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Open questions in open source: Exploring incentives, licensing and competition

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Abstract

Open Source Software (OSS) has grown in importance over the last few decades and now constitutes an important part of the software market particularly in mobile and web technology. In this paper, we provide a preliminary theoretical framework for analyzing currently unexamined issues regarding developer participation in OSS and competition between open source and proprietary software. We start by looking at the upstream market where developers voluntarily contribute effort into the development of OSS without claiming any copyright for their contributions. We explore the relationship between open source licensing and developers' effort provision and suggest an empirical test of developer motivation based on the results of our model. We also look at issues of competition in the software market downstream and provide some conjectures of how proprietary competition can influence upstream OSS development under different license regimes.

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1 Introduction

Open Source Software (OSS) has always presented a special dilemma to economists because of the way software development occurs in the OSS production model. Developers make voluntary contributions to developing the software. But traditional copyright protection is absent from OSS so that no single developer can claim ownership. This calls into question the incentives for developers' participation in OSS when they cannot appropriate the value of their investment. While the lack of copyright protection poses an existential question for OSS, it raises an interesting paradox for the survival of proprietary software at the other end. Typically, OSS is distributed freely to all users. One would expect then the presence of an OSS should foreclose any proprietary software from competing in the market. Yet we find several instances where proprietary and open source products compete and coexist.

Over the decades, the competitive landscape of software technology has changed considerably with OSS gaining steady significance in mobile and cloud computing technology. The leading webserver, Apache's HTTP, is open source and it controls more than 50 percent of the market, far ahead of proprietary Microsoft which has a slim 20 percent share. Linux, which pioneered the OSS movement, holds a near monopoly in the operating system used in high powered supercomputers. More recently, Google's Android OS which is also released under the Apache license controls more than 90 percent of the market. With open source playing a more and more important role in the future of software, it is crucial to understand its implications for the market landscape. Research on this is particularly important because the nature of competition with OSS is substantively different than with a proprietary seller. Whereas in a proprietary product, the downstream market is delinked from the upstream market where production occurs, in the OSS model, production and consumption often occur simultaneously as many developers who produce the software also assume the role of consumers. Hence the term "user-developer" is often applied in the context of OSS. This fundamentally changes the way in which OSS interacts with proprietary software both upstream where software development takes place and downstream where software distribution occurs. In this paper, we examine issues on both ends of the OSS process and provide some interesting avenues for empirical and theoretical research.

1.1 Motives and Licensing

What motivates software developers to participate and exert effort into OSS development? This question has intrigued researchers because there is no explicit monetary mechanism to compensate OSS developers for their contribution. Research on OSS developers has identified three possible motivations. First is simply an ideological or altruistic motive to participate in the "free and open software" movement (Lakhani and Wolf, 2005). Richard Stallman, a computer programmer who has forcefully pushed the philosophy that all software development should be open, has a strong following among open source developers. Beyond ideological or altruistic motivations, however, there are two economic benefits that open source provides to developers. First is a direct user benefit to developers as they improve the quality of a software program for their own use (Lakhani and von Hippel, 2003; Johnson, 2002). In such user-based models of open source development, OSS is viewed as a public good the value of which is determined by the aggregate voluntary contributions by individual developers. Such models inevitably predict that developers' effort under OSS will be less than that under proprietary production which internalizes effort externalities. Second, there is also some evidence to suggest that OSS developers use their contributions to OSS as a signal to potential employers about their programming abilities or to gain reputational benefits from solving complex programming problems (Lerner and Tirole, 2002; Spiegel, 2009). Unlike the public good model, such labor market signaling and reputational gains are private benefits to the programmer and hence do not entail any public good under-provision problems.

The question of incentives and developers' participation is also closely tied to software licensing. OSS is often equated with a copyleft license where all users and developers have the right to modify and distribute the software without worrying about copyright claims. Yet, this applies only to the most restrictive form of OSS licensing. In reality, there are many different OSS licensing regimes that vary in the extent to which developers must surrender their copyright claims on their modifications. Restrictive licenses, such as the GNU Public License (GPL), force all derivative works to use the same open source license thus ensuring that the software is never made proprietary at any stage of its development. But there are less restrictive licenses, such as the Apache license and the Berkeley Software Distribution (BSD) license, that allow modified software to be made proprietary. The licensing terms provide interesting trade-offs between the incentives for developers to participate in open source projects and this in turn has implications for the quality of the final software product. A few papers have empirically examined the relationship between open source license and software development and there is a general consensus among them that restrictive licenses are associated with lower levels of participation and performance compared to non-restrictive licenses (Subramaniam et al., 2009; Comino et al., 2007; Lerner and Tirole, 2005; Fershtman and Gandal, 2007).

