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A multidimensional approach to measure health

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Abstract

In this paper, we borrow the methodology of the so-called "fuzzy approach to multidimensional poverty measurement" (see Cerioli and Zani 1990) in order to propose a different approach to measuring health. This multidimensional indicator satisfies three decomposition properties: by group, by health items, and the multidimensional decomposition. These techniques allow to better evaluate the structure of the population's global health. An empirical application of this synthetic indicator is proposed using the fifth wave of the SHARE survey in order to analyze the health status of the Luxembourgish population.

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1. Introduction

There exist several ways of measuring health though each refers to one of the three dimensions of an individual's health status (Sermet and Cambois 2002 and Blaxter 1985). The first one is the 'medical health', usually measured using the mortality rate and/or the morbidity rate which is the proportion of patients with a particular disease during a given year. The second one is the 'functional health' which is measured *via* the limitation on activities. More precisely, this measure is based on those restrictions in activities of daily living imposed by the individuals' health. Finally, the third dimension of an individual's health status is the 'subjective health' measured as self-assessment health, symptoms and quality of life. In particular, the self-assessment health is one of the most common collected measurement of health in surveys (see Tubeuf *et al*. 2008). This categorical variable is obtained on the basis of the respondents' subjective assessment of their health status often coded on a four or five point scale. Even if this method is a very good predictor of mortality (Idler and Benyamini 1997) or health care utilization (DeSalvo *et al*. 2005), and can be multidimensional (it aggregates the perception about different items of health), the perception of health status and the way it is declared by individuals is affected by individual characteristics (e.g. age, education, gender, nationality, see Barnay and Debrand 2006).

In order to redress the subjectivity of the declared indicators, this paper proposes a composite indicator of health following the methodology of the so-called "fuzzy approach to multidimensional poverty measurement" (see Cerioli and Zani 1990). The general idea of this indicator is that aggregates different items of health reflecting both the mental and physical dimensions of health. In addition, the fuzzy approach to the measurement of health allows considering that there are certainly cases in which an individual's health is such that he may be considered, without any doubt, as not healthy. There are also cases where one can easily conclude that the individual is healthy. There are however, also instances where it is not clear whether a given individual is healthy while according to other variables the individual should not be regarded as healthy. Additionally, for some items of health, individuals can be affected with different intensities.[1]

The suggested composite indicator includes different items of health evaluated by diagnosed or reported diseases and limitations related to mental and physical dimensions of health. This indicator has some interesting mathematical properties. Particularly, it can be decomposed by group of population in order to identify the groups that contribute the most to explain the health status of the whole population. It also satisfies the decomposition by item of health in order to know the contribution of each health variable to the overall health status. Finally, it belongs to a class of indexes that are simultaneously decomposable by item and by group. This property was first introduced by Chakravarty *et al.* (1998). Then, all the couples 'item of health/group' can be calculated.

This paper is organized as follow: Section 2 presents the multidimensional index of health based on the fuzzy set theory. The three decomposition properties satisfied by this indicator are described in Section 3. The SHARE database is presented in Section 4. The health items

[1] The use of fuzzy set theory in health-related issues is not entirely new. In order to compare patient's leakages in the Canadian and U.S. health care systems Makdissi *et al.* (2011) use a fuzzy-fuzzy stochastic dominance approach. In others, Pi Alperin and Berzosa (2011) use the fuzzy set theory to propose a different way to measure overweight taking into consideration the extent and the intensity level of overweight and obesity.

included in the multidimensional indicator are also discussed in this section. Section 5 exposes the main results of the analysis of the health status of the Luxembourgish population. Finally, Section 6 concludes the paper.

2. The multidimensional index of health

In the theory of conventional sets, the health status of an individual would be represented by a dichotomous function that can take the values 0 and 1, following the idea that an individual is affected or not by a specific disease. The fuzzy set theory allows us to consider a more general membership function for each individual according to each health status variable, $\emptyset_P: R_+^1 \rightarrow [0, 1]$. Therefore, the fuzzy logic breaks the constraint of only two possible states of health (i.e. good or bad health) by allowing for a gradual transition between one health outcome to another one.

Let $A = \{1, \dots, i, \dots, N\}$ be the population set, where N is the cardinality of A and n_i the number of individuals that the sample observation i represents in the population.

Let consider that each observed individual is represented by an M-vector of items of health, $\vec{h}_i = (h_{i1}, \dots, h_{ij}, \dots, h_{iM}) \in R_+^M$, where R_+^M stands for the non-negative orthant of the M-dimensional Euclidean space. The vector $\vec{h}_i$ is the i-th row of an $N \times M$ matrix $H^N$and $Q^N$is the set of all $N \times M$ matrix whose entries are positive real numbers. Each column of $H \in Q^N$gives the distribution of the j-th item of health $(j = 1, \dots, M)$ among N individuals.

