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Does Asymmetry or Incomplete Information on Firms' Costs Yield Spatial Agglomeration?

Sung-chi Lin
Department of Economics, National Taipei University

Hsiao-chi Chen
Department of Economics, National Taipei University

Shi-miin Liu
Department of Economics, National Taipei University

Abstract

This paper extends Hotelling's (1929) spatial game by allowing firms to have asymmetric costs or incomplete information about their rivals' costs. In both cases, there exist equilibria under specific conditions. At the equilibria, the cost-efficient firm will locate at the center of the market and earn positive profit, but the less efficient firm may or may not locate at the market center and produces zero output. Thus, our results do not support the findings of Hotelling (1929) and d'Aspremont et al. (1979).

The first author is a Ph.D. candidate, and the last two authors are Professors in the Department of Economics at National Taipei University, 151, University Road, San-Shia District, New Taipei City 23741, Taiwan, ROC. We would like to thank Associate Editor Parimal Bag and an anonymous referee for their valuable comments.
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Contact: Sung-chi Lin - s89761104@webmail.ntpu.edu.tw, Hsiao-chi Chen - hchen@mail.ntpu.edu.tw, Shi-miin Liu - shimiin@mail.ntpu.edu.tw.
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1. Introduction

This paper expands Hotelling's (1929) spatial game by allowing firms to have asymmetric costs or incomplete information about their rivals' costs. We find that the equilibria will exist under specific conditions. At the equilibria, the cost-efficient firm will locate at the market center and capture the entire market with positive profits, while the less efficient firm may or may not locate at the market center and produces zero output. Our results suggest neither Hotelling's (1929) nor d'Aspremont et al.'s (1979) equilibrium existent in spatial games if firms have asymmetric costs and/or incomplete information about their rivals' costs.

The relationships between this study and the relevant literature are as follows. Hotelling (1929) introduces a spatial variable into a duopoly model and constructs a location-then-price game. He claims that the two firms would agglomerate at the market center, set product prices higher than their marginal costs, and have positive profits, which differ from Bertrand's (1883) outcomes. However, d'Aspremont et al. (1979) prove that Hotelling's (1929) equilibrium does not exist because the agglomeration will result in a tougher price competition and firms' location dispersion. Hotelling's (1929) paper assumes that consumers are uniformly distributed over a line with unit length and buy only one unit goods from one firm, the transport cost is a linear function of distance, and firms produce homogeneous products and compete in prices. To show the existence of Hotelling's (1929) equilibrium, the subsequent researchers relax these assumptions and obtain equilibria similar to Hotelling's (1929) or not. For instance, d'Aspremont et al. (1979) replace the linear transport cost function with a quadratic one, and show that both firms would survive with positive profits and locate at the opposite ends of a line. Hamilton and Thisse (1989), Anderson and Neven (1991), and Pal (1998) demonstrate that when competing in quantities, the firms would agglomerate at the market center at equilibria. Assuming price competition and circle location, Eaton and Wooders (1985), Kats (1995), and De Frutos et al. (2002) display that firms would locate at equal distance in a circle at equilibria. All the models above presume that firms' cost are the same and known to everyone, but we consider asymmetry and/or incomplete information about firms' costs.

On the other hand, the works of Dastidar (1995), Spulber (1995), Wang and Yang (2004), Neary (1994), Clarke and Collie (2006), and Lofaro (2002) explore the impacts of firms' asymmetric costs or incomplete information about firms' costs under various set-ups. In contrast, we explore the same issue in spatial games.

2. The Model

We consider a typical Hotelling (1929) model on a line with length one and firms *A* and *B* producing a homogeneous product. Firm *A*'s marginal cost, c_1, is common information, while firm *B*'s marginal cost is known to itself only. Firm *A* can perceive that firm *B*'s marginal cost is either c_2^H with probability θ or c_2^L with probability $(1-\theta)$, where $\theta \in [0,1]$. Without loss of generality, we assume $c_2^H > c_1 > c_2^L > 0$. If $\theta = 0$ or $\theta = 1$, then our model has complete information with asymmetric costs of firms, while the model has incomplete information for $\theta \in (0,1)$.

Firm *A* locates at distance a from the left end of the line, firm *B* with c_2^H and c_2^L locates respectively at distances b^H and b^L from the right end of the line, where $0 \le a,\ b^H, b^L \le 1/2$. Consumers are evenly distributed along the line. Each consumer purchases one unit of the homogeneous product in per unit time from the seller offering a lower delivered price, which equals the product price plus the transportation cost. The transportation cost is a linear function of the distance, and $t > 0$ represents the transportation rate per unit distance.