One of the limitations of existing research has been that, while the issue of motives has largely been confined to theoretical modelling, the effect of licensing has primarily been studied empirically. Even the empirical research on licensing only draws correlations between participation and OSS licensing. To our knowledge, there are no structural models of licensing and participation that would explain observed empirical regularities and shed light on the mechanism by which the choice of license affects software outcomes. On the other hand, although there is a rather well-developed theoretical literature on different motives governing OSS development, data has not been brought to bear the identity and isolate the motives governing OSS participation. Further, existing research, both theoretical and empirical, ignores crucial interactions between licensing and motives by studying them in isolation of each other. Atal and Shankar (2015) is the only paper to our knowledge that provides an integrated model of motives and OSS licensing. That paper examines both software quality and optimal licensing choice. The paper models different stages of software development and assigns differing motivations to developers at each stage. So developers receive reputational benefits in the early stages of software design and then user-benefits at a later stage after the software is (commercially) usable. Software licensing determines whether they receive reputational benefits in the design stage and how much of the software value can be appropriated as user-surplus at a later stage.

In Section 2 of the current paper, we set up a simple model that simultaneously incorporates developers' motives (both reputational and user-driven) and different OSS licenses.

Although preliminary, we provide a theoretical framework to explain empirical evidence on the effects of licensing on software development; and we also propose a way to elicit developers' motives based on observed software outcomes and license. Unlike Atal and Shankar (2015), we do not make any assumptions on motives governing different stages of software development. Hence the current model is intended to be a more robust specification that can be applied to a software at any stage of development. Another important distinction is that we incorporate the effects of product market competition into effort provision under different OSS licenses. In doing so, we find that the value of an incumbent software qualitatively influences the difference in effort provision across OSS licenses. Thus the preliminary findings in the current paper suggest the need for adequate controls on software market characteristics relating to competition when empirically testing the influence of licensing on developers' effort into OSS. We discuss our contribution on product market contribution below.

1.2 Product Market Competition

A growing economic literature deals with the relationship between OSS and traditional proprietary firms. Casadesus-Masanell and Ghemawat (2006), Mustonen (2003) and Athey and Ellison (2014) look at the presence of OSS affecting price and quality choices for the proprietary firm. Jaisingh et al. (2008-9) and Atal and Shankar (2014) look at how the nature of competition changes when the proprietary firm faces an OSS competitor compared to when it faces another proprietary competitor.

As with the research on modelling developers' motives, OSS licensing has generally not been addressed in these models of software competition. Since developers' participation and OSS outcomes depend on the type of license adopted, competition between a proprietary software and OSS with a non-restrictive license is likely to yield a different market outcome than competition between proprietary software and OSS with a restrictive license. Llanes and de Elejalde (2013) do dig into the issue of restrictiveness of OSS licenses in looking at software competition; however, they do not model developers' participation.

We extend our model of developers' participation to include subsequent product market competition. In doing so, we examine how the choice of license affects product market competition with an existing proprietary good. Further, we also provide some conjectures about how competition can influence developers' participation under different OSS licenses. Our preliminary model shows that competition affects OSS developers' participation differently depending on the type of license it employs.

2 The Model

Consider an OSS project with $N > 1$ user-developers who participate in developing the software by exerting effort and then consume the final software product. Software development process has two stages. First, the N developers design the software by exerting effort $e_i \in [0, 1]$ with $i = 1, 2, ..., N$. Marginal cost of effort for each developer is $c > 0$. There is a stochastic process that determines how effort translates into output. The value generated by developer i's effort is $d_i = \alpha e_i \varepsilon_i$, where $\varepsilon_i \overset{i.i.d.}{\sim} U[0, 2]$ is the error in the developer's effort transforming into her contribution and $\alpha > 0$ is the marginal value of the developer's contribution. The value of the software (v) is the total contribution from all developers, i.e., $v = \sum_{i=1}^{N} d_i$. Hence the expected value $E(v) = \left(\sum_{i=1}^{N} \alpha e_i\right)$. Once developed, the resulting software is then commercialized and distributed to users under the terms of the software license. If the license is non-restrictive, denoted by NR, i.e., it allows the software to be made proprietary (such as BSD, or Apache), the final product is sold at a positive price. If, however, the OSS license is restrictive, denoted by R, all the software code including its commercialization is kept open to users resulting in a zero price of the software.