Let P be the fuzzy subset of sick individuals in A, where each $i \in P$ presents a degree of disease in at least one health item. The degree of membership of the i-th individual $(i = 1, \dots, N)$ to fuzzy subset *P*, h_{ij}, is defined as the health status of individual *i* with respect of the j-th item of health $(j = 1, \dots, M)$. In particular: (*i*) $h_{ij} = 1$ if the i-th individual is completely affected by the j-th disease; (*ii*) $h_{ij} = 0$ if the i-th individual is not affected by the j-th disease; and (*iii*) $0 < h_{ij} < 1$ if the i-th individual is affected to some extent by the j-th disease (but not completely affected).

In order to aggregate the health items in one single indicator, we use the system of weights proposed by Betti and Verma (1998). The weights take into account the intensity of the *j*-th diseases and limit the influence of those diseases that are highly correlated. The weight of any item of health is defined as follows:

$$\widetilde{w_j} = w_j^a . w_j^b, \qquad (1)$$

where w_j^a only depends on the distribution of the j-th item, whereas w_j^b depends on the correlation between j and the other items. In particular, w_j^a is determined by the coefficient of variation of h_{ij},

$$w_j^a = \sum_{i=1}^{N} (h_{ij} - \overline{h_j})^2 / (\overline{h_j}.N)^{1/2}. \qquad (1')$$

were $\overline{h_j}$ is the arithmetic average of the *j*-th item among the whole population.

The weights w_j^b are computed as follows:

$$w_j^b = \left[1 + \sum_{j=1}^{M} \rho_{j,j'} F\left(\rho_{j,j'} < \rho_H\right)\right]^{-1} . \left[\sum_{j=1}^{M} \rho_{j,j'} F\left(\rho_{j,j'} \geq \rho_H\right)\right]^{-1}, \quad (1'')$$

where $\rho_{j,j'}$ is the correlation coefficient between items j and j' and $F(.)$ is an indicator function valued to be 1 if the expression in brackets is true and 0 otherwise. ρ_H is a pre-determined cut-off correlation level between the two items.[2] In other terms, ρ_H separates high and low correlations. w_j^b is the inverse of a measure of average correlation of item j with the others. The largest the average correlation with item j, the lowest the resulting weight for that item.

The multidimensional health index of the i-th individual is thus written as a weighted average of h_{ij},

$$\emptyset_i = \sum_{j=1}^{M} h_{ij} w_j / \sum_{j=1}^{M} w_j. \quad (2)$$

In particular, (*i*) $\emptyset_i = 0$ if the i-th individual is completely healthy; (*ii*) $\emptyset_i = 1$ if the i-th individual is totally affected by all the M diseases; and (*iii*) $0 < \emptyset_i < 1$ if the i-th individual is partially, or totally, affected by some diseases but not fully affected by all of them.

It is also possible to construct an unidimensional index for each one of the j-th item of heath. In particular, $\emptyset_j$ measures the degree of illness of the entire population of N individuals with respect to the j-th health item,

$$\emptyset_j = \sum_{i=1}^{N} h_{ij} n_i / \sum_{i=1}^{N} n_i. \quad (3)$$

Finally, the multidimensional health index for the entire population can be written as a weighted average of the unidimensional index of health, $\emptyset_j$ (eq. "(3)"):[3]

$$\emptyset_P = \sum_{j=1}^{M} \emptyset_j w_j / \sum_{j=1}^{M} \emptyset_j w_j. \quad (4)$$

$\emptyset_P$ measures the global health outcomes considering all the M diseases for the entire population. Specifically, (*i*) $\emptyset_P = 0$ if all the individuals of the population are healthy; (*ii*) $\emptyset_P = 1$ if all the individuals are fully affected in all of the M items; and (*iii*) $0 < \emptyset_P < 1$ if some individuals are partially, or totally, affected by some diseases but not all of them.

3. Decomposition properties

This section relies on a previous paper by Mussard and Pi Alperin (2007) and briefly reviews the decomposition properties of the multidimensional index of health. In particular, the decomposition techniques allow to evaluate the structure of the population's global health. These techniques are the decomposition by sub-population groups, the decomposition by items of health and, the multidimensional decomposition which is obtained as a mixture of the first two decomposition methods. Other mathematical properties of the multidimensional health index are presented in Appendix A.1.

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[2] Betti and Verma (1998) suggest setting ρ_H as to divide the ordered set of correlations at the point of the largest gap.

