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### Modelling Oil Price Volatility with the Beta-Skew-t-EGARCH Framework

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#### Abstract

This paper employs the Beta-Skew-t-EGARCH framework proposed by Harvey and Succarat (2014) to model oil price volatility. It utilizes two prominent oil proxies and also accounts for structural break to gauge the robustness of results. In all, it finds that the approach seems more suitable than the standard symmetric and asymmetric GARCH models if the oil price return exhibits fat tails, leverage and skewness.

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# Modelling Oil Price Volatility with the Beta-Skew-t-EGARCH Framework

## 1.0 Introduction

The evidence of volatility clustering in oil price has been validated in the literature (see Li et al., 2002 for a review of earlier evidence and Salisu and Fasanya, 2013 for recent findings). Certainly, there are policy and investment implications of modelling oil price volatility. These implications are well highlighted by Narayan and Narayan (2007) and Salisu and Fasanya (2013). For instance, investors in the oil market are affected by oil price volatility due to the associated risk and uncertainty. Thus, information about the persistence of oil price shocks and significance of leverage effect may offer useful insights to profit maximizing investors regarding the extent and severity of oil price volatility. Similarly, oil dependent and oil exporting countries are also concerned about oil price volatility due to its attendant implications on external reserves and budget implementation, among others.

The search for the best volatility framework has remained unabated despite the proliferation of literature in this regard (see Francq and Zakoïan, 2010 for a survey). The choice of volatility model in general appears to be influenced by the underlying statistical features of the series under examination and validated by relevant diagnostic tests. Recently, Harvey and Chakravarty (2008) and Harvey (2013) propose an extension to the asymmetric version defined as the Beta-t-EGARCH model. The model has been further extended by Harvey and Sucarrat (2014) to account for the skewed case and is consequently referred to as Beta-Skew-t-EGARCH model.

Sucarrat (2013) highlights a number of attractions of the latter model when dealing with volatility as follows: First, the model is robust to jumps or outliers, and performs very well empirically for a variety of financial returns when compared with other GARCH models. Second, the model accounts for prominent characteristics of time-varying financial volatility such as leverage, conditional fat-tailedness, conditional skewness and a decomposition of volatility into a short-term and a long-term component. Although, other volatility models deal with these characteristics but the EGARCH does it more easily. Third, the unconditional moments of return exist (if the conditional moments exist), which is important in long-term forecasting and for the computation of the autocorrelation function of returns. Fourth, there are no positivity constraints on the ARCH, GARCH and leverage parameters. Fifth, the asymptotic properties are much easier to derive compared to the earlier version such as the Nelson's (1991) EGARCH including the two-component GARCH model of Engle and Lee (1999).

In this present paper, the extended framework (i.e. the Beta-Skew-t-EGARCH model) is used to model oil price volatility.<sup>1</sup> First, the paper utilizes two prominent oil price indices namely Brent and

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<sup>1</sup> The computational procedure for Simulation, estimation and inference of first-order process of the model is readily available in the R package (using the `betategarch` package) and written by Sucarrat (2013). Nonetheless, the program file including data used in this study can be provided on request.

West Texas Intermediate (WTI) and their daily data are obtained over a considerable period of time. Second, it considers different sub-samples based on the results of Perron (2006) test that determines structural breaks endogenously. Third, the results from the Beta-Skew-t-EGARCH model using both the full sample as well as the sub-samples are then compared with standard symmetric and asymmetric GARCH models (GARCH and GJR-GARCH models respectively). With these innovations, this paper is able to gauge the robustness and significance of the Beta-Skew-t-EGARCH model when dealing with oil price volatility.

The considered model including relevant preliminary statistics is described in section 2 while section 3 presents and discusses the results including diagnostic tests. Section 4 concludes the paper.