2.1 Effort Provision and Licensing

Suppose effort provision by developers is potentially motivated by two factors. First, developers get user value from consuming the final software. Second, in OSS, since developer's effort into software design is kept open and public, the developer who produces the highest design value d_i also receives a reputational prize ($S \geq 0$) for her innovation. This prize can be interpreted as a reputational payoff to the developer from solving a coding problem or it could be a labor market prize in terms of signaling her ability to prospective employers.

Let us start by looking at the outcome in a proprietary model where a proprietary firm determines developers' effort in order to maximize profits. Since the firm does not make developers' contributions public, by definition, there is no reputational payoff to developers. To keep the labor market dynamics simple, assume that the developer's reservation wage is zero. Hence the firm pays every developer a wage exactly equal to her cost of effort. The total wage cost is then $\sum_{i=1}^{N} ce_i$. In the consumers' market, once the software has been produced, the expected monopoly price is the value to each user, i.e., $\sum_{i=1}^{N} \alpha e_i$. Thus, developers do not get any surplus as users or workers. Profits are $\pi = N\sum_{i=1}^{N} \alpha e_i - \sum_{i=1}^{N} ce_i$. In a symmetric equilibrium, effort provision under a proprietary license is $e_p = 0$ if $\alpha < \frac{c}{N}$; if $\alpha \geq \frac{c}{N}$, then $e_p = 1$.

Next let us move to the most restrictive OSS license where the market price of the software is trivially zero. Here developers receive a user benefit from their contribution to the software. Moreover, the winning contributor receives the reputational prize, S. So effort, e_i, is chosen to maximize developer i's expected payoff $\left[P\left(e_i, e_{j\neq i}\right) S + \alpha \left(e_i + \sum_{j\neq i} e_j\right) - ce_i\right]$, where $P\left(e_i, e_{j\neq i}\right)$ is developer i's probability of winning S, given that the other $(N-1)$ developers choose effort $e_{j\neq i}$.[1] The symmetric equilibrium in effort is as follows. For $S \geq \frac{N}{N-1}c$ or $\alpha \geq c - \frac{N-1}{N}S$, we obtain a corner solution and the symmetric equilibrium in effort is $e_r = 1$. For $S < \frac{N}{N-1}c$ and $\alpha < c - \frac{N-1}{N}S$, we derive an interior equilibrium in effort where $e_r = \frac{N-1}{N}\frac{S}{(c-\alpha)}$.

Comparing efforts across the proprietary and R licenses, e_r could be higher or less than e_p. For example, if $S = 0$ so that e_r is driven solely by user-benefits, $e_r \leq e_p$ and the value of

[1] The derivation of this expression is provided in the Appendix.

the OSS is constrained by the standard public good under-provision problem. But if $\alpha < \frac{c}{N}$ and $S \geq c$, then $e_r > e_p$. In other words, if user-based motives are weak while reputational motives are strong, a restrictive OSS can outperform the proprietary model.

Finally, let us consider the NR license. Here the design of the software is kept open. But once it has been developed, the final commercial product can be made proprietary and sold at a positive monopoly price to users. Assume that the developer with the highest value of effort makes the software proprietary. Now each developer's payoff, if she has the highest d_i, is the reputational prize plus expected monopoly profit from commercializing the product, i.e. $P\left(e_i, e_{j\neq i}\right)\left[S + N\alpha\left(e_i + \sum\limits_{j\neq i} e_j\right)\right] - ce_i$. The symmetric equilibrium effort is then given by $e_{nr} = 1$ if $S \geq \frac{N}{N-1}c$ or $\alpha \geq \frac{1}{(N^2-N+1)}\left(c - \frac{N-1}{N}S\right)$. If $S < \frac{N}{N-1}c$ and $\alpha < \frac{1}{(N^2-N+1)}\left(c - \frac{N-1}{N}S\right)$, then an interior equilibrium effort exists with $e_{nr} = \frac{N-1}{N}\frac{S}{c-\alpha(N^2-N+1)}$.