[3] The multidimensional health index can also be written as a weighted average of the multidimensional index of health of each individual, $\emptyset_i$, then; $\emptyset_P = \sum_{i=1}^{N} \emptyset_i n_i / \sum_{i=1}^{N} n_i$.

3.1 Group decomposition

The group decomposition property involves the principle of "consistency of sub-groups". In other words, when the disease level increase (decrease) in any of one of the population groups, overall disease level increases (decreases) (see Foster *et al.* 1984, and Foster and Shorrocks 1991). This technique was first axiomatized by Bourguignon (1979) and Shorrocks (1980, 1984).

Let us divide the total population into S groups of size N_k $(k = 1, \dots, S)$, e.g. regions, gender, etc. The health status of the i-th individual in group k is given by the following equation:

$$\emptyset_i^k = \sum_{j=1}^{M} h_{ij}^k w_j / \sum_{j=1}^{M} w_j, \quad (5)$$

where h_{ij}^k represents the degree of membership related to the fuzzy sub-set P of the i-th individual in sub-group k with respect to the j-th item of health.

The multidimensional index of health in group k can be written as a weighted average of $\emptyset_i^k$:[4]

$$\emptyset_P^k = \sum_{i=1}^{N_k} \emptyset_i^k n_i^k / \sum_{i=1}^{N_k} n_i^k. \quad (6)$$

Following the previous equations, the multidimensional index of health for the entire population can be written as a weighted average of the multidimensional index for each group:

$$\emptyset_P = \sum_{k=1}^{S} \sum_{i=1}^{N_k} \emptyset_i^k n_i^k / \sum_{k=1}^{S} \sum_{i=1}^{N_k} n_i^k. \quad (7)$$

Consequently, the contribution of the k-th group of population to the global state of health can be written as follows:

$$C_P^k = \sum_{i=1}^{N_k} \emptyset_i^k n_i^k / \sum_{i=1}^{N} n_i. \quad (8)$$

In this respect, the groups that contribute the most to explain the health status of the entire population can be identified.

3.2 Item decomposition

The synthetic indicators aggregate a set of items selected to study the population health status. The indicators that satisfy the item decomposition property are those that can be written as a weighted average of unidimensional indices for each item of health. Shorrocks (1982) first axiomatized this technique.

From eq. "(3)", it is possible to deduce the contribution level of the j-th item to $\emptyset_P$:

$$C_P^j = \emptyset_j w_j / \sum_{j=1}^{M} w_j. \quad (12)$$

The decomposition by items allows to identify both the relative distribution of each health item and the diseases that have the largest contribution to the state of global health.

[4] Of the *k*-th group is divided into *x* sub-groups, then a multilevel decomposition is possible.

3.3 Multidimensional decomposition

The multidimensional decomposition aims at obtaining a mixture of both decomposed components, *i.e.* groups and items (see *e.g.* Shorrocks 1990, and Yitzhaki 2002). Particularly, this method yields the contribution of all couples "item of health/group of population" to the health status of the global population. This multi-decomposition rule is respected by the indicators relying on fuzzy set theory (see Mussard and Pi Alperin 2007).

Let us consider the unidimensional index of the j-th item of health in the k-th group,

$$\emptyset_j^k = \sum_{i=1}^{N_k} h_{ij}^k n_i^k / \sum_{i=1}^{N_k} n_i^k. \tag{10}$$

From the previous equation, it is also possible to compute the multidimensional index of health for the k-th group,

$$\emptyset_P^k = \sum_{j=1}^{M} \emptyset_j^k w_j / \sum_{j=1}^{M} w_j. \tag{11}$$

The synthetic indicator of health for the entire population can be derived from equations "(10)" and "(11)". In particular, it is a weighted average of the multidimensional indicators of health for each sub-population,

$$\emptyset_P = \sum_{k=1}^{S} \sum_{j=1}^{M} \emptyset_j^k w_j / \sum_{j=1}^{M} w_j. \tag{12}$$

Finally, the contribution of the j-th item of health of the k-th group to the global state of health for the entire population P is :

$$C_P^{jk} = \emptyset_j^k w_j / \sum_{j=1}^{M} w_j. \tag{13}$$

4. Data and descriptive statistics

This paper uses data from wave 5 Release 1.0.0 of the Survey of Health, Ageing and Retirement in Europe (SHARE; Börsch-Supan *et al.* 2013; Börsch-Supan 2015, and Malter and Börsch-Supan 2015). SHARE is a multidisciplinary and cross-national panel database collecting micro data on health, socio-economic status and social and family networks with the objective of better understand the ageing process. The wave 5 was collected in 2013 in fourteen European countries (Austria, Belgium, Czech Republic, Denmark, Estonia, France, Germany, Italy, the Netherlands, Luxembourg, Slovenia, Spain, Sweden, and Switzerland) and Israel.

