Economics Bulletin

Volume 36, Issue 1

Privatization neutrality theorem in a mixed oligopoly with firm asymmetry

Kojun Hamada
Faculty of Economics, Niigata University

Abstract

This paper revisits the privatization neutrality theorem, which states that welfare is exactly the same before and after privatization when the government optimally subsidizes firms. Contrary to the existing literature that shows that this theorem does not hold when there is an asymmetry between a public firm and private firms in a mixed oligopoly, we prove that this theorem holds by appropriately setting the discriminatory subsidies. We show that, even if there exist differences in cost and/or the timing of production among firms, adopting the discriminatory subsidy policy leads to welfare neutrality.

We would like to thank Minoru Kunizaki and Mitsuyoshi Yanagihara for valuable comments and suggestions. We also thank seminar participants of the 40th Nagoya International Economic Study Group (NIESG) held at Matsuyama University, and those at the 1st, 2nd, and 3rd seminars on the theory of mixed oligopoly held at Nagoya University for their helpful comments and suggestions. This study was supported by JSPS Grant-in-Aid for Scientific Research (C) No. 25380286. The author is solely responsible for any errors.
Citation: Kojun Hamada, (2016) "Privatization neutrality theorem in a mixed oligopoly with firm asymmetry", *Economics Bulletin*, Volume 36, Issue 1, pages 395-400
Contact: Kojun Hamada - khamada@econ.niigata-u.ac.jp.
Submitted: September 29, 2015. **Published:** March 17, 2016.

1. Introduction

The privatization neutrality theorem states that when the government gives optimal subsidies to both public and private firms in a mixed oligopoly, welfare is exactly the same before and after privatization. Since White (1996) first showed welfare neutrality existed before and after privatization, numerous articles have confirmed that the privatization neutrality theorem holds in various generalized situations (see Poyago-Theotoky 2001, Myles 2002, Tomaru and Saito 2010, and Tomaru 2006). On the other hand, it is well-known that firm asymmetry among public and private firms does not result in privatization neutrality. For example, Fjell and Heywood (2004) considered a sequential-move situation in which a public firm acts as a leader irrespective of privatization and they showed that the sequential move of firms does not result in privatization neutrality.[1]

However, following the setting of White (1996), almost all of the existing literature has examined only the optimal uniform subsidy to both the public and private firms. If the government recognizes firm heterogeneity, it will be quite natural to give the discriminatory subsidies to firms. By introducing the discriminatory subsidies into the model, this note investigates whether the privatization neutrality theorem holds even when there is firm asymmetry between public and private firms. Challenging the conventional wisdom that firm asymmetry does not imply welfare neutrality, we demonstrate that the privatization neutrality theorem holds even with firm asymmetry.

We demonstrate that if the government gives the optimal discriminatory subsidies to both public and private firms, the privatization neutrality result always holds even when there is firm asymmetry. In particular, we show that even if the cost of a public firm differs from those of private firms, privatization neutrality holds and welfare is maximized. In addition, we show that even if the public firm acts as a Stackelberg leader before and after privatization and there exist cost differences between firms, privatization neutrality is satisfied and welfare is maximized.

2. The model

We consider the oligopolistic market of a homogeneous good in which a public firm and n private firms engage in quantity competition. The public firm is indexed by firm 0 and each private firm by firm $i = \{1, 2, \cdots, n\}$. All n private firms except the public firm are identical.[2] The public firm before privatizing maximizes welfare and the privatized one maximizes its profit. Each private firm maximizes its profit. q_i denotes firm i's output. Because all private firms are identical, their outputs are also identical, i.e., $q = q_i,\ i = \{1, \cdots, n\}$. The inverse demand function is denoted by $p = p(Q)$, where Q is the total output and p is the price. We assume that $p' < 0$ and $p' + p''Q < 0$ for all $Q > 0$. Firm i's cost function is denoted by $C_i(q_i)$, assuming that $C_i(0) \geq 0$, $C_i'(q_i) > 0$, and $C_i''(q_i) > 0$. $C_0(q_0)$ and $C(q_i) \equiv C_i(q_i)$ denote the cost functions of the public firm and the private firms, respectively. s_i denotes the discriminatory specific subsidy given to firm i. Thus, s_0 denotes the public firm's subsidy and $s_i \equiv s$ private firm i's subsidy. Firm i's profit is $\pi_i = (p(Q) + s_i)q_i - C_i(q_i)$. Consumer surplus and producer surplus are

[1] Recently, in contrast to the conclusion by Fjell and Heywood (2004) that privatization is not welfare neutral, Matsumura and Okumura (2013) found that privatization neutrality holds when an output floor is introduced. However, an output floor regulation seems to be a bit artificial.

