A tractable cost pass-through benchmark

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Abstract

Under the assumptions of constant marginal costs, linear demand and symmetric cross-price effects, the equilibrium rates at which firms pass cost changes through to prices can be calculated from diversion ratios. The resulting pass-through benchmark makes it possible to capture market asymmetries and can easily be extended to cover not just the conventional cases of firm-specific and industrywide cost changes but also cases where some but not all firms face a cost change and where the degree of exposure to the cost change varies between firms. As such the benchmark is more accurate and adaptable than existing benchmarks and also has a number of practical advantages vis-à-vis more complex approaches.
1. Introduction

The rates at which cost changes are passed through to prices are a key aspect in the quantification of damages resulting from, e.g., competition law infringements. This is because the damages incurred by downstream firms as a result of a price overcharge (the ‘overcharge effect’) are counteracted to the extent that the higher costs are passed through to downstream prices (the ‘pass-through effect’).\(^1\)

If, for example, a cartel raises its prices and cartel outsiders respond by also raising prices, cartel members may be liable for resulting damages incurred by the customers of both cartel members and cartel outsiders. Price increases by cartel outsiders follow from the strategic complementarity of prices and are referred to as ‘umbrella effects’.\(^2\) Since downstream firms may differ in their ability and/or willingness to switch between cartel members and outsiders or in their degree of exposure to the cartelized products, they may also differ in the degree to which they are affected by the cartel overcharge.

Existing theoretical approaches to estimating pass-through are often difficult to apply to real-life damages cases for at least two reasons. First, they tend to focus on the extreme cases of firm-specific and industrywide cost increases. This is often not the case in practice, where it frequently occurs that some but not all firms face cost changes and that the extent of these cost changes varies between firms.

Second, existing approaches focus on characterizing the effects of market features like the shapes of demand and cost curves, on which data is rarely available, while making the simplifying assumption of symmetry between firms, which is demonstrably false in most real market settings. For example, Stennek and Verboven (2001) derive pass-through as a function of supply elasticities and elasticities of elasticities of demand. Similarly RBB (2014) and the literature cited therein derive pass-through rates as functions of elasticities of the slopes of the demand curve. While the curvature of the demand curve no doubt has an impact on pass-through, in damages cases point estimates of demand elasticities and estimates of fixed and marginal costs are usually as good as it gets in terms of data availability.

For this reason applied analyses often resort to the following well-known existing benchmarks:

- In the case of a monopolist pass-through is 50% if demand is linear.
- In the case of perfect competition pass-through is 0% for firm-specific cost changes and 100% for industrywide cost changes.
- In the case of imperfect competition, pass-through is between 0% and 50% for firm-specific cost changes and between 50% and 100% for industrywide cost changes.

While these benchmarks are reasonable they are also crude. The analysis in this article shows that more precise yet still tractable benchmarks can be achieved by making simplifying assumptions

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\(^1\) Of course pass-through also creates additional damages as a result of higher prices leading to reduced sales (the ‘volume effect’). While this is certainly relevant to the quantification of damages, the scope of the present analysis is limited to estimating pass-through rates, which are used to quantify both the pass-through and volume effects.

\(^2\) This article focuses on price competition and differentiated products. In the case of quantity competition and homogeneous products, there is only one market price and customers of cartel outsiders are affected by the same price change as are customers of cartel members. For a discussion of umbrella effects see Inderst et al. (2014).
that are more in line with the data that is likely to be available. In particular, the analysis in this article drops the following assumptions:

- **Cost changes are either firm-specific or industrywide; a firm is either fully affected or not affected by cost changes**—The below analysis instead allows for a subset of firms to be affected by cost changes and to be affected to varying degrees.
- **Symmetry between firms**—The below analysis instead allows for the incorporation of possibly asymmetric diversion ratio data to account for differences between firms in terms of the degree of substitutability between their products.\(^3\)

In addition, to avoid the data issues discussed above, the analysis in this article makes the following assumptions:

- **Linear demand**—This is a sensible default assumption since data on the curvature of the demand curve is rarely available.
- **Constant marginal costs**—Again, this is a sensible default assumption since firms are often unable to provide details about their cost structure beyond a distinction between fixed and variable costs.
- **Symmetric cross-price effects**—This assumption is required for tractability and is common in the literature.\(^4\) This says that switching between firms depends on their relative prices but not their absolute prices, or in other words, that price changes do not have an income effect. This assumption is plausible in many industries and, where it is not, it may still be an appropriate approximation provided that price changes are not too large.