In particular, we analyze the health status of individuals living in Luxembourg. After excluding all individuals with missing values on any of the variables used in our empirical analysis, our estimation sample includes 1610 individuals aged 50 and older and their partners. Among the whole sample, 53% of the studied population are females, 55% have less than 65 years old and only 10% are 80 years old or more. Concerning the perception of the individuals about their health status, 28% of the population declared that their health is excellent or very good, 63% good or fair and only 9% think that they have poor health (see Table 1).

4.1. Health items

One of the most important steps in the measurement of a synthetic indicator of health is the selection of those items that will be included in the multidimensional index. We exploit nine single or composite items reflecting different aspects of the mental and physical health status of the individuals.[5]

Depression and *memory* are two main items of mental health. In particular, *depression* is defined on the basis of the Euro-D symptom scale that measures current depression covering twelve different aspects. The index is then scored by summing binary items. The *memory* item is constructed using one test from the cognitive functions module of SHARE. In particular, this test is about the ability of people to think about things as the day of the month, the month, the year and the day of the week.

Seven items are included in the physical dimension of health. Among them, the item *long term illness* takes into consideration two different aspects. First, having or not any long-term health problem, illness, disability or infirmity; and second, the fact of being limited because of a health problem in those activities that people usually do. The item *other illnesses* includes a list of fourteen health conditions.

Two different items are constructed to outline the limitation of activities: *limitation activities 1* and *limitation activities 2*. Each one of them tries to understand those difficulties people may have with various activities because of a health problem excluding any difficulty that the individual expects to last less than three months.

The *weight problems* item involves individuals with overweight and obesity, but also underweight problems (see Pi Alperin and Berzosa, 2011). Finally, the two last items are the *eyesight* and the *hearing* problems.

5. Results

We have constructed unidimensional health indicators for each one of the nine health items in order to obtain multidimensional health measures.[6] Table 2 reports these results. Among the health items, two of them emerge as the most important causes of the health status of the Luxembourgish population, these are *chronic illness* and *other illness*. Additionally, we have computed the contribution level of each health item to global health. The *weight problems* appear as the item explaining the most the health status of the entire population (24.26%). It is followed by *other illness* (11.90%) and *chronic illness* (11.45).

One way of having more detailed information is dividing the whole population in different groups (see Table 3). Three different decompositions by groups are proposed: by gender (male and female), by age (50-64 years old; 65-79 years old; and 80 or more years old) and by the perception of individuals about their health status (excellent or very good health; good or fair health; and poor health). The female group not only is the most affected group (0.2954) but also the group that contributes the most to explain global health (54.46%). Of course, the group of the oldest is the most affected by the health problems, however the youngest (50-64 years old) explains 47.61% of global health.

[5] See in Appendix A.2 a complete description of the construction of each item.
[6] All the indicators were computed using the *MDEPRIV* program (see Pi Alperin and Van Kerm, 2009).

Table 4, 5 and 6 show in more detail: the unidimensional health items by sub-population group (4th column), the contribution of each item of health in each group (5th column), and the contribution level of the items in each group to explain the health status of global population (6th column) for the gender, age and self-assessment health decomposition, respectively. In all the eight sub-population groups, the *other illness* and *chronic illness* are the two main causes of general health status. Nevertheless, the same items do not explain the global health status of each group. Hence, the health status of the 'men' group is explained by the *weight problems* (27.07%) and *other illness* (12.08%), whereas the 'female' group is mainly explained by *weight problems* (21.94%) and *limitation activities 1* (11.97%). The couples *weight problems/men* and *weights problems/female* contributed the most to the level health status of the entire population (13.03% and 11.38%, respectively).

Concerning the age decomposition and the contribution level of each item to each group, the groups '50-64 years old' and '65-79 years old' are mainly explained by *weights problems* (26.91% and 24.25%, respectively) and by *other illness* (12.27% and 12.01%, respectively). In contrast, the group '80 or more years old' is mainly explained by the items *limitation activities 2* (18.27%) and *weight problems* (16.90%). Finally, the couples *weights problems/65-79, limitation activities 2/80 or more* and *weight problems/80 or more* contribute the most to understand the health level status of the entire population (7.76%, 7.75% and 7.17%, respectively).