Since $N > 1$, a nonrestrictive OSS weakly outperforms both a restrictive OSS as well as the proprietary software in terms of effort. But it is interesting to note that the difference between the two OSS licenses disappears when S is high (i.e., for $S \geq \frac{N}{N-1}c$); and the OSS licenses provide the same effort as under a proprietary license when α is high. This result can be used empirically to elicit developers' motivations in any software project. If all else equal, difference in efforts between a proprietary license and a non-restrictive OSS license is significant, while developers' efforts are similar between the two OSS licenses, it implies that software development is motivated more by reputational benefits from participating rather than user benefits from the software.[2]

2.2 Product Market Competition and OSS Licensing

Our model of effort provision can be extended to analyze how licensing influences entry and competition in the software market. In this section, we provide a preliminary analysis of such a model. Assume that there is already an incumbent proprietary firm selling software B of value v_B to users. N developers exert effort $e_i \in [0, 1]$ into developing a new open source software which we denote by A. As before, software development in A can occur under a restrictive or non-restrictive OSS license. The realized value of this software is v_A. The game now has two stages. The model described in the previous subsection occurs in the first stage with developers exerting effort to produce software A. In the second stage, there is price competition between software A and B. Assume that users buy the higher valued software if indifferent and if they are indifferent between A and B, they choose A. To focus on the effects of competition on OSS development, we set $S = 0$ (i.e., developers are only driven by user benefits) and $\alpha = 1$.[3]

[2] Various measures of developers' participation or effort provision have been used in empirical studies of OSS. Fershtman and Gandal (2007) use lines of code entered by developers into an OSS project to measure developer's effort. Subramaniam et al. (2009) use number of active contributors to an OSS project as their measure of developers' participation. Both these papers use data from Sourceforge.net which is a large database of open source projects.

[3] There is no reason to expect that the size of the reputational prize, S, will be affected by competition. Hence, the presence of an incumbent does not alter the relationship between S and relative effort provision under the two OSS licenses. In other words, holding v_B constant, the difference in effort provision between R and NR licenses disappears for S high. Thus we expect the empirical tests proposed in the previous section

Looking at the second stage of the game, we can now have two different competitive situations in the software market - (1) where the incumbent software is proprietary while the new software is developed under a restrictive OSS license in stage 1 and hence is sold for free in stage 2; or (2) both the incumbent and the new software are sold at positive prices in stage 2 because the new software adopted an NR license in stage 1.

Let us start with the first case. If A is developed under a restrictive OSS license, its price is $p_A = 0$. If $v_A \geq v_B$, then all users buy the new software and B is effectively foreclosed from the market. Users get a surplus of v_A. If, however, $v_A < v_B$, then users will buy B as long as $(v_B - p_B) \geq v_A$. This means that the proprietor of B charges $p_B = (v_B - v_A)$ and all users consume B. Note however that the user surplus is still v_A. This gives us a striking result that since user surplus does not depend on v_B, developers' effort in the first stage is unaffected by the presence of proprietary competition. Thus equilibrium effort in stage 1 is identical to what we derived in the previous subsection.

This, however, is not the case for the other competitive match-up. In case (2), where both software are sold at positive price in stage 2 (because the new software adopted an NR license in stage 1), with Bertrand price competition, if $v_A \geq v_B$, then everyone uses software A and $p_A = (v_A - v_B)$. The winning developer who is the proprietor of A consumes A herself and also sells it to the remaining $(N - 1)$ developers at p_A to get a surplus of $v_A + (N - 1)(v_A - v_B)$; while the remaining developers who buy A at p_A receive a surplus of v_B. On the other hand, if $v_A < v_B$, then everyone uses B at $p_B = (v_B - v_A)$ and receives a user surplus of v_A. Thus the expected stage 2 surplus to developers in software A is no longer just v_A, but it rather depends on the value of the incumbent software, v_B. This is the case as long as there is a positive probability that $v_A > v_B$. Thus we find that the presence of an incumbent proprietary software has no effect on OSS development when it adopts a R license, but it *can* affect developers' effort when it adopts an NR license.