## 2.0 The Model and Data

Essentially, I consider the first order one-component of the Beta-Skew-t-EGARCH model which is comparable with the standard symmetric GARCH model (GARCH (1, 1)) and standard asymmetric GARCH model (GJR-GARCH (1, 1)). The martingale difference version of the first order one-component version is described below (see Sections 4 and 6 in Harvey and Sucarrat, 2014; and Page 138 in Sucarrat, 2013):

$$r_t = \exp(\lambda_t) \varepsilon_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim \text{st}(0, \sigma_\varepsilon^2, \nu, \gamma), \quad \nu > 2, \quad \gamma \in (0, \infty), \quad (1)$$

$$\lambda_t = \omega + \lambda_t^+, \quad (2)$$

$$\lambda_t^+ = \phi_1 \lambda_{t-1}^+ + \kappa_1 u_{t-1} + \kappa^* \text{sgn}(-r_{t-1})(u_{t-1} + 1), \quad |\phi_1| < 1, \quad (3)$$

$$\varepsilon_t = \varepsilon_t^* - \mu_{\varepsilon^*} \quad (4)$$

Where  $r_t$  is the oil price returns measured by taking the first difference of log of crude oil price;  $\lambda_t$  is the logarithm of the scale;  $\varepsilon_t$  is the conditional error;  $\sigma_t$  is the conditional scale or volatility;  $\sigma_\varepsilon^2$  is the variance of  $\varepsilon_t$  (implying that  $\varepsilon_t$  is not standardised to have variance one);  $\nu$  is the degrees of freedom;  $\gamma$  is the skewness parameter;  $\omega$  is the log-scale intercept which captures the long-term volatility;  $\phi_1$  is the persistence parameter (and it is assumed that the more clustering the return series, the bigger the value of the parameter);  $\kappa_1$  is the ARCH parameter (and it is expected that the greater the response to shocks, the bigger the parameter in absolute value);  $u_t$  is the conditional score which is obtained by taking the derivative of the log-likelihood of  $r_t$  at  $t$  with respect to  $\lambda_t$  and  $\kappa^*$  is the leverage parameter. As depicted in equation (1),  $\varepsilon_t$  is distributed as a skewed  $t$  with zero mean, scale  $\sigma_\varepsilon^2$ ,  $\nu$  degrees of freedom and skewness parameter  $\gamma$ . However,  $\varepsilon_t^*$  which is defined as an uncentred skewed  $t$  variable is distributed with mean  $\mu_{\varepsilon^*}$ ,  $\nu$  degrees of freedom and skewness parameter  $\gamma$ . Also, a centred and symmetric  $t$ -distributed variable with mean zero is

obtained when  $\gamma = 1$ , in which  $\mu_{\varepsilon^*} = 0$ , whereas a left-skewed (right-skewed)  $t$ -variable is obtained when  $\gamma < 1$  ( $\gamma > 1$ ) (Sucarrat, 2013). To ensure stability in  $\lambda_t$ , the  $|\phi_1| < 1$  condition must hold.

Since the one-component Beta-Skew-t-EGARCH model is comparable with GJR-GARCH(1,1) and GARCH (1,1), it does suffice to also specify the models and they are given respectively below:

$$r_t = \sigma_t \varepsilon_t, \quad \varepsilon_t \sim st(0, 1, \nu, \gamma), \quad (5)$$

$$\sigma_t^2 = \omega + \phi_1 \sigma_{t-1}^2 + \kappa_1 r_{t-1}^2 + \kappa^* I(r_{t-1} < 0) r_{t-1}^2 \quad (6)$$

Equations (5) and (6) characterize the GJR-GARCH model and the parameters are as previously defined. Note that GARCH(1,1) is obtained when  $\kappa^*$  is restricted to zero.

In all, I estimate 4 models based on different testable assumptions derived from the specifications above: (i) Model 1 is the standard GARCH model (which is the restricted GJR-GARCH model as leverage and skewness are restricted to zero); (ii) Model 2 is the GJR-GARCH model that allows for both leverage and skewness; (iii) Model 3 is the restricted Beta-Skew-t-EGARCH as leverage and skewness are restricted to zero; and (iv) Model 4 is the full Beta-Skew-t-EGARCH with both leverage and skewness. This arrangement presents a hierarchy of models, from simple to complex which facilitates the comparison of the models in the results tables.