[2] The assumption that all private firms are identical is just for brevity. The similar result on welfare neutrality can be obtained even if private firms are not identical.

$CS \equiv \int_0^Q p(x)dx - p(Q)Q$ and $PS \equiv \pi_0 + n\pi_i$, respectively. Welfare is the sum of the consumer and producer surpluses, net of the subsidy, i.e., $W = \int_0^Q p(x)dx - \sum_{i=0}^n C_i(q_i)$.

The timing of the mixed oligopoly consists of a two- or three-stage game. In the first stage, the government determines the optimal discriminatory subsidy levels (s_0, s) to maximize welfare. Before entering the next stage, all firms observe (s_0, s). In the following stage(s), firms engage in a Cournot or a Stackelberg competition.

The first-order condition for welfare maximization yields the following equation:

$$p(q_0 + nq) = C_0'(q_0) = C'(q). \tag{1}$$

The optimal output levels maximizing the welfare, (q_0^*, q^*), satisfy (1). Marginal cost pricing is satisfied.[3]

3. Results

3.1 Cournot equilibrium

In the second stage before privatization, by solving the welfare maximization problem for the public firm and the profit maximization problem for private firms, we confirm that the Cournot equilibrium outputs before privatization, (q_0^{CB}, q^{CB}), satisfy the following first-order conditions:

$$p(q_0^{CB} + nq^{CB}) = C_0'(q_0^{CB}), \tag{2}$$

$$p(q_0^{CB} + nq^{CB}) + s + p'(q_0^{CB} + nq^{CB})q^{CB} = C'(q^{CB}). \tag{3}$$

Note that (q_0^{CB}, q^{CB}) does not depend on s_0. To obtain the optimal outputs (q_0^*, q^*), suppose that the government gives the following subsidy to private firms:

$$s_B^* = -p'(q_0^* + nq^*)q^*. \tag{4}$$

If s_B^* is given to private firms, (q_0^{CB}, q^{CB}) is equal to (q_0^*, q^*). Thus, subsidizing s_B^* to private firms maximizes welfare.

By solving the profit maximization problem for a privatized public firm and for private firms, respectively, we obtain that the Cournot equilibrium outputs after privatization, (q_0^{CA}, q^{CA}), satisfy the following first-order conditions:

$$p(q_0^{CA} + nq^{CA}) + s_0 + p'(q_0^{CA} + nq^{CA})q_0^{CA} = C_0'(q_0^{CA}), \tag{5}$$

$$p(q_0^{CA} + nq^{CA}) + s + p'(q_0^{CA} + nq^{CA})q^{CA} = C'(q^{CA}). \tag{6}$$

Note that (q_0^{CA}, q^{CA}) depends on s_0. To obtain (q_0^*, q^*), suppose that the government gives the following subsidies to the firms:

$$s_{0A}^{C*} = -p'(q_0^* + nq^*)q_0^*, \tag{7}$$

$$s_A^* = -p'(q_0^* + nq^*)q^*. \tag{8}$$

[3] It should be noted that if there is a cost difference between the public firm and private firms when the marginal cost of firm i is constant, i.e., $C_i'' = 0$, (1) is never satisfied. Therefore, when the marginal cost is constant, the firm with the higher marginal cost withdraws from producing. Throughout the paper, we assume that the cost function is strictly convex, i.e., $C_i'' > 0$. This assumption guarantees that there uniquely exist output levels (q_0^*, q^*).

If s_{0A}^{C*} and s_A^* are given to the public firm and the private firms, respectively, (q_0^{CA}, q^{CA}) is equal to (q_0^*, q^*). Thus, subsidizing s_{0A}^{C*} and s_A^* to the public firm and the private firms, respectively, achieves welfare maximization. From (7) and (8), the firm that produces higher output receives more subsidy, i.e., $s_{0A}^{C*} \gtrless s_A^*$ if and only if $q_0^* \gtrless q^*$. As marginal cost pricing always holds, the low-cost firm can produce more than the high-cost firm. It implies that under the discriminatory subsidy policy, the government should give more subsidy to the low-cost firm than to the high-cost firm.

