms' demand functions and solving for q_{is}, we can obtain

$$q_{is} = \frac{1}{1+b} - \frac{p_i}{1-b^2} + \frac{bp_j}{1-b^2}, \quad i \neq j = 1, 2.$$

Similarly, firm i's demand in the weak market is given by

$$p_i = a - q_{iw} - bq_{jw},$$

[6]This is different from the well understood market foreclosure under monopoly. The latter has been established in second-degree price discrimination models (e.g., Mussa and Rosen (1978)) or standard models of uniform pricing across two markets (e.g., Schmalensee (1981)). In these two settings, the monopolist may find it optimal to focus on the strong market, by charging a price so high that there is no positive demand in the weak market. In contrast, in our model, the foreclosed market may have positive demand but firms choose not to satisfy such demand.

[7]This is a linear demand system with two products. It can be generated from a representative consumer model with utility function $u_s = (q_{1s} + q_{2s}) - \frac{1}{2}(q_{1s}^2 + q_{2s}^2) - bq_{1s}q_{2s} + [I - (p_1 q_{1s} + p_2 q_{2s})]$, where I is the consumer's income and p_1, p_2 are the prices. If there is only one firm (say firm 1) in the market, then we need to substitute $q_{2s} = 0$ into the utility function. Utility maximization then leads to a demand function of $p_1 = 1 - q_{1s}$ which we will use later.

[8]b also measures the degree of product homogeneity, the opposite of product differentiation. Product differentiation is maximized (independent products) when $b = 0$ but minimized (homogeneous products) when $b = 1$. Other papers also use a single parameter to measure the degree of product differentiation as well as competition intensity. See, for example, the unit transport cost t in Dai, Liu and Serfes (2014). We do not consider the case of $b < 0$, i.e., when the two products are complements.

which leads to

$$q_{iw} = \frac{a}{1+b} - \frac{p_i}{1-b^2} + \frac{bp_j}{1-b^2}, \quad i \neq j = 1, 2.$$

We assume $a \in (0, 1]$ to be consistent with the notion that market w is the weak market. Both firms have constant marginal costs which we normalize to 0. We allow either market to be served by only one firm. In this case, demand become $p = 1 - q_s$ and $p = a - q_w$ in the strong market and weak market respectively.

We analyze the following two-stage game. In stage 1, firms make foreclosure decisions, or equivalently, decisions on which markets to serve. In stage 2, after observing each other's foreclosure decisions, firms make pricing decisions. From the managerial perspective, pricing decisions are made more frequently than foreclosure decisions, supporting a two-stage foreclosure-then-pricing game. Before a firm is able to serve a market, it needs to take certain actions which its rival can observe before the pricing stage starts. For example, the firm may need to open a store in that market which can easily be observed by its rival. In the case of drugs, the fact that a drug cures one or both diseases can clearly be seen by its rival.

3 Analysis

We solve the game backwards, starting with stage 2.

3.1 Stage 2: Pricing decisions

Let $(D_1 - D_2)$ denote firms' foreclosure decisions in stage 1 where $D_i \in \{N, W, S\}$ represents firm $i = 1, 2$'s decision of foreclosing no market (N), foreclosing the weak market (W) or foreclosing the strong market (S) respectively.[9] Each firm has 3 choices so there are a total of 9 subgames as listed below:

- $(N - N)$: no foreclosure
- $(W - N)$ and $(N - W)$: one firm forecloses the weak market
- $(S - N)$ and $(N - S)$: one firm forecloses the strong market
- $(W - W)$: both firms foreclose the weak market.
- $(S - W)$ and $(W - S)$: one firm forecloses the weak market while the other forecloses the strong market.
- $(S - S)$: both firms foreclose the strong market.

Next, we analyze these subgames.

Subgame 1: $(N - N)$

In this subgame, neither firm forecloses any market. Firm i's profit is given by

$$\pi_i = p_i(q_{iw} + q_{is}), \quad i = 1, 2.$$

[9] It is obvious that neither firm has an incentive to foreclose both markets.