Further, comparing the outcome to what would occur under a proprietary production process for software A, we find that even without any signaling benefits, it is possible for a restrictive OSS license to beat the proprietary production model in terms of effort provision when there is a high value incumbent software proprietor in the market. To see how, consider the case where v_B is so high that software A can never exceed the value of B, i.e., $v_B > 2N \geq v_A$ for all $e_i \in [0, 1]$ and $\varepsilon_i \in [0, 2]$. If A were developed by a proprietary firm in stage 1, profits to the proprietor are always zero in stage 2, so that development will not occur. On the other hand, under an OSS license (both R and NR licenses), the stage 2 user surplus from effort is v_A. In this situation, both OSS licenses induce greater effort than proprietary production. Moreover, among the two OSS licenses, there is no difference in effort provision. This result highlights the importance of controlling for competition when looking at the relationship between developers' effort and OSS licensing. In particular, we see that even in the absence of reputational motivations, the difference in effort between the two licenses disappears when there is a high valued incumbent in the market.

Our findings here provide some preliminary predictions regarding how competition affects OSS outcomes. However, our analysis is incomplete in a few different ways. First, a more complete analysis to determine first stage effort levels will allow comparative statics analysis

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to be robust to a model with competition as long as the competitive effects can be adequately controlled in the regression.

into the interactions between licensing and competition. For example, it will enable us to understand how v_B affects effort provision under an NR license. It will also allow us to compare the total welfare generated by the two licenses. Finally, we can also consider the impact of competition from an incumbent OSS. We leave this for future research.

3 Conclusion and Further Research

The basic model laid out in this paper provides a starting point for analyzing a number of theoretical and empirical issues that surround OSS development. First, even our simple model provides a good empirical test of developers' motivation. If, all else equal, the difference in effort between restrictive and non-restrictive OSS licenses is small, it is likely that reputational concerns rather than user-based motives are incentivizing developers' participation. A further confirmation of this can be achieved if, simultaneously, effort invested into proprietary software is significantly lower than under OSS licenses. Second, we also show that product market competition plays a crucial role in determining which type of license performs the best. This result provides a note of caution about interpreting empirical results on developers' participation under different OSS licenses. None of the empirical research on developers' participation in OSS controls for competition and hence in our view greater attention needs to be paid to the control variable.

From a theoretical perspective, our basic model can benefit from a number of generalizations. For example, while we have assumed that the incumbent software producer is proprietary, it is likely that some of our results on how competition changes effort provision will change if the existing software had an OSS license. Second, one could also simultaneously model the development of the incumbent software B to get a better insight into how developers choose to participate among software with different licenses.

Our model can also be extended to look into other interesting issues in OSS licensing, particularly, license compatibility, viral licenses and dual licensing. The terms of restrictive licenses, such as GPL, usually create a conflict when the software is combined with other differently licensed codes. If the incumbent software B is a complement rather than a substitute for the new software A, then the license for B will have an important effect on the value of A beyond effort incentives. So if B has a GPL license, A will have a higher value if it also adopts a GPL license. This is why GPL is sometimes referred to as a viral license.

One way around the problem of license compatibility is to release the software under a dual license, i.e., both a proprietary license as well as an open source license. Typically, use of the proprietary license requires a payment to the license owner, while the open source version requires all modifications to be kept open as with the restrictive license. A dual license allows the license owner to appropriate some of the future proprietary profits generated from the dual licensed software and to allow license compatibility with other software released under restrictive terms (Valimaki, 2003; Comino and Manenti, 2011). On the other hand, the restrictive license poses a competitive threat to the proprietary version especially if it generates a better software product to users. Incorporating both competitive and complementarity effects of software development into our model will help to disentangle these trade-offs in a dual license.

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A Appendix

In the following subsections, we derive equilibrium effort provision under restrictive and non-restrictive OSS licenses. We then introduce competition from an existing proprietary software to derive some preliminary results about the effect of competition on effort provision under each type of license.