Finally, let us have a look to the results of Table 6. In the first two groups, those who declared their health as 'excellent or very good' and 'good or fair', we find that the same items, but not with the same intensity, explained the health level in these groups. These items are *weight problems* (30.76% and 24.43% respectively for each group) and *other illness* (13.88% and 11.98%, respectively). Regarding the last group 'poor health', *limitation activities 2* (19.28%) and *weight problems* (16.66%) are the two most contributing items of health. To conclude, the global health status of the entire population is mostly explained by the couples *limitation activities 2/poor health* (9.67%), *weight problems/poor health* (8.35%) and *limitation activities 1/poor health* (7.74%).

6. Concluding remarks

This paper proposes to adapt the methodology based on the fuzzy set theory to measure multidimensional poverty to construct a composite indicator of the health status of the population. This indicator satisfies three decomposition properties: by group, by health items, and the so-called multidimensional decomposition.

We used the fifth wave of the SHARE database to analyze the health status of the Luxembourgish population older than 50 years old. Nine different items of health are selected and included in the composite indicator. Two of them characterize the mental dimension of health and the others seven the physical dimension. Three-group decompositions where proposed: by gender, age and self-assessed health. The results show that in each case the items *chronic illness* and *other illness* are the cause of the health level of the sub-populations. In contrast, when analyzing the contributions levels, most of them are mainly explained by the *weight problems*. Finally, the contribution of the physical dimension of health is more important to explain the global population health status in all decompositions. This application illustrates how useful this methodology can be for health policy analysis.

Table 1: *Sample composition by subgroup of population*

Group decomposition		Sample size	In percentage
Gender	Men	755	47
	Women	855	53
Age	< 65	877	55
	65-79	566	35
	80 or +	167	10
Self assessment health	Excellent, Very good	456	28
	Good, Fair	1002	63
	Poor	152	9
Education level	None, primary	595	37
	Secondary	718	45
	Post-secondary	297	18
Professional status*	1	319	20
	2	637	40
	3	207	12
	4	321	20
	5	126	8
Nationality	Luxembourgish	1184	74
	Portuguese	169	10
	EU-15	222	14
	Other non EU-15	35	2
Total		1610	100

* 1: Legislator, senior official or manager; Professional. 2: Technician or associate professional; Clerk; Service worker and shop and market sales worker; Armed forces. 3: Skilled agricultural or fishery worker; Craft and related trades worker. 4: Plant and machine operator or assembler; Elementary occupation. 5: Never worked.

Table 2: *Unidimensional indicators of health*

Health items	Unidimensional index	Contribution to MHI (in %)
Depression	0.2746	6.79
Memory	0.3292	6.33
Chronic illness	0.7264	11.45
Other illness	0.7557	11.90
Limitation activities 1	0.2810	10.09
Limitation activities 2	0.0961	8.15
Weight problems	0.4483	24.26
Eyesight	0.2444	11.11
Hearing	0.1736	9.93
Total	**0.2834**	**100**

Table 3: *Multidimensional indicators of health by subpopulation groups*

Group decomposition		MHI by group	Contribution to MHI
Gender	Men	0.2742	45.54
	Women	0.2954	54.46
Age	50-64	0.2460	47.61
	65-79	0.3075	35.54
	80 or +	0.4080	16.86
Self assessment health	Excellent, Very good	0.1759	17.33
	Good, Fair	0.3047	66.62
	Poor	0.4831	16.05

Table 4: *Unidimensional indicators of health by gender*

Sub-population group		Health Items	Index	Cont. group	Cont. pop
Gender	Men	Depression	0.2384	6.09	2.93
		Memory	0.3146	6.25	3.01
		Chronic illness	0.7166	11.68	5.62
		Other illness	0.7428	12.08	5.81
		Lim. Act. 1	0.2113	7.84	3.78
		Lim. Act. 2	0.0742	6.50	3.13
		Weight probl.	0.4843	27.07	13.03
		Eyesight	0.2312	10.86	5.23
		Hearing	0.1967	11.63	5.60
	Women	Depression	0.3077	7.31	3.79
		Memory	0.3464	6.39	3.31
		Chronic illness	0.7306	11.06	5.73
		Other illness	0.7626	11.51	5.97
		Lim. Act. 1	0.3475	11.97	6.21
		Lim. Act. 2	0.1233	10.02	5.20
		Weight probl.	0.4228	21.94	11.38
		Eyesight	0.2591	11.29	5.86
		Hearing	0.1550	8.51	4.41