Substituting the demand functions and solving FOCs, we can obtain

$$q_{iw} = \frac{3a + b - ab - 1}{2(1+b)(2-b)}, \quad i = 1, 2.$$

$$\pi_{N-N} = \frac{(a+1)^2(1-b)}{2(1+b)(2-b)^2}. \tag{1}$$

For the weak market to be actually served, we need

$$q_{iw} > 0 \Longleftrightarrow a > \frac{1-b}{3-b}.$$

Subgames 2 and 3: $(W-N)$ and $(N-W)$

Due to symmetry, we only consider $(W-N)$ where firm 1 forecloses the weak market. Firms' profits are given by

$$\pi_1 = p_1 q_{1s}, \quad \pi_2 = p_2(q_{2w} + q_{2s}).$$

Solving the first-order conditions, we obtain

$$q_{2w} = \frac{(3ab^2 - 6a + 2 - b - b^2)}{(5b^2 - 8)},$$

$$\pi_{1,W-N} = \frac{-(ab^2 - 2b^2 + ab + b + 4)^2(b-1)}{(1+b)(5b^2-8)^2}, \tag{2}$$

$$\pi_{2,W-N} = \frac{(b-1)(2ab + b + 2a + 2)^2(-2 + b^2)}{(1+b)(5b^2-8)^2}. \tag{3}$$

It can be shown that $q_{2w} > 0$ if and only if

$$a > \frac{(2+b)(1-b)}{3(2-b^2)}.$$

By symmetry, profits in the $(N-W)$ subgame are given by

$$\pi_{i,N-W} = \pi_{j,W-N}, \quad i \neq j = 1, 2.$$

Subgames 4 and 5: $(S-N)$ and $(N-S)$

Due to symmetry, we consider $(S-N)$ only. Firms' profits are given by

$$\pi_1 = p_1 q_{1w}, \quad \pi_2 = p_2(q_{2w} + q_{2s}).$$

Solving firms' first-order conditions, we can obtain

$$q_{1w} = \frac{2ab^2 - b^2 - ba - b - 4a}{(1+b)(5b^2-8)}, \quad q_{2w} = \frac{2b^3a - b^3 + 4ab^2 - b^2 + 2b - 3ba - 6a + 2)}{(1+b)(5b^2-8)},$$

$$\pi_{1,S-N} = \frac{(1-b)(-b^2+2ab^2-ba-b-4a)^2}{(1+b)(5b^2-8)^2}, \tag{4}$$

$$\pi_{2,S-N} = \frac{(1-b)(2b+ba+2a+2)^2(2-b^2)}{(1+b)(5b^2-8)^2}. \tag{5}$$

It can be shown that $\min\{q_{1w}, q_{2w}\} > 0$ if and only if

$$a > \frac{(-2+b^2)(1+b)}{2b^3+4b^2-3b-6}.$$

By symmetry, profits in the $(N-S)$ subgame are given by

$$\pi_{i,N-S} = \pi_{j,S-N}, \quad i \neq j = 1, 2.$$

Subgame 6: $(W-W)$

Now both firms foreclose the weak market. Firm i's profit becomes

$$\pi_i = p_i q_{is}, \quad i = 1, 2.$$

Solving first-order conditions, each firm earns a profit of

$$\pi_{W-W} = \frac{1-b}{(1+b)(2-b)^2}. \tag{6}$$

Subgames 7 and 8: $(S-W)$ and $(W-S)$

Due to symmetry we consider $(S-W)$ only where firm 1 forecloses the strong market and firm 2 forecloses the weak market. Note that each firm is a monopolist in one market. Straightforward calculation shows that

$$\pi_{1,S-W} = \frac{a^2}{4}, \quad \pi_{2,S-W} = \frac{1}{4}. \tag{7}$$

Subgame 9: $(S-S)$

Now both firms foreclose the strong market. Firm i's profit becomes

$$\pi_i = p_i q_{iw}, \quad i = 1, 2.$$

Solving the first-order conditions, each firm earns a profit of

$$\pi_{S-S} = \frac{a^2(1-b)}{(1+b)(2-b)^2}. \tag{8}$$

3.2 Stage 1: Foreclosure decisions

Having derived firms' profits for all subgames, we now move on to stage 1 and analyze firms' foreclosure decisions. The results are summarized in the next Proposition.