A.1 Effort and OSS license

In both types of OSS licenses, developer i receives the reputational payoff if and only if $d_i > \max\{d_j\}_{j \neq i}$. Since we are looking for a symmetric equilibrium, let us fix the effort of all developers $j \neq i$ as e_L, where $L \in \{r, nr\}$ represents the type of the OSS license. Then the probability that $d_i > \max\{d_j\}_{j \neq i}$ is $P(e_i, e_L) = \Pr\left[e_i \varepsilon_i > e_L \max\{\varepsilon_j\}_{j \neq i}\right]$. Using the uniform distribution for ε_i and ε_j for all $j \neq i$, this probability can be derived as:

$$P(e_i, e_L) = \begin{cases} 0 & \text{if} \quad e_i = 0 \text{ and } e_L \geq 0, \\ \int\limits_0^2 \left(\frac{e_i}{e_L}\frac{\varepsilon_i}{2}\right)^{N-1} \frac{d\varepsilon_i}{2} & \text{if} \quad e_L \geq e_i > 0, \\ \int\limits_0^{\frac{2e_L}{e_i}} \left(\frac{e_i}{e_L}\frac{\varepsilon_i}{2}\right)^{N-1} \frac{d\varepsilon_i}{2} + \int\limits_{\frac{2e_L}{e_i}}^2 \frac{d\varepsilon_i}{2} & \text{if} \quad e_i \geq e_L > 0, \\ 1 & \text{if} \quad e_L = 0 \text{ and } e_i > 0. \end{cases} \tag{1}$$

For $e_L = 0$, $\lim\limits_{e_i \to 0} \frac{dP(e_i, e_L)}{de_i} = \infty$. For $e_L > 0$, differentiating with respect to e_i, we get

$$\frac{dP(e_i, e_L)}{de_i} = \begin{cases} \frac{N-1}{N}\frac{1}{e_L}\left(\frac{e_i}{e_L}\right)^{N-2} & \text{if} \quad e_L \geq e_i > 0, \\ \frac{N-1}{N}\frac{e_L}{(e_i)^2} & \text{if} \quad e_i \geq e_L > 0. \end{cases} \tag{2}$$

$\frac{d^2 P(e_i, e_L)}{de_i^2} > 0$ for $e_i < e_L$. Hence $P(e_i, e_L)$ is concave only in the range $e_i \geq e_L > 0$ and we are going to look for a symmetric equilibrium for OSS licenses in this range only.

A.1.1 Restrictive OSS License

Suppose the other $N-1$ developers exert e_r in equilibrium, the payoff to developer i from e_i is

$$v_r^d = P\left(e_i, e_r\right) S + \alpha \left(e_i + \left(N-1\right) e_r\right) - ce_i,$$

where the expression for $P\left(e_i, e_r\right)$ is given in equation (1) with $e_L = e_r$.

Differentiating with respect to e_i and setting $e_L = e_r$ in equation (2), we get

$$\frac{dv_r^d}{de_i} = \frac{N-1}{N} S \frac{e_r}{\left(e_i\right)^2} + \alpha - c.$$

In a symmetric equilibrium where $e_i = e_r$, if $S \geq \frac{N}{N-1} c$ or if $\alpha \geq c - \frac{N-1}{N} S$, then $\frac{dv_r^d}{de_i} \mid_{e_i=1} > 0$ so that $e_i = e_r = 1$. For $S < \frac{N}{N-1} c$ and $\alpha < c - \frac{N-1}{N} S$, we have $e_i = e_r = \frac{N-1}{N} \frac{S}{(c-\alpha)}$.

Comparing effort across proprietary and restrictive OSS, we have that $e_p > e_r$ if and only if $0 \leq S < c$ and $\frac{c}{N} \leq \alpha < c - \frac{N-1}{N} S$. In all other cases, $e_r \geq e_p$.

A.1.2 Non-restrictive OSS License

If developer i exerts effort i and the remaining $N-1$ developers exert effort e_{nr}, the payoff to i is:

$$v_{nr}^d = P\left(e_i, e_{nr}\right) \left[S + N\alpha \left(e_i + \left(N-1\right) e_{nr}\right)\right] - ce_i,$$

where $P\left(e_i, e_{nr}\right)$ is the same probability described in (1) with $e_L = e_{nr}$.

