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Evolving assortativity and social conventions

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Abstract

This paper considers a stochastic evolutionary model in which agents can vote for different policies through majority voting that induce different assortative matching patterns. We find that in coordination games, agents either vote for complete segregation policy or maximal integration policy. The Pareto-dominant social convention is always stochastically stable and it is possible that both risk-dominant and Pareto-dominant social conventions are stochastically stable. This contrasts to risk-dominant social convention being uniquely stochastically stable predicted in conventional stochastic stability analysis where matching is uniformly random.

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1. Introduction

The literature of stochastic stability analysis provides powerful tools for studying the emergence of social conventions (for example, Foster and Young 1990, Young 1993, and Kandori *et al*. 1993, among many others). One of the most remarkable findings in this literature is the selection of risk-dominant social convention over Pareto-dominant social convention in coordination games, which are commonly used for modeling the coordination on languages, forms of money and credits, industrial and technological standards, accounting rules, business contracts and etc.

Most models in this literature are of decentralized nature: social conventions evolve through a process of individual trial and error, experimentation and adaptation. However, in most modern societies, people also engage in collective decision making processes. Therefore, it is of interest to understand how such processes influence the emergence of social conventions. To our limited knowledge, not many works contribute to this research direction.

Majority voting is one of the most common forms of collective decision making processes. In this paper, we study how majority voting would affect the emergence of social conventions in coordination games. More specifically, agents in a population vote for different policies that induce different assortative matching patterns. We consider a 2×2 coordination game in which one strategy is risk-dominant while the other is Pareto-dominant. There are two possible scenarios: 1) All agents prefer to match with those who play the same strategies. In this case, they always vote for complete segregation policy and Pareto-dominant social convention is uniquely stochastically stable. 2) All agents prefer to match with agents playing Pareto-dominant strategy. In this case, if agents playing risk-dominant strategy are the majority, they vote for maximal integration policy, if agents playing Pareto-dominant strategy are the majority, they vote for complete segregation policy. Then both risk-dominant and Pareto-dominant social conventions are stochastically stable. Our findings sharply contrasts to the conventional finding in the literature of stochastic stability that risk-dominant social convention is uniquely stochastic stable when matching is uniformly random. This demonstrates that collective decision making processes such as majority rule have non-negligible impacts on the emergence of social conventions.

Assortative matching has long been considered in evolutionary game theory. The first assortative mechanisms date back to Wright (1921, 1922). Well-known examples include kin selection (Hamilton 1964a,b; Bergstrom 1995), spatial interactions (Nowak and May 1992) and homophily in preferences (Alger and Weibull 2012, 2013). Bergstrom (2003, 2013) propose a general framework for studying assortativity. However, most of these works consider exogenous assortative levels and they mainly apply assortative matching to study the phenomenon of cooperation in social dilemmas instead of the emergence of social conventions in coordination games. The closest paper to our work is Nax and Rigos (2016) who consider a model in which assortative matching is endogenously determined by democratic consensus and agents interact in social dilemmas. Izquierdo *et al*. (2014) allows agents to voluntarily leave their matches in prisoner dilemmas, Newton (2016) also consider endogenous assortative matching but in a preference evolution context.

The paper is organized as follows. Section 2 delivers the model. Section 3 provides the stochastic stability analysis. Section 4 concludes.

2. The Model

Consider a population consisting of $n < \infty$ identical agents who are randomly matched to play a stage game in each period $t = \{1, 2, \ldots\}$, where n is an even number. The stage game is a 2×2 symmetric coordination game with strategy space $S = \{0, 1\}$. Let $x \in X \equiv \{0, \frac{1}{n}, \frac{2}{n}, \ldots, 1\}$ denote the fraction of players playing 1. The coordination game is illustrated in table 1.

Table 1: A coordination game

		2	
		0	1
1	0	a, a	b, c
	1	c, b	d, d

The payoffs satisfy $a > c$ and $d > b$. $(0,0)$ and $(1,1)$ are two symmetric Nash equilibria.

The matching process is assortative. Let the population size n be sufficiently large. Let $\Pr[i|j,x]$ denote the probability that an agent playing j is matched with an agent playing i, where $i, j \in \{0, 1\}$. Let $\theta(x)$ be the index of assortativity of the matching, such that

$$\theta(x) = \Pr[0|0,x] - \Pr[0|1,x]. \tag{1}$$

In other words, $\theta(x)$ is the difference between the probability that an agent playing 0 is matched with an agent playing 0 and the probability that an agent playing 1 is matched with an agent playing 0.