Proposition 1 *(i) $(N-N)$ is an equilibrium if and only if $a \geq a_{N-N}$ where*

$$a_{N-N} = \frac{-176b^2 + 128 + 8b^6 + 46b^4 - 28b^5 - 64b + 80b^3 + 2\sqrt{2}(-64 + 32b + 88b^2 - 44b^3 - 30b^4 + 15b^5)}{2(-4b^5 + 88b^2 - 64 - 31b^4 + 8b^3 + 2b^6)}. \tag{9}$$

Moreover, $a_{N-N} \leq 1$ if and only if $b \leq b_{N-N} \approx .912$ with b_{N-N} defined by

$$\pi_{N-N} = \pi_{1,W-N}\big|_{a=1,\ b=b_{N-N}}.$$

(ii) $(W-W)$ is an equilibrium if and only if $a \leq a_{W-W}$ where

$$a_{W-W} = \frac{-4b^4 + 24b^2 - 32 + 4\sqrt{-25b^6 + 130b^4 - 224b^2 + 128}}{2(-8 - 8b + 4b^2 + 4b^3)(-2 + b)}. \tag{10}$$

(iii) $(W-N)$ and $(N-W)$ are equilibria if and only if $a \in [a_{W-W}, a_{N-N}]$. Moreover, $a_{W-W} \leq a_{N-N}$ holds if and only if $b \geq b_{W-N} \approx 0.123$ with b_{W-N} defined by

$$a_{N-N} = a_{W-W}\big|_{b=b_{W-N}}.$$

(iv) $(S-W)$ and $(W-S)$ are equilibria if and only if

$$a \geq a_{S-W} = \frac{\left(16b^4 - 48b^2 + 32 + 4\sqrt{25b^8 - 155b^6 + 354b^4 - 352b^2 + 128}\right)(b+2)}{2(-32b^2 + 32b + 32 - 32b^3 + 9b^4 + 9b^5)}. \tag{11}$$

Moreover, $a_{S-W} \leq 1$ if and only if $b \geq b_{S-W} \approx 0.826$ where b_{S-W} is defined by

$$\pi_{1,S-W} = \pi_{1,N-W}\big|_{a=1,\ b=b_{S-W}}.$$

(v) $(S-N)$, $(N-S)$ and $(S-S)$ cannot be supported as part of a subgame perfect Nash equilibrium.

Proof. See Appendix. ■

The results in Proposition 1 are quite intuitive. Both firms will foreclose the weak market when it is weak ($a \leq a_{W-W}$) but not when it is strong ($a \geq a_{N-N}$). When the strength of the weak market is in the intermediate range ($a_{W-W} \leq a \leq a_{N-N}$), only one firm will foreclose the weak market ($W-N$ and $N-W$) so as to alleviate competition. A firm may also have an incentive to foreclose the strong market if it can be a monopolist in the weak market ($W-S$ and $S-W$). This occurs when competition intensity is high ($b \geq b_{S-W}$) and the weak market is not too weak ($a \geq a_{S-W}$). Foreclosing the strong market yet still facing competition in the weak market ($S-N$, $N-S$ and $S-S$) can never be optimal – the firm would be better off foreclosing the weak market instead.

3.3 Competition intensity and market foreclosure

Next, we investigate how firms' incentives for market foreclosure vary with competition intensity, focusing on symmetric subgames which can be supported as SPNE: $(N-N)$ and $(W-W)$.[10]

Consider a_{W-W} and a_{N-N}, the two threshold values of a for $(W-W)$ and $(N-N)$ to be an equilibrium respectively. In particular, $(W-W)$ can be supported as part of SPNE if and only if $a \leq a^{W-W}$, while $(N-N)$ can be an equilibrium if and only if $a \geq a_{N-N}$.