While incorporating the effects of demand curvature, (dis)economies of scale and income effects is not straightforward, it is possible to give an idea of the direction in which one ought to depart from the benchmark in order to capture these effects. For example, pass-through may be exacerbated by economies of scale as a cost increase leads to higher prices and lower output, causing the firm to lose scale and hence reinforcing the initial cost increase and pass-through. Likewise, if customers become more price sensitive when prices rise, this may dampen pass-through.

### 2. The model and result

Suppose \( N \) firms, indexed by \( i \), are competing à la Bertrand, with constant marginal cost \( c_i \) and firm-specific demand as a function of the vector of all firms’ prices given by \( q_i(p) \). Suppose that cross-price effects are symmetric, i.e. that \( \frac{\partial q_i}{\partial p_i} = \frac{\partial q_j}{\partial p_j} \) for all firms \( i \) and \( j \). Suppose moreover that \( c_i = \alpha_i + \beta_i c \), where \( c \) is an input of interest that is common to all firms, \( \beta_i \in [0,1] \) is the firm-specific degree of exposure to this input and \( \alpha_i \) denotes other firm-specific costs.

Firm-specific profit is given by \( \pi_i = (p_i - c_i)q_i(p) \). Equilibrium is defined by the set of first-order conditions:

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\(^3\) Diversion ratios are frequently used in the analysis of the competitive effects of mergers and can be estimated from customer switching data. Customer switching is tracked by many companies, using customer surveys and other market research.

\(^4\) See for example Verboven and Van Dijk (2009).
\[ q_i + (p_i - c_i) \frac{\partial q_i}{\partial p_i} = 0 \]

Totally differentiating this with respect to \( c \) gives

\[
\sum_j \frac{\partial q_i}{\partial p_j} \frac{\partial p_j}{\partial c} + (p_i - c_i) \sum_j \frac{\partial^2 q_i}{\partial p_j \partial p_j} = \sum_j \frac{\partial q_i}{\partial p_j} \frac{\partial p_j}{\partial c} + (p_i - c_i) \frac{\partial}{\partial c} \frac{\partial q_i}{\partial p_i} = 0,
\]

where the first equality follows from the assumption that demand is linear.

Dividing by the slope of the demand curve of firm \( i \) gives

\[
\sum_j \frac{\partial q_i}{\partial p_j} \frac{\partial p_j}{\partial c} - \frac{\partial q_i}{\partial p_i} - \frac{\partial^2 q_i}{\partial p_j \partial p_j} = \sum_j d_{ij} \frac{\partial p_j}{\partial c} + \beta_i = 0,
\]

where \( d_{ij} = \frac{\partial q_j}{\partial p_i} / -\frac{\partial q_i}{\partial p_i} \) is the diversion ratio from \( i \) to \( j \), and where the first equality follows from the assumption that cross-price effects are symmetric.\(^5\) Equivalently,

\[
\frac{\partial p_i}{\partial c} - \sum_j d_{ij} \frac{\partial p_j}{\partial c} = \beta_i
\]

In matrix form this can be written as \((I - D)P = B\), where \( I \) is the \( n \times n \) identity matrix, \( D \) is the \( n \times n \) diversion ratio matrix with entries \( d_{ij} \), \( P \) is the \( n \times 1 \) pass-through vector with entries \( \frac{\partial p_i}{\partial c} \) and \( B \) is the \( n \times 1 \) vector of cost exposure rates with entries \( \beta_i \). Left-multiplying both sides by \((I - D)^{-1}\) proves the following proposition.

**Proposition 1:** In a differentiated Bertrand setting with linear demand, constant marginal costs and symmetric cross-price effects, the vector of pass-through rates is given by \( P = (I - D)^{-1}B \).