When $x \in [0, \frac{1}{2}]$, the range of $\theta(x)$ is $[\frac{1-2x}{1-x} - 1, 1]$; when $x \in (\frac{1}{2}, 1]$, the range of $\theta(x)$ is $[-\frac{1-x}{x}, 1]$. Notice that $\theta(x) = 0$ implies uniformly random matching; $\theta(x) = 1$ implies that the agents are completely segregated according to the strategies they are playing. When $\theta(x)$ takes the lowest value in its range, agents are most integrated in the matching. For example, suppose $x < \frac{1}{2}$, and $\theta(x) = \frac{1-2x}{1-x} - 1$, then all agents playing strategy 1 are matched with agents playing strategy 0.

The index of assortativity $\theta(x)$ was first introduced by Bergstrom (2003, 2013) to study strategy evolution and later adopted by Alger and Weibull (2012, 2013) to study preference evolution. It is endogenized in Izquierdo *et al.* (2014), Nax and Rigos (2016) and Newton (2016) in ways different from our approach.

The following balancing condition ensures that the expected number of agents playing 0 matched with agents playing 1 equals the expected number of agents playing 1 matched with agents playing 0,

$$(1-x)(1 - \Pr[0|0,x]) = x\Pr[0|1,x]. \tag{2}$$

From (1) and (2), we get

$$\Pr[0|0,x] = (1-x)(1-\theta(x)) + \theta(x); \tag{3}$$

$$\Pr[0|1,x] = (1-x)(1-\theta(x)); \tag{4}$$

and $\Pr[1|0,x] = 1 - \Pr[0|0,x]$, $\Pr[1|1,x] = 1 - \Pr[0|1,x]$.

Given the probabilities of matching, the payoffs corresponding to the two strategies are given by

$$\pi_0(x) = \Pr[0|0,x]a + \Pr[1|0,x]b, \tag{5}$$

$$\pi_1(x) = \Pr[0|1,x]c + \Pr[1|1,x]d. \tag{6}$$

The index of assortativity $\theta(x)$ is determined by majority voting. When $x < \frac{1}{2}$, agents playing 0 are the majority, so they decide the assortativity level. When $x > \frac{1}{2}$, agents playing 1 are the majority, so they decide the assortativity level. When $x = \frac{1}{2}$, both types of agents have 50% chance to determine the assortativity level. In the case that the majority is indifferent among any assortativity level, the assortativity level is determined by the minority's preferences. We call $\theta(x) = 1$ the complete segregation policy. When $\theta(x)$ takes the lowest value in its range, we call it the maximal integration policy.

The timing of the dynamic process is as follows. In each period, first an agent is chosen at random to have a strategy revision opportunity.[1] Second, agents vote for the index of assortativity through majority voting. Finally, agents are simultaneously rematched according to the index of assortativity.

When a strategy revision opportunity arrives, an agent employs the following noise best response revision protocol:

$$\rho_{01}(x) = \sigma^\eta(\pi_1(x) - \pi_0(x)), \tag{7}$$

$$\rho_{10}(x) = \sigma^\eta(\pi_0(x) - \pi_1(x)), \tag{8}$$

where $\eta > 0$ denotes the noise level and $\sigma^\eta : \mathbb{R} \to [0,1]$ satisfies

$$\lim_{\eta \to 0} \sigma^\eta(k) = \begin{cases} 1 & \text{if } k > 0, \\ 0 & \text{if } k \leq 0. \end{cases} \tag{9}$$

If an agent playing strategy 0 receives a revising opportunity, she switches to strategy 1 with probability $\rho_{01}(x)$. If an agent playing strategy 1 receives a revising opportunity, she switches to strategy 0 with probability $\rho_{10}(x)$.

In particular, we adopt the widely used protocol from Kandori *et al.* (1993), best response with mutations (BRM) protocol, which is defined by

$$\sigma^\eta(k) = \begin{cases} 1 - \exp(-\eta^{-1}) & \text{if } k > 0, \\ \exp(-\eta^{-1}) & \text{if } k \leq 0. \end{cases} \tag{10}$$

[1] Blume (2003) argues that such a discrete time dynamic process is equivalent to a continuous time one in which agents receive strategy revision opportunities according to some independent Poisson alarm clock. Alternatively, one can assume that at the beginning of each period, each agent is awarded a strategy revision opportunity with some fixed probability independent of the situation of all other agents as in Kandori *et al.* (1993). Our main results remain unchanged if we adopt this alternative model specification.