Before the estimation of the models, I offer some preliminary statistics to strengthen the justification for the application of the Beta-Skew-t-EGARCH model. The statistical features evaluated are the skewness (for right or left tail), kurtosis (for fat or thin tail), Jarque-Bera (for normality), ARCH-LM tests (for heteroscedasticity or volatility), Ljung-Box test (for serial correlation or long memory) and other relevant statistics. Daily data on spot price of Brent and WTI covering the period June 01, 1987 to February 09, 2015 are sourced from US Energy Information Administration (EIA).<sup>2</sup> In all, about 7089 observations are used for the analysis and it is believed that the dataset is quite long enough to distil the historical behaviour of oil price. Table I below shows the computed summary statistics for the oil price returns. The skewness statistics show that the return series are negatively skewed (left-tailed). Also, the kurtosis statistics offer evidence of fat-tailedness of the return series implying that the series are heavier than normal. In addition, both series are non-normal based on the Jarque-Bera statistics, thus reinforcing the values of kurtosis and skewness which lie outside the acceptable range for a normally distributed series. The Ljung-Box test also confirms the presence of serial correlation in the return series likewise the ARCH-LM test indicates the presence of heteroscedasticity and therefore supporting the existence of time varying volatility (i.e. ARCH effects). There is also evidence of volatility clustering as shown in Figure 1; this further attests to the results of ARCH-LM test. Based on these statistical features, it does suffice to assume that the Beta-Skew-t-EGARCH model would offer insightful results. In the section that follows, the empirical results are presented and discussed.

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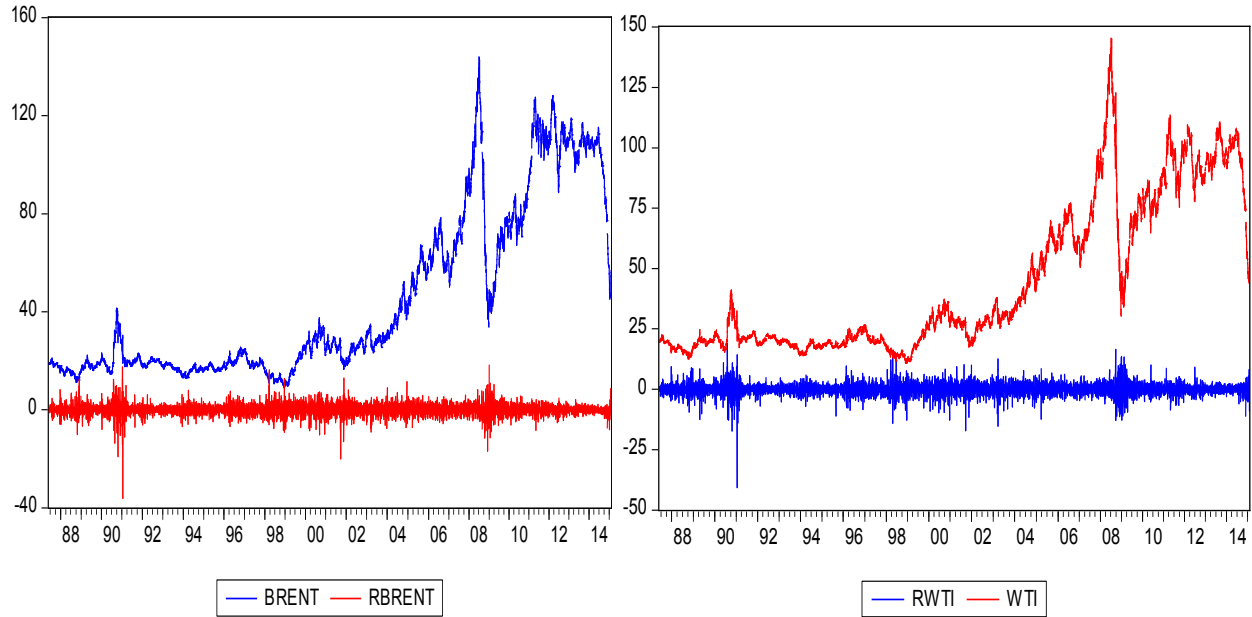
<sup>2</sup> Data is freely downloadable from US EIA website using the link: [http://www.eia.gov/dnav/pet/pet\\_pri\\_spt\\_s1\\_d.htm](http://www.eia.gov/dnav/pet/pet_pri_spt_s1_d.htm)