Both a_{W-W} and a_{N-N} are functions of b only, where b measures the intensity of competition. A natural question then is how a_{W-W} and a_{N-N} vary with b. The results are shown in Figure 1.

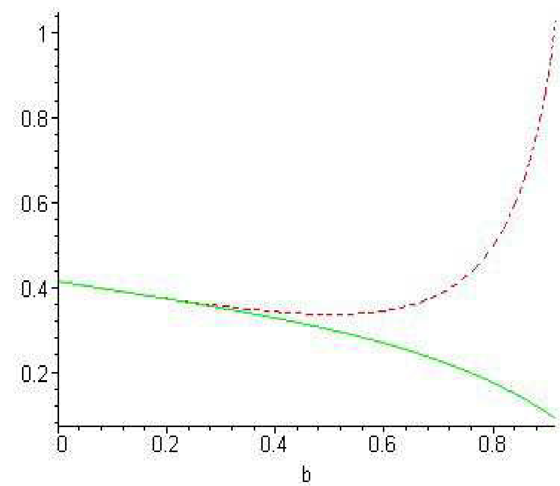

Figure 1: Solid: a_{W-W}; Dashed: a_{N-N}.

From Figure 1, we can see that a_{W-W} decreases with b, while a_{N-N} is U-shaped in b. Let us see why. First, consider $(W-W)$ where both firms foreclose the weak market so their profits come solely from the strong market. Due to symmetry, we only check firm 1's incentive to deviate. Suppose that firm 1 deviates and serves the weak market as well. This deviation has two effects on firm 1's profit. The direct effect is that it will earn monopoly profit from the weak market which is independent of b. The indirect effect is that deviation will also induce firms to adjust their prices which in turn will affect firm 1's competitive profit from the strong market. When b increases, competitive profit in the strong market decreases, thus becomes less important relative to monopoly profit in the weak market. This gives firm 1 more incentive to deviate, i.e., a_{W-W} decreases with b (recall that $(W-W)$ is an equilibrium when $a \leq a_{W-W}$).

How about $(N-N)$? A deviation, say by firm 1, also has a direct effect and an indirect effect which is similar to above. However, the direct effect now also decreases with b since

[10] While not considered, the asymmetric subgames ($S-W$ or $W-S$) are also relevant from the anti-trust perspective. These two subgames are in the same spirit as exclusive territory arrangements, and are more likely to occur when there is little product differentiation across the firms and the two markets (strong and weak) are similar.

what is lost from the weak market is competitive profit. Since both direct and indirect effects decrease with b, it is unclear how an increase in b affects how the two effects compare with each other. Our results suggest that when b is small, the threshold a_{N-N} decreases with b but the result is opposite when b is large.

3.4 Strategic market foreclosure

It is well known that under uniform pricing, a firm may choose price sufficiently high so as not to serve the weak market. Next, we take one step further and show that a firm may choose not serve a market even when its demand there is positive. That is, the firm commits not to serve the weak market. We call this *strategic market foreclosure*.

We will use subgame $(W-W)$ as an example and show that strategic market foreclosure can be an equilibrium feature.[11]

Fixing firms' prices at the equilibrium level under $(W-W)$ but allowing both firms to serve the weak market, we have

$$\begin{aligned} q_{1w} &= \frac{a}{1+b} - \frac{p_1}{1-b^2} + \frac{bp_2}{1-b^2} \\ &= \frac{a}{1+b} - \frac{1-b}{(1+b)(2-b)}, \end{aligned}$$

which is positive if and only if $a > \frac{1-b}{2-b}$.

Recall that when $a \leq a_{W-W}$, $(W-W)$ is an equilibrium. It can be verified that when b is sufficiently large, $a_{W-W} > \frac{1-b}{2-b}$ holds so the weak market has positive demand. Therefore, strategic market foreclosure is an equilibrium feature when $\frac{1-b}{2-b} < a \leq a_{W-W}$.