The derivative with respect to e_i is

$$\begin{aligned} \frac{dv_{nr}^d}{de_i} &= \frac{N-1}{N} \frac{e_{nr}}{\left(e_i\right)^2} \left[S + N\alpha \left(e_i + \left(N-1\right) e_{nr}\right)\right] + P\left(e_i, e_{nr}\right) N\alpha - c \\ &= \frac{N-1}{N} \frac{e_{nr}}{\left(e_i\right)^2} \left[S + N\alpha \left(e_i + \left(N-1\right) e_{nr}\right)\right] + N\alpha \left(1 - \frac{N-1}{N} \frac{e_{nr}}{e_i}\right) - c. \end{aligned}$$

Again, as with the restrictive OSS license, in a symmetric equilibrium, for $S \geq \frac{N}{N-1} c$ or $\alpha \geq \frac{1}{(N^2-N+1)} \left(c - \frac{N-1}{N} S\right)$, $\frac{dv_{nr}^d}{de_i} \mid_{e_i=1} > 0$ so that $e_i = e_{nr} = 1$. For $S < \frac{N}{N-1} c$ and $\alpha < \frac{1}{(N^2-N+1)} \left(c - \frac{N-1}{N} S\right)$, we have $e_{nr} = \frac{N-1}{N} \frac{S}{c - \alpha(N^2-N+1)}$.

Comparing e_{nr} with e_r and e_p, we get that $e_{nr} \geq e_r$ and $e_{nr} \geq e_p$ always.

A.2 Product Market Competition

We assume that $S = 0$ and $\alpha = 1$. In the absence of any competition, i.e., if $v_B = 0$, the results from the previous model with the new parameter assumptions yield the following effort levels in equilibrium.

Under proprietary software development,

$$e_p = \begin{cases} 1 & \text{if} \quad c \leq N, \\ 0 & \text{if} \quad c > N. \end{cases}$$

If software A adopts a restrictive OSS license,

$$e_r = \begin{cases} 1 & \text{if} \quad c \leq 1, \\ 0 & \text{if} \quad c > 1. \end{cases} \tag{3}$$

As discussed above, in the absence of reputational benefits, the restrictive OSS always under-performs relative to proprietary software.

Finally, if A adopts a non-restrictive OSS license,

$$e_{nr} = \begin{cases} 1 & \text{if} \quad c \leq N^2 - N + 1, \\ 0 & \text{if} \quad c > (N^2 - N + 1). \end{cases}$$

Again, the non-restrictive OSS license provides the highest software value of all licenses since $N^2 - N + 1 > N > 1$.

A.2.1 Restrictive OSS

If the new software is distributed under a restrictive OSS license, its price is zero. This means that if $v_A < v_B$, then the proprietary price for B will be $p_B = (v_B - v_A)$ and all users buy software B. If $v_A \geq v_B$, then all users will use A. In both cases, the user surplus to each developer is v_A. Note that since developer's surplus is independent of v_B, the existence of proprietary competition has no effect on the development of restrictive OSS. So $e_r^B = e_r$ given in equation (3).

A.2.2 Proprietary

First let us derive the Bertrand Nash Equilibrium in prices when A and B are both proprietary.

Case 1: $v_B > v_A$

Given p_A, B's best response is $p_B = (v_B - v_A + p_A)$ if $p_A \leq v_A$ and $p_B = v_B$ if $p_A > v_A$.

If $p_B = v_B$, then A's best response is $p_A = v_A - \phi$ where ϕ is arbitrarily close to zero. So this is not an equilibrium.

If $p_B = (v_B - v_A + p_A)$, then A's best response is $p_A = v_A - v_B + p_B - \phi$ where ϕ is arbitrarily close to zero as long as $v_A - v_B + p_B > 0$.

Solving the best response functions for A and B, we get a price equilibrium where $p_B = v_B - v_A$ and $p_A = 0$. This means that proprietor A makes zero revenue.

Case 2: $v_A \geq v_B$

Then working in the same manner as in Case 1, we get $p_A = v_A - v_B$ and $p_B = 0$. The total revenue to proprietor A is then $N(v_A - v_B)$.

A.2.3 Non-Restrictive OSS License

Now let us look at what happens under a non-restrictive OSS license. Price competition yields the same Bertrand price equilibrium as in the proprietary case. However, the winning developer's payoffs are now different.

If $v_A < v_B$, then each developer gets a user surplus of v_A. If $v_A > v_B$, then the losing developer gets a user surplus of v_B which is independent of effort; the winning developer gets a user surplus of v_A and revenue from selling software A to the remaining $N-1$ users, totaling a revenue of $v_A + (N-1)(v_A - v_B)$.