When firm *A* and firm *B* with c_2^H locate at the same point, i.e., $a + b^H = 1$, the demand functions faced by them are respectively

$$q_1^H\left(a+b^H=1\right)=\begin{cases}0 & \text{if } p_1 > p_2^H, \\ 1 & \text{if } p_1 \le p_2^H,\end{cases} \quad \text{and} \quad q_2^H\left(a+b^H=1\right)=\begin{cases}1 & \text{if } p_2^H < p_1, \\ 0 & \text{if } p_2^H \ge p_1,\end{cases} \tag{1}$$

where p_1 and p_2^H are the product prices set by firm *A* and firm *B* with c_2^H. As in Jehle and Reny (2011, p.190), if two firms with different costs set the same product price, the entire market will belong to the efficient one. However, if equally efficient firms set the same product price, they will share the market equally.[1] By contrast, if firm *A* and firm *B* with c_2^H do not locate at the same point, i.e., $a + b^H < 1$, firm *A* will serve all the consumers at the left of point a and those with length $x^H \ge 0$ at the right of point a. Similarly, firm *B* with c_2^H will serve all the consumers at the right

[1]The tie-breaking rule is crucial in obtaining the equilibria. If firms with all kinds of cost levels set the same product prices, then they will share the market equally and no equilibrium exists. The proofs are available upon request.

of point b^H and those with length $y^H \geq 0$ at the left of point b^H. A consumer will be indifferent to buying from firm *A* or firm *B* with c_2^H if conditions

$$a + x^H + y^H + b^H = 1 \quad \text{and} \quad p_1 + t \times x^H = p_2^H + t \times y^H \tag{2}$$

hold. Solving (2) yields

$$x^H = \frac{t(1 - a - b^H) - p_1 + p_2^H}{2t} \quad \text{and} \quad y^H = \frac{t(1 - a - b^H) + p_1 - p_2^H}{2t}. \tag{3}$$

Accordingly, equations in (3) imply that the demand functions for firm *A* and firm *B* with c_2^H are

$$q_1^H(a + b^H < 1) = \frac{t(1 + a - b^H) - p_1 + p_2^H}{2t} \quad \text{and} \tag{4}$$

$$q_2^H(a + b^H < 1) = \frac{t(1 - a + b^H) + p_1 - p_2^H}{2t}, \text{ respectively.} \tag{5}$$

Similarly, when firm *A* and firm *B* with c_2^L locate at the same point, i.e., $a + b^L = 1$, the demand functions faced by them are respectively

$$q_1^L(a + b^L = 1) = \begin{cases} 0 & \text{if } p_1 \geq p_2^L \\ 1 & \text{if } p_1 < p_2^L \end{cases} \quad \text{and} \quad q_2^L(a + b^L = 1) = \begin{cases} 1 & \text{if } p_2^L \leq p_1 \\ 0 & \text{if } p_2^L > p_1, \end{cases} \tag{6}$$

where p_2^L is the product price set by firm *B* with c_2^L. If firm *A* and firm *B* with c_2^L do not locate at the same point, i.e., $a + b^L < 1$, the demand functions faced by them are respectively

$$q_1^L(a + b^L < 1) = \frac{t(1 + a - b^L) - p_1 + p_2^L}{2t} \quad \text{and} \tag{7}$$

$$q_2^L(a + b^L < 1) = \frac{t(1 - a + b^L) + p_1 - p_2^L}{2t}. \tag{8}$$

Thus, the (expected) profit functions of firm *A*, firm *B* with c_2^H and firm *B* with c_2^L are

$$\mathrm{E}\pi_1 = (p_1 - c_1) \times \begin{cases} \theta \times q_1^H(a + b^H = 1) + (1 - \theta) \times q_1^L(a + b^L = 1) \\ \theta \times q_1^H(a + b^H < 1) + (1 - \theta) \times q_1^L(a + b^L = 1) \\ \theta \times q_1^H(a + b^H < 1) + (1 - \theta) \times q_1^L(a + b^L < 1) \\ \theta \times q_1^H(a + b^H = 1) + (1 - \theta) \times q_1^L(a + b^L < 1), \end{cases} \tag{9}$$

$$\pi_2^H = (p_2^H - c_2^H) \times \begin{cases} q_2^H(a + b^H = 1) \\ q_2^H(a + b^H < 1) \end{cases} \quad \text{and} \tag{10}$$

$$\pi_2^L = \left(p_2^L - c_2^L\right) \times \begin{cases} q_2^L\left(a + b^L = 1\right) \\ q_2^L\left(a + b^L < 1\right), \end{cases} \text{ respectively.} \tag{11}$$

Based on the above, our two-stage Bayesian game for $\theta \in (0,1)$ proceeds as follows. In the first stage, the two firms choose locations $\left(a^*, b^{H*}, b^{L*}\right)$ to maximize their (expected) profits independently and simultaneously. Given the locations, both firms then choose prices $\left(p_1^*, p_2^{H*}, p_2^{L*}\right)$ to maximize their (expected) profits independently and simultaneously in the second stage. The concept of Bayesian Nash equilibrium (BNE) is adopted to characterize firms' equilibrium behaviors. By contrast, if $\theta = 0$ or $\theta = 1$, the two-stage game remains the same, but the concept of subgame perfect Nash equilibrium (SPNE) will be employed. All the equilibria are derived in the next section by backward induction.