Table 5: *Unidimensional indicators of health by age*

Sub-population group		Health Items	Index	Cont. group	Cont. pop
Age	50-64	Depression	0.2728	7.77	1.99
		Memory	0.2940	6.51	1.67
		Chronic illness	0.6444	11.71	3.00
		Other illness	0.6766	12.27	3.14
		Lim. Act. 1	0.2181	9.02	2.31
		Lim. Act. 2	0.0472	4.60	1.18
		Weight probl.	0.4318	26.91	6.88
		Eyesight	0.2198	11.51	2.94
		Hearing	0.1471	9.70	2.48
	65-79	Depression	0.2685	6.12	1.96
		Memory	0.3556	6.30	2.02
		Chronic illness	0.8057	11.71	3.75
		Other illness	0.8283	12.01	3.84
		Lim. Act. 1	0.3033	10.04	3.21
		Lim. Act. 2	0.1134	8.85	2.83
		Weight probl.	0.4866	24.25	7.76
		Eyesight	0.2532	10.60	3.39
		Hearing	0.1916	10.10	3.23
	80 or +	Depression	0.3018	5.18	2.20
		Memory	0.4381	5.85	2.48
		Chronic illness	0.8679	9.51	4.03
		Other illness	0.9022	9.86	4.18
		Lim. Act. 1	0.5297	13.22	5.61
		Lim. Act. 2	0.3105	18.27	7.75
		Weight probl.	0.4498	16.90	7.17
		Eyesight	0.3475	10.97	4.65
		Hearing	0.2575	10.24	4.34

Table 6: *Unidimensional indicators of health by self-assessment health*

Sub-population group		Health items	Index	Cont. group	Cont. pop
Self-assessment health	Excellent, Very good	Depression	0.1888	7.52	1.37
		Memory	0.2588	8.02	1.46
		Chronic illness	0.5030	12.78	2.33
		Other illness	0.5473	13.88	2.53
		Lim. Act. 1	0.0755	4.37	0.80
		Lim. Act. 2	0.0066	0.91	0.17
		Weight probl.	0.3530	30.76	5.61
		Eyesight	0.1574	11.53	2.10
		Hearing	0.1110	10.23	1.87
	Good, Fair	Depression	0.2852	6.56	2.07
		Memory	0.3502	6.26	1.98
		Chronic illness	0.7906	11.60	3.67
		Other illness	0.8182	11.98	3.79
		Lim. Act. 1	0.3080	10.29	3.25
		Lim. Act. 2	0.0984	7.75	2.45
		Weight probl.	0.4855	24.43	7.72
		Eyesight	0.2672	11.30	3.57
		Hearing	0.1847	9.83	3.11
	Poor	Depression	0.4616	6.70	3.36
		Memory	0.4223	4.76	2.39
		Chronic illness	0.9404	8.70	4.36
		Other illness	0.9361	8.64	4.33
		Lim. Act. 1	0.7325	15.43	7.74
		Lim. Act. 2	0.3880	19.28	9.67
		Weight probl.	0.5250	16.66	8.35
		Eyesight	0.3677	9.80	4.91
		Hearing	0.2984	10.02	5.02

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Appendix A.1. Other mathematical properties of the multidimensional health index

The health synthetic indicator is adapted from the construction of the poverty measures based on the fuzzy set theory. In this way, the axiomatic properties associated to this health index are founded from those properties establish for this kind of multidimensional poverty measures. Chakravarty (2006) proposes some axiomatic properties adapted to fuzzy set indexes to measure multidimensional poverty. In the same context, Pi Alperin (2007) extends this list of properties.

Let be P the fuzzy subset of individuals with health problems, P_j the fuzzy subset of individuals affected by the j-th diseases, and F be the set of possible membership function vectors for the set of health items $\vec{z} = (z_1, \dots, z_j, \dots, z_M)$.

The multidimensional index of health is: $\emptyset_P: QxF \to R^1_+$. For each $i \in A,\ H \in Q^N$ and $\vec{z} \in F$, the index $\emptyset_P(H, \vec{z})$ measures the health outcome associated to the H-matrix (composed by N individuals and M items of health).

The multidimensional index of health satisfies the following mathematical properties.

Axiom 1: Focus, $\forall i \in A,\ H, \widehat{H} \in Q^N,\ \vec{z} \in F$, if $P_j(H) = P_j(\widehat{H})$, with $j = 1, \dots, M$ and $h_{ij} = \hat{h}_{ij}$, $\forall i \in P_j(H), j = 1, \dots, M$, then $\emptyset_P(H, \vec{z}) = \emptyset_P(\widehat{H}, \vec{z})$.

Given a population size N, the multidimensional health index only depends on the state of health of those individuals with health problems.