This pass-through benchmark incorporates data on the degree of substitutability between different products as well as the different degrees to which firms are exposed to changes in the given cost component.

The case of an industrywide cost change is given if \( B \) is a vector of ones. On the other hand a firm-specific cost increase for firm \( i \) is given if \( B \) is a vector of zeros, except for a one in the \( i \)th entry.

The following three-firm example illustrates how Proposition 1 can be applied in practice. There is no difficulty performing similar analyses for markets with more firms and different inputs.

**3. An illustrative example**

Suppose there are three firms, 1, 2 and 3, with firm 1 fully exposed to the cost increase (\( \beta_1 = 1 \)), firm 2 partially exposed to the cost increase (with the degree of exposure \( \beta_2 \) varying over the interval \([0,1]\)) and firm 3 not exposed to the cost increase (\( \beta_3 = 0 \)). Suppose that the degree of

\[^5\] Here the definition of diversion ratio is not restricted to the case \( i \neq j \), so that \( d_{ii} = -1 \).
substitutability between firms 1 and 2 is given by $d_{12} = d_{21} = a$ and suppose that the degree of substitutability between these two firms on the one hand and firm 3 on the other is given by $d_{13} = d_{31} = d_{23} = d_{32} = b$.

By Proposition 1 the equilibrium pass-through rates are given by

$$
\begin{bmatrix}
PTR_1 \\
PTR_2 \\
PTR_3
\end{bmatrix} = \left(\begin{bmatrix} 1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & a & b \\
 a & -1 & b \\
b & b & -1 \end{bmatrix}\right)^{-1} \begin{bmatrix} 1 \\
\beta_2 \\
0 \end{bmatrix} = \begin{bmatrix} 2 & -a & -b \\
-a & 2 & -b \\
-b & -b & 2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\
\beta_2 \\
0 \end{bmatrix}
$$

It follows that

$$
PTR_1 = \frac{4 - b^2 + 2a\beta_2 + b^2\beta_2}{8 - 2a^2 - 4b^2 - 2ab^2}
$$

$$
PTR_2 = \frac{4\beta_2 + 2a + b^2 - b^2\beta_2}{8 - 2a^2 - 4b^2 - 2ab^2}
$$

$$
PTR_3 = \frac{2b\beta_2 + ab\beta_2 + ab + 2b}{8 - 2a^2 - 4b^2 - 2ab^2}
$$

As expected, all pass-through rates are increasing in the extent of exposure of firm 2 to the cost change:

$$
\frac{\partial PTR_1}{\partial \beta_2} = \frac{2a + b^2}{8 - 2a^2 - 4b^2 - 2ab^2}
$$

$$
\frac{\partial PTR_2}{\partial \beta_2} = \frac{4 - b^2}{8 - 2a^2 - 4b^2 - 2ab^2}
$$

$$
\frac{\partial PTR_3}{\partial \beta_2} = \frac{2b + ab}{8 - 2a^2 - 4b^2 - 2ab^2}
$$

The numerators indicate that a change in firm 2’s exposure to the cost change has the biggest impact on firm 2, while the impact on firm 1 depends most strongly on the degree of substitutability between 1 and 2 (as measured by $a$) and the impact on firm 3 depends most strongly on the degree of substitutability between 2 and 3 (as measured by $b$). This is illustrated in Figure 1, which considers low and high levels of both parameters.
Figure 1: Pass-through rates if one firm is fully affected, one firm partially affected and the third firm not affected by a cost change for different parameter values

The left-hand side of these four graphs, where $\beta_2 = 0$, gives pass-through rates for the case of a firm-specific cost increase for firm 1. The right-hand side of these graphs, where $\beta_2 = 1$, gives pass-through rates when a cost increase fully affects two of three firms.

4. Conclusion

Changing the set of assumptions used to model pass-through makes it possible to derive pass-through as a function of the degree to which firms are exposed to cost changes and diversion ratios, for which data is more likely to be available, instead of as a function of the curvature of cost and demand curves, for which data is less likely to be available. The resulting benchmark is tractable and allows for cases where only some firms are affected by a cost change (and to varying degrees) and where there are asymmetries between firms in terms of the degree of substitutability between the products they produce.
References