Note that strategic market foreclosure cannot happen in equilibrium under monopoly. The incentive to foreclose a market is to reduce competition in the other market. With no monopoly, there is no competition in any market.

4 Market foreclosure and the welfare impacts of price discrimination

In this section, we show that the welfare impacts of price discrimination depends on whether market foreclosure is allowed under uniform pricing. In particular, we illustrate that the impacts of price discrimination on profit can be the opposite when market foreclosure is allowed vs. when it is not allowed.

Holmes (1989) derives conditions under which price discrimination raises firms' profits, when market foreclosure is not allowed.[12] Next, we use similar setup but allow for market foreclosure. We construct an example with the following features: (i) Price discrimination raises profits if market foreclosure is not allowed; but lowers profit if market foreclosure is allowed, (ii) Equilibrium exhibits market foreclosure.

[11]It is easy to see that $(S-W)$ and $(W-S)$ also feature strategic market foreclosure. Consider the firm which serves only the weak market. Its demand in the strong market must be positive.

[12]Dastidar (2006) also characterizes similar sufficient conditions.

Suppose that firms' demand functions in market $j = s, w$ are given by

$$q_{1j} = 1 - a_j p_1 - b_j(p_1 - p_2), \quad q_{2j} = 1 - a_j p_2 - b_j(p_2 - p_1), \quad j = s, w,$$

from which we can obtain

$$p_1 = \frac{1}{a_j} - \frac{a_j + b_j}{a_j(a_j + 2b_j)} q_{1j} - \frac{b_j}{a_j(a_j + 2b_j)} q_{2j},$$

$$p_2 = \frac{1}{a_j} - \frac{b_j}{a_j(a_j + 2b_j)} q_{1j} - \frac{a_j + b_j}{a_j(a_j + 2b_j)} q_{2j}.$$

If a market is served by one firm only, say the strong market is served by firm 1 only, then we substitute $q_{2s} = 0$ into the p_1 expression,

$$p_1 = \frac{1}{a_s} - \frac{a_s + b_s}{a_s(a_s + 2b_s)} q_{1s} \Rightarrow q_{1s} = \frac{a_s + 2b_s}{a_s + b_s}(1 - a_s p_1).$$

This will be firm 1's demand function in the strong market.

We choose the following parameter values: $a_w = 1.5$, $a_s = 1$, $b_w = b_s = 10$. Under uniform pricing, in the absence of market foreclosure, each firm makes a profit of $\pi^{U1} \approx 0.1440$. Next, we allow market foreclosure, for example, firm 1 serves the strong market only and firm 2 serves the weak market only.[13] It is easy to show that they make profits of $\pi^{U2} \approx 0.4773$ and $\pi^{U3} \approx 0.3116$ respectively. Moreover, neither firm has an incentive to deviate. If firms price discriminate, it can be shown that both markets will be served and each firm will make a profit of $\pi^{PD} \approx 0.1444$. We can see that

$$\min\{\pi^{U2}, \pi^{U3}\} > \pi^{PD} > \pi^{U1}.$$

That is, price discrimination raises profits relative to uniform pricing if and only if market foreclosure is ruled out.[14]

5 Conclusion

In this paper, we analyze how firms' incentives for market foreclosure vary with competition intensity, which has important implications when investigating the welfare impacts of third-degree price discrimination. We illustrate that the welfare comparison of price discrimination

[13] If both firms foreclose the weak market under uniform pricing, then the uniform price must be the same as the discriminatory price in the strong market. In this case, price discrimination would raise firms' profits for sure.