**Table I: Descriptive Statistics**

Descriptive Statistics	RBENT	RWTI
Skewness	-0.660827	-0.814625
Kurtosis	18.38453	19.23487
Jarque-Bera	70406.51***	78614.13***
Ljung-Box Q Stat. (5)	13.03**	24.78***
Ljung-Box Q Stat. (10)	28.97***	40.01***
ARCH-LM test (5)	471.75***	306.58***
ARCH-LM test (10)	504.94***	357.17***
No. of Observations	7087	7087

Note: RBENT and RWTI denote return series for Brent and WTI respectively.

\*\*\*, \*\*, & \* represent 1%, 5% & 10% levels of statistical significance.

**Figure 1: Trends in Oil prices and their returns**

### 3.0 Empirical Results

Table II contains the results for the four models using full sample. The table is partitioned into two: Panel A and Panel B for WTI returns and Brent returns respectively. We employ the Bayesian Information Criterion (BIC) and the Log-Likelihood (LogL) as model selection criteria. The model with the best fit is expected to have the lowest BIC and the highest LogL values. We also consider relevant post-estimation diagnostics namely the ARCH-LM test and the Ljung-Box (LB) test. The former test is used to test for the remaining ARCH effects in the model after estimation to know whether they are still significant or not and therefore, a non-rejection of the null hypothesis implies the absence of ARCH effects. The non-rejection is also an indication that the estimated volatility model is adequate in dealing with the ARCH effects. The LB test evaluates the adequacy of the dynamics captured in the model particularly in the mean equation of the volatility model and

therefore, a non-rejection of the null hypothesis implies the adequacy of the dynamics incorporated in the model; otherwise, the considered dynamics remain inadequate.

Looking at the overall results, the full Beta-Skew-t-EGARCH model that accounts for both asymmetry as well as skewness, out-performs all the other models for both oil return series judging by the Log-Likelihood and the Bayesian Information Criterion. In other words, the superior performance of this model is consistent across the two return series. Thus, for the full sample, accounting for asymmetry and skewness is important for modelling oil price volatility. This is in line with the descriptive statistics that support evidence of heavy tails and skewness in the return series. In essence, for a heavy-tailed and skewed oil price returns, the Beta-Skew-t-EGARCH model seems more appropriate than the Standard symmetric or asymmetric GARCH model. Also, parameter estimates for the two returns series are consistent in terms of the sign, size and significance; thus, confirming the robustness of the estimates.

Turning to the estimates of the Beta-Skew-t-EGARCH model (see Table II), as expected, the conditional oil price returns are heavy tailed based on the positive and significant value of the degree of freedom parameter in the Skewed  $t$  which hovers around 6.0 on average. There is also evidence of significant negative skewness which is estimated to be 0.95 on average ( $\hat{\gamma} < 1$ ) indicating that large negative returns impose a higher risk than large positive returns. The volatility in oil price returns also tends to persist when it is evident ( $\hat{\phi}_1$  is close to 1) and negative shocks seem to accentuate the volatility than positive shocks of the same magnitude ( $\hat{\kappa}^*$  is positive and significant).