Axiom 2: Normalization, If $\forall i \in A,\ H \in Q^N,\ \vec{z} \in F$, if $P_j(H) = \emptyset$, then $\emptyset_P(H, \vec{z}) = 0$. On the other hand, if $P_j(H) = Ker(A)$, and $Ker(A) = A, \forall j,$ then $\emptyset_P(H, \vec{z}) = 1$.[7]

The normalization refers to the cardinality's property of the health index. This property establishes that if all members of a society are healthy, then the multidimensional health index will be equal to 0. Otherwise, if all members of a society are completely affected by all the M diseases, then, the index will be equal to 1. Then, $\emptyset_P(H, \vec{z}) \in [0,1]$.

Axiom 3: Monotonicity, $\forall i \in A,\ H, \widehat{H} \in Q^N,\ \vec{z} \in F$, if $h_{fl} = \hat{h}_{fl}$, $\forall f = \{1, \dots, N\} - \{i\}$ and $\forall l \in \{1, \dots M\}$ and if $h_{il} = \hat{h}_{il}, \forall l \in \{1, \dots M\} - \{j\}$ but $h_{ij} > \hat{h}_{ij}$, where $i \in P_j$, then $\emptyset_P(H, \vec{z}) > \emptyset_P(\widehat{H}, \vec{z})$.

This axiom establishes that the multidimensional health index shall reduce its value if the state of health of an individual against a specific disease improves. This property includes the possibility that an individual can became healthy with respect of that disease.

Axiom 4: Symmetry, $\forall i \in A,\ H \in Q^N,\ \vec{z} \in F$, then $\emptyset_P(H, \vec{z}) = \emptyset_P(\Pi H, \vec{z})$ where Π is a permutation matrix of NxN order.[8]

[7] The core of a fuzzy set $A \in U$ is an ordinary subset of U in which each element has a degree of membership equal to 1. We note $Ker(A) = \{h \in U;\ \emptyset_A(h) = 1\}$. The core consists of elements that fully satisfy the fuzzy characteristics defined by A. When the core is not empty, i.e. $Ker(A) \neq \emptyset$, we say that the fuzzy set is normal. If A is classic $Ker(A) = A$.

[8] A square matrix of any order with entries 0 and 1 is called a permutation matrix if each of its rows and columns sums to 1.

The symmetry of the multidimensional health index indicates that the measure is invariant after swapping the order of the individuals. Each individual maintains its anonymity.

Axiom 5: Continuity, $\forall i \in A,\ H \in Q^N,\ \vec{z} \in F$, then $\emptyset_P(H, \vec{z})$ is continuous in Q^N.

This principle means that little changes in the health status of individuals in a population does not lead to a sharp jump of the value of the multidimensional health index.

Axiom 6: Increasing membership function, $\forall i \in A,\ H \in Q^N,\ \vec{z}, \hat{\vec{z}} \in F$, if $z_l = \hat{z}_l,\ \forall l \in \{1, \dots M\} - \{j\}$, so $P_l(H) = P_l(\widehat{H})$, and if $z_j(h_{ij}) > \hat{z}_j(h_{ij}),\ \forall i \in P_j$, then $\emptyset_P(H, \vec{z}) > \emptyset_P(H, \hat{\vec{z}})$.

Suppose two identical surfaces, *B* and *C*. We associate a membership function, with respect to the *j*-th item, higher in *B*. Then, individuals in *B* must therefore have a lower level of health because individuals who live there have a higher potential probability of been sick in this item, compare to individuals in *C*.

Axiom: Principle of population, $\forall i \in A,\ H \in Q^N,\ \vec{z} \in F$, $\emptyset_P(H, \vec{z}) = \emptyset_P(\widehat{H}, \vec{z})$ where $\widehat{H}$ is the *h*-th concatenation of *H*.[9]

The principle of population provides an index which remains unchanged when the population is growing equally. This axiom allow constructing indexes whose values are comparable even if the distributions are of different sizes.

Axiom 8: Health growth, $\forall i \in A,\ H \in Q^N,\ \vec{z} \in F$, if $\widehat{H}$ is obtained by adding a completely healthy individual, then $\emptyset_P(H, \vec{z}) > \emptyset_P(\widehat{H}, \vec{z})$.

The inclusion of this axiom leads a higher level of health, and a smaller value of the multidimensional health index, if a healthy individual is incorporated into the society.

Axiom 9: Scale invariance: $\forall i \in A,\ H \in Q^N,\ \vec{z} \in F$, $\emptyset_P(H\Omega, \vec{z}) = \emptyset_P(H, \vec{z})$ where Ω is the diagonal matrix $diag(\omega_1, \dots, \omega_m)$, $\omega_j > 0, \forall j = 1, \dots, m$.

The multidimensional health index is invariant to scale transformation of the quantities of items. It is homogeneous of degree zero. Therefore, the index is independent of the unit of measurement of the items of health.