This dynamic model defines a discrete time Markov process on the discrete state space X. The transition probabilities are given by

$$P^n_{xy} = \begin{cases} (1-x)\rho_{01}(x) & \text{if } y = x + \frac{1}{n}, \\ x\rho_{10}(x) & \text{if } y = x - \frac{1}{n}, \\ 1-(1-x)\rho_{01}(x) - x\rho_{10}(x) & \text{if } y = x, \\ 0 & \text{otherwise.} \end{cases} \tag{11}$$

We focus on the stationary distribution μ of the Markov process when the noise level approaches zero and the population size approaches infinity. A state $x \in [0,1]$ is ***stochastically stable*** in double limit if for every open set $O \subseteq X$ containing x,

$$\lim_{n\to\infty} \lim_{\eta\to 0} \mu(O) > 0. \tag{12}$$

See Sandholm (2010) for a textbook treatment of the related concepts.

3. Analysis

Without loss of generality, let $a+b > c+d$ and $a < d$. Hence, 0 is the risk-dominant strategy and 1 is the Pareto-dominant strategy (Harsanyi and Selten (1988)) and we call the all-0 state ($x = 0$) the risk-dominant social convention and all-1 state ($x = 1$) the Pareto-dominant social convention. $a > c$ together with $d > a$ implies that $d > c$. Hence, agents playing 1 always prefer to match with agents playing 1. We have the following main results of the paper:

Proposition 1. *If $a \geq b$, then Pareto-dominant social convention is uniquely stochastically stable.*

Proof: When $a > b$, agents playing 0 always prefer to match with agents playing 0. Therefore, regardless of which type of agents constitute the majority, $\theta(x) = 1$ for any $x \in [0,1]$. When $a = b$, agents playing 0 are indifferent among any assortativity level. Hence, we still have $\theta(x) = 1$ for any $x \in [0,1]$. The payoffs corresponding to the two strategies satisfy $\pi_0(x) = a < \pi_1(x) = d$. 1 is always the best response and all-1 state is thus uniquely stochastically stable. *Q.E.D.*

Proposition 1 portrays a scenario in which agents with different strategies have incentive to use their political power to segregate themselves from others whenever they are the majority. Given segregation, because strategy 1 is the Pareto-dominant strategy, eventually, all-1 state is uniquely stochastically stable. This result sharply contrasts to Young (1993) and Kandori *et al*. (1993), who support the prediction that the risk-dominant equilibrium is uniquely stochastically stable.

Proposition 2. *If $a < b$, then both the risk-dominant and the Pareto-dominant social conventions are stochastically stable.*

Proof: When $a < b$, agents playing 0 always prefer to match with agents playing 1. Therefore, when $x < \frac{1}{2}$, they vote for $\theta(x) = \frac{1-2x}{1-x} - 1$. The payoffs of the two strategies are given by

$$\pi_0(x) = \frac{1-2x}{1-x}a + \frac{x}{1-x}b, \text{ and } \pi_1(x) = c.$$

Since $c < a$ and $a < b$, we must have $\pi_0(x) > \pi_1(x)$ and 0 is the best response.

On the other hand, when $x > \frac{1}{2}$, agents playing 1 are the majority and they vote for $\theta(x) = 1$. Hence, the payoffs corresponding to the two strategies satisfy $\pi_0(x) = a < \pi_1(x) = d$ and 1 is the best response.

Since n is even, $\frac{1}{2}$ does not belong to the state space and it should not be counted in the basin of attraction of either all-0 state or all-1 state. The basins of attraction of all-0 state and all-1 state thus have equal sizes, implying that all-0 state and all-1 state are both stochastically stable. *Q.E.D.*

Proposition 2 portrays a scenario in which agents playing the risk-dominant strategy 0 benefits from integration policy while agents playing the Pareto-dominant strategy 1 benefits from segregation policy. When agents playing 0 are the majority, they use the maximal integration policy to exploit agents playing 1. Therefore, 0 serves as the best response. When agents playing 1 are the majority, they use the complete segregation policy to protect themselves from agents playing 0. Therefore, 1 serves as the best response. This gives rise to the coexistence of two distinct social conventions.

4. Conclusion

This paper demonstrates how majority voting can influence the selection of social conventions in stochastic evolutionary models. We propose a model in which agents vote for different policies through majority voting that lead to different assortative matching patterns. We find that agents either vote for maximal integration policy or complete segregation policy. Complete segregation policy helps agents playing Pareto-dominant strategy to earn a high payoff so that the Pareto-dominant convention becomes stochastically stable. We hope that our attempt to incorporate collective decision making processes sheds some new insights to the study of social conventions.

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