I also consider subsamples based on the Perron (2006) unit root test that allows for the determination of structural break endogenously. Like the previous estimations, the oil price return series for both Brent and WTI are also utilized for the endogenous structural break test with unit root. The most significant break dates are presented in Table III. These dates coincide with the period of Iraqi/Kuwait conflict that impacted significantly on the supply of crude oil and by extension culminated into higher volatility in oil price (see also, Salisu and Fasanya, 2013). The dates also reflect the behaviour of the two oil price return series as the most notable spikes are recorded in 1991 (see Figure 1). Consequently, two subsamples are formed (i.e. pre- and post-break sub-periods for each of the series). On the basis of these subsamples, I re-estimate the four models and the results obtained are presented in tables IV and V for WTI and Brent returns respectively. The analyses are restricted to the break for the Gulf war in order to more carefully evaluate the performance of the Beta-Skew-t-EGARCH model in the presence of structural breaks.

Some interesting and striking features are discernible from the results. First, like the full sample results, the full Beta-Skew-t-EGARCH model is the preferred model for the two oil price returns as it gives the best fit judging by the same model selection criteria – the log-likelihood and the Bayesian Information Criterion. Thus, our results are robust to structural breaks. Second, there may be problem of under-fitting when oil price returns that exhibit significant leverage effect, heavy tails and skewness are modelled with the restricted Beta-Skew-t-EGARCH model or the restricted GARCH model. Even with the GJR-GARCH model with leverage effect and skewness, the Beta-Skew-t-EGARCH appears to increase the log-likelihood and lower the BIC than the former model. Third, other estimates such as the volatility persistence, degree of freedom and skewness parameters

are consistent with the results of the full sample. Finally, the diagnostics for the two return series suggest the non-existence of any ARCH effects after modelling with the GARCH-type models. More noticeably, the diagnostic results for the Brent improved when the structural break is accounted for. However, additional dynamics may be required for Brent returns when modelling the series in future. On the whole, the choice of Beta-Skew-t-EGARCH for modelling oil price returns remains superior with and without structural break. However, some modifications may be required based on the results of the diagnostics. Some of the additional modifications may include accounting for structural breaks and extending the mean equation to include more dynamics such as the autoregressive moving average process (ARMA) among others.

In addition to the result tables, the performance of the various models is also evaluated by plotting the fitted values for the conditional standard deviations of these models with the absolute returns. Figures 2 and 3 reveal how these models are able to track the absolute returns of the oil price proxies. As evident in the figures, all the models reasonably track the oil price return series. However, although not clearly visible to the eye, a closer examination of the fitted values suggests that the Beta-Skew-t-EGARCH models better track the return series than other variants. Instructively, the most notable spike in the figures coincides with the break date earlier reported. This is a good cross-check on the reliability of the test conducted to determine the presence of structural breaks endogenously in oil price returns. Thus, the selected break date in this paper is consistent with the statistical behaviour of the return series.

Table II: Beta-Skew-t-EGARCH, GJR-GARCH and GARCH specifications fitted to Oil price returns  
(Full Sample: June 01, 1987 to February 09, 2015)