[14] Existing studies, including Holmes (1989) and Dastidar(2006), characterize conditions for third-degree price discrimination to raise profits, for more general demand functions and under the implicit assumption of full market coverage. Our focus in this paper is not to replicate their analysis, but rather to analyze when market foreclosure will take place and how that varies with market structure and competition intensity. We also point out that their sufficient conditions is based on full market coverage. Once market foreclosure is allowed, such sufficient conditions may be insufficient anymore. In the above example, their sufficient conditions would be comparing π^{PD} and π^{U1}. But profits under uniform pricing can be π^{U2} and π^{U3}, when market foreclosure takes place.

vs. uniform pricing depends on whether market foreclosure is allowed. On foreclosure incentives, we find that an increase in competition intensity makes complete foreclosure less likely to be an equilibrium. On the other hand, an increase in competition intensity makes complete coverage more likely to be an equilibrium when competition intensity is low but the result is opposite when competition intensity is high. Firms may choose to foreclose a market even when it faces positive demand there, *strategic market foreclosure*. Firms choose not to satisfy the positive demand in anticipation that doing so leads to less intense competition in the other market. The profit increase from reduced competition can more than offset the lost profit from the foreclosed market.

Appendix

Proof of Proposition 1

(i) $(N-N)$. Due to symmetry, we only check firm 1's deviation. For firm 1 not to have an incentive to deviate, we need $\pi_{N-N} \geq \pi_{1,W-N}$ and $\pi_{N-N} \geq \pi_{1,S-N}$. Using the profit functions in section 3.1, it can be shown that $\pi_{N-N} \geq \pi_{1,W-N}$ holds if and only if

$$a \geq a_{N-N} =$$

$$\frac{-176b^2+128+8b^6+46b^4-28b^5-64b+80b^3+2\sqrt{2}(-64+32b+88b^2-44b^3-30b^4+15b^5)}{2(-4b^5+88b^2-64-31b^4+8b^3+2b^6)}.$$

It can be verified that (1) a_{N-N} increases with b; (2) when $a \geq a_{N-N}$, the weak market is indeed served and $\pi_{N-N} \geq \pi_{1,S-N}$.

(ii) $(W-W)$. Due to symmetry, we only check firm 1's deviation. For firm 1 not to have an incentive to deviate, we need

$$\pi_{W-W} \geq \max\{\pi_{1,N-W}, \pi_{1,S-W}\}.$$

$\pi_{W-W} \geq \pi_{1,N-W}$ if and only if

$$a \leq a_{W-W} = \frac{-4b^4+24b^2-32+4\sqrt{-25b^6+130b^4-224b^2+128}}{2(-8-8b+4b^2+4b^3)(-2+b)}.$$

It can be verified that $\pi_{W-W} \geq \pi_{1,S-W}$ always holds when $a \leq a_{W-W}$.

(iii) $(W-N)$. In this asymmetric subgame, we need to check both firms' deviations. For firm 1 not to deviate, we need

$$\pi_{1,W-N} \geq \max\{\pi_{N-N}, \pi_{1,S-N}\}.$$

This holds if and only if $a \leq a_{N-N}$.

For firm 2 not to deviate, we need

$$\pi_{2,W-N} \geq \max\{\pi_{W-W}, \pi_{2,W-S}\},$$

and this holds if and only if $a \geq a_{W-W}$. Moreover, it can be verified that $q_{2w} > 0$ when $a \geq a_{W-W}$.

We find that $a_{N-N} - a_{W-W}$ increases with b – negative (positive) when b is small (large).

(iv) $(S - W)$. For firm 1 not to deviate, we need

$$\pi_{1,S-W} \geq \max\{\pi_{W-W}, \pi_{1,N-W}\}.$$

This holds if and only if

$$a \geq a_{S-W} = \frac{1}{2}\frac{(16b^4 - 48b^2 + 32 + 4\sqrt{(25b^8 - 155b^6 + 354b^4 - 352b^2 + 128)}(b+2)}{(-32b^2 + 32b + 32 - 32b^3 + 9b^4 + 9b^5)}.$$

It can be verified that (1) when $a \geq a_{S-W}$, $\pi_{2,S-W} \geq \max\{\pi_{S-S}, \pi_{2,S-N}\}$ so firm 2 has no incentive to deviate either. (2) a_{S-W} decreases with b.

(v) $(S - S)$ is not an equilibrium. Firms would have an incentive to deviate to either $(S - W)$ or $(W - S)$. Similarly, in $(S - N)$ firm 1 always has an incentive to deviate to $(W - N)$.■

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