The above result is summarized in the following proposition.

Proposition 1. *Suppose that the government gives the optimal discriminatory subsidies to the public and the private firms that have different costs. The private firms' subsidies, the public firm's output, the private firms' outputs, and the welfare are the same before and after privatization.*

Proposition 1 implies that if the government optimally sets the discriminatory subsidies to the public and the private firms that have different costs, privatization neutrality holds. Unlike the existing literature focusing only on the uniform subsidy, this result suggests that the economic situation in which privatization neutrality holds is enlarged if the government can implement the discriminatory subsidy policy.

3.2 Stackelberg equilibrium

We now show that even if there exist differences among firms, not only in costs but also in the timing of production, the adoption of the discriminatory subsidy policy is sufficient to ensure privatization neutrality.

In the third stage, each private firm as a follower maximizes its profit given the public firm's output level. In the second stage before privatization, the public firm determines its output level as follows:

$$\frac{\partial W}{\partial q_0} + n\frac{\partial W}{\partial q}r'(q_0) = p(Q) - C_0'(q_0) + n(p(Q) - C'(q))r'(q_0) = 0, \tag{9}$$

where $q = r(q_0)$ denotes a private firm's reaction function. The Stackelberg equilibrium outputs before privatization, (q_0^{SB}, q^{SB}), satisfy the following equations:

$$p(q_0^{SB} + nq^{SB}) - C_0'(q_0^{SB}) + n(p(q_0^{SB} + nq^{SB}) - C'(q^{SB}))r'(q_0^{SB}) = 0, \tag{10}$$

$$p(q_0^{SB} + nq^{SB}) + s + p'(q_0^{SB} + nq^{SB})q^{SB} = C'(q^{SB}), \tag{11}$$

where (q_0^{SB}, q^{SB}) does not depend on s_0. To obtain (q_0^*, q^*), suppose that the government gives the following subsidy to private firms:

$$s_B^* = -p'(q_0^* + nq^*)q^*. \tag{12}$$

If s_B^* is given to private firms, (q_0^{SB}, q^{SB}) is equal to (q_0^*, q^*). Thus, subsidizing s_B^* to private firms achieves welfare maximization.

On the other hand, in the second stage after privatization, the privatized firm determines its output as follows:

$$\frac{\partial \pi_0}{\partial q_0} + n\frac{\partial \pi_0}{\partial q}r'(q_0) = p(Q) + s_0 + p'(Q)q_0 - C_0'(q_0) + np'(Q)q_0r'(q_0) = 0. \tag{13}$$

The Stackelberg equilibrium outputs after privatization, (q_0^{SA}, q^{SA}), satisfy the following equations:

$$p(q_0^{SA} + nq^{SA}) + s_0 + p'(q_0^{SA} + nq^{SA})q_0^{SA}\big(1 + nr'(q_0^{SA})\big) = C_0'(q_0^{SA}), \tag{14}$$

$$p(q_0^{SA} + nq^{SA}) + s + p'(q_0^{SA} + nq^{SA})q^{SA} = C'(q^{SA}). \tag{15}$$

In contrast to the case before privatization, (q_0^{SA}, q^{SA}) depends on s_0. To obtain (q_0^*, q^*), suppose that the government gives the following subsidies to the public and private firms, respectively:

$$s_{0A}^{S*} = -p'(q_0^* + nq^*)q_0^*\big(1 + nr'(q_0^*)\big), \tag{16}$$

$$s_A^* = -p'(q_0^* + nq^*)q^*. \tag{17}$$

If s_{0A}^{S*} and s_A^* are given to the public firm and the private firms, respectively, (q_0^{SA}, q^{SA}) is equal to (q_0^*, q^*). Thus, subsidizing s_{0A}^{S*} and s_A^* to the public firm and the private firms, respectively, achieves the maximized welfare. From (16) and (17), $s_{0A}^{S*} \gtrless s_A^*$ if and only if $q_0^*(1 + nr') \gtrless q^*$. As $r' < 0$, only if q_0^* is sufficiently larger than q^*, the public firm receives more subsidy than the private firms. In a Stackelberg equilibrium, the public firm as a Stackelberg leader is handicapped in the sense that even if the cost condition is identical between a public firm and private firms, the public firm receives less subsidy than private firms. The intuitive reason is that since the public firm with the first-mover advantage produces more output than private firms, the government needs to restrict its output by reducing the subsidy.