Panel A: WTI Returns												
Model	$\hat{\omega}$ [ser]	$\hat{\phi}_1$ [ser]	$\hat{\kappa}_1$ [ser]	$\hat{\kappa}^*$ [ser]	$\hat{\nu}$ [ser]	$\hat{\gamma}$ [ser]	LogL	BIC	LB (10) [p value]	LB (20) [p value]	ARCH (10) [p value]	ARCH (20) [p value]
Standard GARCH	5.04*10 <sup>-6***</sup> [0.000]	0.930*** [0.006]	0.062*** [0.006]	-	5.763*** [0.379]	-	17534.73	-4.943	8.887 [0.543]	17.633 [0.612]	0.308 [1.000]	0.561 [1.000]
GJR-GARCH	4.98*10 <sup>-6***</sup> [0.000]	0.933*** [0.007]	0.062*** [0.006]	0.048 [0.040]	5.766*** [0.380]	0.934*** [0.014]	17545.22	-4.944	8.960 [0.536]	17.811 [0.600]	0.279 [1.000]	0.500 [1.000]
Restricted Beta-Skew-t-EGARCH	-4.085*** [0.067]	0.991*** [0.002]	0.040*** [0.004]	-	6.053*** [0.400]	-	17551.37	-4.948	9.873 [0.452]	18.312 [0.567]	1.499 [0.999]	1.620 [1.000]
Full Beta-Skew-t-EGARCH	-4.056*** [0.067]	0.991*** [0.002]	0.038*** [0.004]	0.009*** [0.002]	5.999*** [0.394]	0.931*** [0.014]	17566.55	-4.950	10.657 [0.385]	18.987 [0.523]	1.125 [1.000]	1.213 [1.000]
Panel B: Brent Returns												
Standard GARCH	3.03*10 <sup>6***</sup> [0.000]	0.935*** [0.006]	0.061*** [0.006]	-	5.984*** [0.419]	-	17976.34	-5.068	24.656*** [0.006]	40.334*** [0.005]	21.095** [0.020]	22.036 [0.339]
GJR-GARCH	2.90*10 <sup>6***</sup> [0.000]	0.936*** [0.006]	0.060 [0.006]	0.071** [0.036]	6.021*** [0.424]	0.953 [0.015]	17893.65	-5.068	24.655*** [0.006]	40.264*** [0.005]	27.421*** [0.002]	28.646* [0.095]
Restricted Beta-Skew-t-EGARCH	-4.156*** [0.071]	0.992*** [0.002]	0.037*** [0.004]	-	6.297*** [0.449]	-	17984.81	-5.070	24.052*** [0.007]	41.0235*** [0.004]	30.115*** [0.001]	30.560* [0.061]
Full Beta-Skew-t-EGARCH	-4.136*** [0.072]	0.992*** [0.002]	0.036*** [0.004]	0.007*** [0.002]	6.307*** [0.449]	0.954*** [0.014]	17994.17	-5.071	24.188*** [0.007]	40.9595*** [0.004]	43.319*** [0.000]	44.058*** [0.001]

Notes: ser is the standard error and p value is the probability value. \*\*\*, \*\* and \* denote 1%, 5% and 10% levels of statistical significance. The models have been previously defined. BIC is the Bayesian Information Criterion computed as  $BIC = -2 \ln(L) + k \ln(T)$ . T = sample size/number of observations; k = the number of free parameters to be estimated; L = the maximized value of the likelihood. LogL is the Log Likelihood. ARCH (LM test) and LB (Ljung-Box test) are the post-estimation tests for ARCH effects and serial correlation.

Table III: Perron (2006) Test

Series	WTI Returns	Brent Returns
Break Date	Jan 14, 1991	Jan 14, 1991
No. of Obs	7088	7088

Table IV: Beta-Skew-t-EGARCH, GJR-GARCH and GARCH specifications fitted to WTI Returns



Panel A: Post-Break (Jan 16, 1991 - Feb 09, 2015)												
Model	$\hat{\omega}$ [ser]	$\hat{\phi}_1$ [ser]	$\hat{\kappa}_1$ [ser]	$\hat{\kappa}^*$ [ser]	$\hat{\nu}$ [ser]	$\hat{\gamma}$ [ser]	LogL	BIC	LB (10) [p value]	LB (20) [p value]	ARCH (10) [p value]	ARCH (20) [p value]
Standard GARCH	4.25*10 <sup>-6</sup> *** [0.000]	0.938** * [0.006]	0.054** * [0.005]	-	5.932** * [0.416]	-	15328.54	-4.978	4.472 [0.924]	13.344 [0.862]	2.810 [0.986]	4.124 [1.000]
GJR-GARCH	4.2410 <sup>-6</sup> *** [0.000]	0.938** * [0.006]	0.053** * [0.005]	0.080 [0.045]	5.973** * [0.423]	0.929** * [0.015]	15339.58	-4.978	4.591 [0.917]	13.899 [0.836]	2.480 [0.991]	3.665 [1.000]
Restricted Beta-Skew-t-EGARCH	-3.332*** [0.470]	0.998** * [0.002]	0.036** * [0.003]	-	6.168** * [0.440]	-	15332.02	-4.979	4.183 [0.939]	14.653 [0.796]	20.157** [0.028]	20.827 [0.407]
Full Beta-Skew-t-EGARCH	-3.876*** [0.123]	0.994** * [0.002]	0.033** * [0.003]	0.013** * [0.002]	6.167** * [0.438]	0.918** * [0.016]	15354.57	-4.983	6.843 [0.740]	17.071 [0.648]	14.414 [0.155]	14.845 [0.785]