[9] Let H be a vector, $H = H_1, \dots, H_m$. The concatenation of order *f* consists to replicate the vector *f* times, then $H^f = \left\{\overbrace{H_1, \dots, H_1}^{f\ times}, \dots, \overbrace{H_m, \dots, H_m}^{f\ times}\right\}$.

Appendix A.2: Health items

Table A.2.I: Depression

Depression scale Euro-d*	Degree of membership
Non depressed (0 items)	0
Between 1 and 11 items	1 - (12-X_i)/12
Very depressed (12 items)	1

*This composite indicator take into consideration the following items: depression, pessimism, suicidality, guilty, sleep, interest, irritability, appetite, tiredness, concentration, enjoyment, tearfulness.

Table A.2.II: Memory

Memory and ability to think about things		Degree of membership
Four questions has been asked*	Knows all	0
	Knows 3 of 4	0.3
	Knows 2 of 4	0.6
	Knows 1 of 4	0.9
	Doesn't know	1

*Which day of the moth it is? Which month it is? Which year it is? Can you tell me which day of the week it is?

Table A.2.III: Chronic illness

Long term illness		Degree of membership
Do you have any long-term health problems, illness, disability or infirmity?	No	0
	Yes	1

Table A.2.IV: Other illnesses

Other illnesses		Degree of membership
Doctor told you have any of the following conditions?*	No	0
	One of these conditions	0.75
	Two or more of these conditions	1

*A heart problem; High blood pressure or hypertension; High blood cholesterol; A stroke or cerebral vascular disease; Diabetes or high blood sugar; Chronic lung disease such as chronic bronchitis or emphysema; Asthma; Arthritis, including osteoarthritis, or rheumatism; Osteoporosis; Cancer or malignant tumor; Stomach or duodenal ulcer; Parkinson disease; Cataracts; Hip fracture or femoral fracture.

Table A.2.V: Limitation activities 1

Health and activities		Degree of membership
Because of a health problem, do you have difficulty doing any of the following activities?*	No	0
	One of these activities	0.15
	Two of these activities	0.25
	Three of these activities	0.50
	Four of these activities	0.75
	Five or more of these activities	1

*Walking 100 meters; Sitting for about two hours; Getting up from a chair after sitting for long periods; Climbing several flights of stairs without resting; Climbing one flight of stairs without resting; Stooping, kneeling or crouching; Reaching or extending your arms above shoulder level; Pulling or pushing large objects like a living room chair; Lifting or carrying weight over 5 kilos, like a heavy bag or groceries.

Table A.2.VI: Limitation activities 2

Health and activities		Degree of membership
Because of a health problem, do you have difficulty doing any of the following activities?*	No	0
	One of these activities	0.15
	Two of these activities	0.25
	Three of these activities	0.50
	Four of these activities	0.75
	Five or more of these activities	1

*Dressing, including putting on shoes and socks; Walking across a room; Bathing or showering; Eating, such as cutting up for your food; Getting in or out of bed; Using the toilet, including getting up or down; Using a map to figure out how to get around in a strange place; Preparing a hot meal; Shopping for groceries; Making telephone calls; Taking medications; Doing work around the house or garden; Managing money, such as paying bills and keeping track or expenses.

Table A.2.VII: Weight problems

Weight problems ≤ 65 years old	*Degree of membership*
IMC[10] < 17.5	1
17.5 ≤ IMC < 18.5	(18.5 – IMC) / (18.5 – 17.5)
18.5 ≤ IMC < 25	0
25 ≤ IMC < 30	(30 – IMC) / (30 – 25)
IMC ≥ 30	1
Weight problems ≥ 66 years old	***Degree of membership***
IMC < 21	1
21 ≤ IMC < 23	(23 – IMC) / (23 – 21)
23 ≤ IMC < 27	0
27 ≤ IMC < 30	(30 – IMC) / (30 – 27)
IMC ≥ 30	1

Table A.2.VIII: Eyesight

Eyesight distance and reading*	Degree of membership
Both are E or VG	0
One is E or VG, the other is G or F	0.15
One is E or VG, the other is P	0.25
Both are G or F	0.30
One is G or F, the other is P	0.60
Both are P	1

*E: excellent; VG: very good; G: good; F: fair; P: poor

Table A.2.IX: Hearing

Hearing		Degree of membership
Is your hearing*	Excellent or Very good	0
	Good or Fair	0.15
	Poor	1

*We have also considered individuals that are using a hearing aid as usual.

[10] The Body Mass Index is calculated as follows: IMC= weight (in kg)/height² (in metters).