The above results are summarized in the following proposition.

Proposition 2. *Suppose that the government gives the optimal discriminatory subsidies to the public and the private firms that have different costs when the public firm acts as a Stackelberg leader. The private firms' subsidies, the public firm's output, the private firms' outputs, and the welfare are the same before and after privatization.*

Proposition 2 implies that if the government optimally sets the discriminatory subsidies to both the public and the private firms, privatization neutrality holds even when there exist differences in both costs and the timing of production. Even if there exists firm asymmetry, the discriminatory subsidies appropriately adjust the firms' outputs and implement the maximized welfare. Our result demonstrates for the first time that irrespective of the difference in the timing of outputs between the public and the private firms, privatization neutrality always holds with a discriminatory subsidy policy.

Finally, comparing the optimal subsidy level of the privatized public firm in Cournot equilibrium with that in Stackelberg equilibrium, we obtain the following proposition.

Proposition 3. *The privatized public firm's subsidy in Stackelberg equilibrium is less than that in Cournot equilibrium.*

Proof. $s_{0A}^{C*} \equiv -p'(q_0^*+nq^*)q_0^* > s_{0A}^{S*} \equiv -p'(q_0^*+nq^*)q_0^*\big(1+nr'(q_0^*)\big)$ if and only if $r'(q_0^*) < 0$. By the definition of $q = r(q_0)$, $p(q_0+nr(q_0))+s+p'(q_0+nr(q_0))r(q_0)-C'(r(q_0)) = 0$. By totally differentiating this equation with respect to q_0, we obtain $r' = -(p'+p''q)/(n(p'+p''q)+p'-C'') < 0$. Its negative slope determines the size relationship between s_{0A}^{C*} and s_{0A}^{S*}. □

The privatized public firm that behaves as a Stackelberg leader loses a part of the first-mover advantage by its discriminatory subsidy being reduced and, as a result, the output levels are always adjusted to the same level as the optimal ones in both Cournot and Stackelberg equilibria.

4. Conclusion

This note revisits the privatization neutrality theorem and demonstrates that if the different subsidy rates are adopted even when there is firm asymmetry between public and private firms, this theorem holds. Even when the cost of a public firm differs from those of private firms, privatization is welfare neutral by using the discriminatory subsidy policy. Furthermore, we show that even when a public firm acts as a Stackelberg leader before and after privatization, the government can maximize welfare and, as a result, privatization neutrality holds.

In this note, we assume for brevity that all private firms are identical with regard to the cost and the timing of production. We can easily extend our results to the situation in which there are not identical private firms. This is because if the government can subsidize the individual private firms in a differentiated manner, they can appropriately control the output level of private firms through the discriminatory subsidy to achieve the first-best result. As another simplification, we assume that there is no subsidy distortion. In a more realistic situation, subsidization can cause the distortion on the income distribution and reduce the social welfare. If subsidy is distortionary, it might be possible that the privatization neutrality theorem no longer holds. It is a future challenge to investigate what happens when subsidy is distortionary.

References

[1] Fjell, K. and J.S. Heywood (2004) “Mixed Oligopoly, Subsidization and the Order of Firm’s Moves: The Relevance of Privatization” *Economics Letters* **83**, 411-416.

[2] Matsumura, T. and Y. Okumura (2013) “Privatization Neutrality Theorem Revisited” *Economics Letters* **118**, 324-326.

[3] Myles, G.D. (2002) “Mixed Oligopoly, Subsidization and the Order of Firms’ Moves: An Irrelevance Result for the General Case” *Economics Bulletin* **12**, 1-6.

[4] Poyago-Theotoky, J. (2001) “Mixed Oligopoly, Subsidization and the Order of Firms’ Moves: An Irrelevance Result” *Economics Bulletin* **12**, 1-5.

[5] Tomaru, Y. (2006) “Mixed Oligopoly, Partial Privatization and Subsidization” *Economics Bulletin* **12**, 1-6.

[6] Tomaru, Y. and M. Saito (2010) “Mixed Duopoly, Privatization and Subsidization in an Endogenous Timing Framework” *Manchester School* **78**, 41-59.

[7] White, M.D. (1996) “Mixed Oligopoly, Privatization and Subsidization” *Economics Letters* **53**, 189-195.