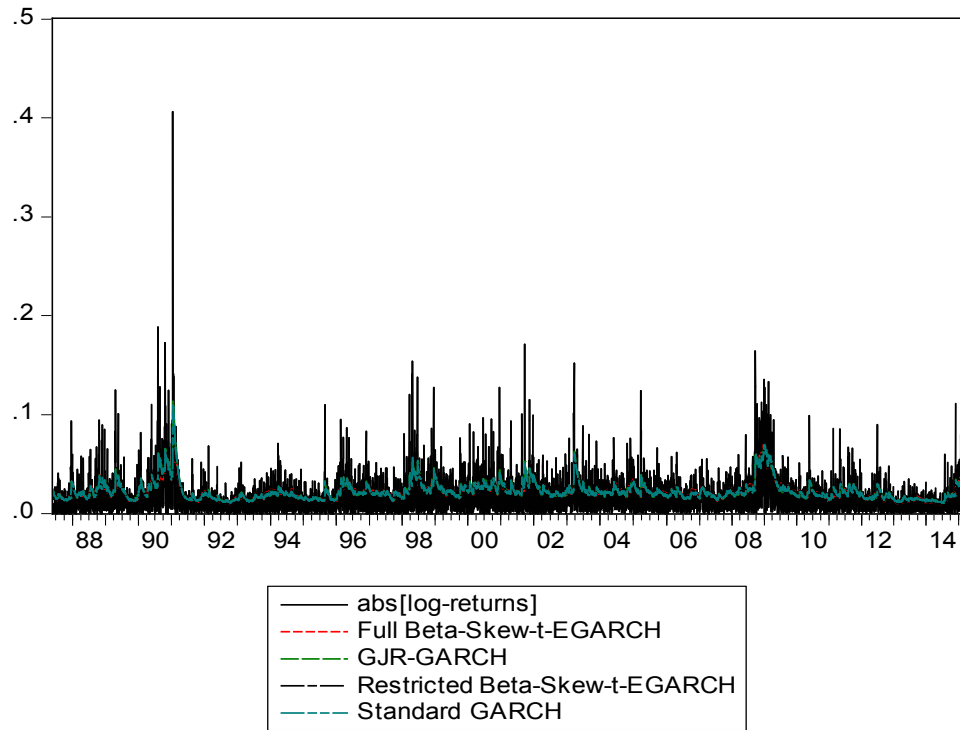
Notes: See Table II

Table V: Beta-Skew-t-EGARCH, GJR-GARCH and GARCH specifications fitted to Brent Returns

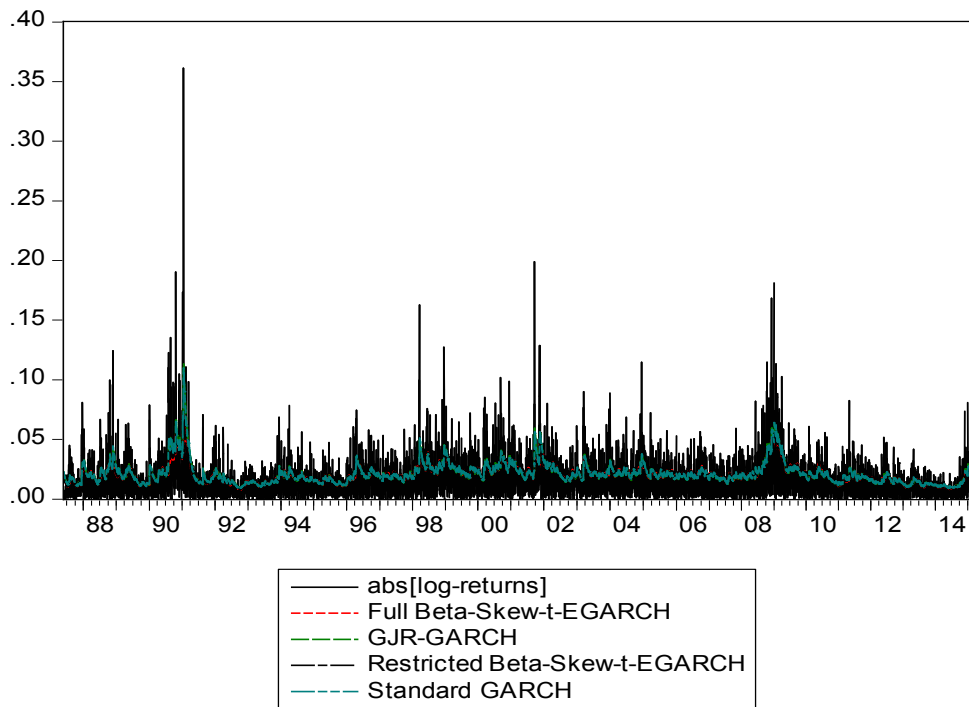
Panel B: Post-Break (Jan 16, 1991 - Feb 09, 2015)												
Model	$\hat{\omega}$ [ser]	$\hat{\phi}_1$ [ser]	$\hat{\kappa}_1$ [ser]	$\hat{\kappa}^*$ [ser]	$\hat{\nu}$ [ser]	$\hat{\gamma}$ [ser]	LogL	BIC	LB (10) [p value]	LB (20) [p value]	ARCH (10) [p value]	ARCH (20) [p value]
Standard GARCH	1.97*10 <sup>-6</sup> *** [0.000]	0.951*** [0.005]	0.046*** [0.005]	-	6.480*** [0.497]	-	15673.18	-5.090	18.215* [0.051]	30.448* [0.063]	12.687 [0.242]	13.092 [0.873]
GJR-GARCH	1.78*10 <sup>-6</sup> *** [0.000]	0.952*** [0.005]	0.044*** [0.005]	0.088* [0.044]	6.581*** [0.514]	0.944*** [0.016]	15681.24	-5.089	17.912* [0.056]	30.141* [0.068]	15.692 [0.109]	16.164 [0.706]
Restricted Beta-Skew-t-EGARCH	-3.980*** [0.121]	0.995*** [0.002]	0.029*** [0.003]	-	6.790*** [0.536]	-	15678.67	-5.091	21.664** [0.017]	34.665** [0.022]	10.968 [0.360]	11.312 [0.938]
Full Beta-Skew-t-EGARCH	-3.878*** [0.163]	0.997*** [0.002]	0.026*** [0.003]	0.009*** [0.002]	6.919*** [0.554]	0.936*** [0.016]	15694.44	-5.094	20.028** [0.029]	32.836** [0.035]	13.152 [0.215]	13.403 [0.859]

Notes: See Table II

**Figure 2: Oil price returns (WTI) with Beta-Skew-t-EGARCH and GARCH filters**



**Figure 3: Oil price returns (Brent) with Beta-Skew-t-EGARCH and GARCH filters**



## 4.0 Conclusion

This paper demonstrates the application of the new EGARCH (referred to as Beta-Skew-t-EGARCH) models developed by Harvey and Sucarrat (2014) to oil price data. More specifically, it considers the first order one-component of the Beta-Skew-t-EGARCH model and then, compares the performance of the latter with the ordinary symmetric (GARCH (1, 1)) and ordinary asymmetric GARCH (GJR-GARCH (1, 1)) models. It considers two oil proxies and also accounts for structural break in the analysis in order to gauge the robustness of results. In all, it finds that the approach seems more suitable for modelling oil price volatility than the standard symmetric and asymmetry GARCH models if the oil price return exhibits fat tails, leverage and skewness.

## Acknowledgments

I thank Professor Genaro Sucarrat for helpful suggestions on estimation. I also appreciate the useful comments by the anonymous reviewer. The usual disclaimer applies.

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