3. The Equilibria

Given locations $\left(a, b^H, b^L\right)$, firm *A*, firm *B* with c_2^H, and firm *B* with c_2^L will choose $\left(p_1^*, p_2^{H*}, p_2^{L*}\right)$ to solve the following problems in the second stage.

$$p_1^* \in \arg \max_{p_1} \mathrm{E}\pi_1 \text{ s.t. } p_1 \geq c_1, \tag{12}$$

$$p_2^{H*} \in \arg \max_{p_2^H} \pi_2^H \text{ s.t. } p_2^H \geq c_2^H, \text{ and} \tag{13}$$

$$p_2^{L*} \in \arg \max_{p_2^L} \pi_2^L \text{ s.t. } p_2^L \geq c_2^L, \tag{14}$$

where $\mathrm{E}\pi_1$, π_2^H and π_2^L are defined in equations (9)-(11), respectively.

After deriving $\left(p_1^*, p_2^{H*}, p_2^{L*}\right)$ and substituting them into firms' (expected) profit functions, these firms will choose $\left(a^*, b^{H*}, b^{L*}\right)$ to solve the problems of

$$a^* \in \arg \max_a \mathrm{E}\pi_1 \text{ s.t. } 0 \leq a \leq \tfrac{1}{2},$$

$$b^{H*} \in \arg \max_{b^H} \pi_2^H \text{ s.t. } 0 \leq b^H \leq \tfrac{1}{2}, \text{ and}$$

$$b^{L*} \in \arg \max_{b^L} \pi_2^L \text{ s.t. } 0 \leq b^L \leq \tfrac{1}{2},$$

in the first stage. The equilibria under asymmetric costs and incomplete information are presented in Proposition 1 and Proposition 2, respectively. Their proofs are provided in Appendix.

Proposition 1. *Suppose* $\theta = 1$. *If* $\left(c_2^H - c_1\right) \geq t\left(2.5 + b^{H*}\right)$ *holds, the subgame perfect Nash equilibria are* $\left(\left(a^*,\, b^{H*}\right),\, \left(p_1^*,\, p_2^{H*}\right)\right) = \left(\left(\frac{1}{2},\, \left[0,\, \frac{1}{2}\right)\right),\, \left(\frac{t\left(1.5 - b^{H*}\right) + c_1 + c_2^H}{2},\, c_2^H\right)\right)$ *with firms' equilibrium outputs* $\left(q_1^*, q_2^{H*}\right) = (1, 0)$ *and equilibrium profits* $\left(\pi_1^*, \pi_2^{H*}\right) = \left(\frac{t\left(1.5 - b^{H*}\right) - c_1 + c_2^H}{2} > 0,\ 0\right)$. *By contrast, if* $\theta = 0$, *the subgame perfect Nash equilibria are* $\left(\left(a^*,\, b^{L*}\right),\, \left(p_1^*,\, p_2^{L*}\right)\right) =$

$\left(\left(\left[0,\, \frac{1}{2}\right),\frac{1}{2}\right),\, \left(c_1,\, \frac{t\left(1.5 - a^*\right) + c_1 + c_2^L}{2}\right)\right)$ *with firms' equilibrium outputs* $\left(q_1^*,\, q_2^{L*}\right) = (0, 1)$ *and equilibrium profits* $\left(\pi_1^*,\, \pi_2^{L*}\right) = \left(0,\, \frac{t\left(1.5 - a^*\right) - c_2^L + c_1}{2} > 0\right)$ *when* $\left(c_1 - c_2^L\right) \geq t\left(2.5 + a^*\right)$ *holds.*

Under Hotelling's (1929) set-up with asymmetric costs of firms, Proposition 1 shows the existence of SPNEs if the marginal cost of firm *B* is larger than that of firm *A*. At equilibria, the efficient firm will locate at the market center and capture the entire market with positive profit, while the less efficient firm will not locate at the market center and produce zero output. This is explained below. If $\left(c_2^H - c_1\right) \geq t\left(2.5 + b^{H*}\right)$ holds, the product price set by firm *A*, $p_1^* = \frac{t\left(1.5 - b^{H*}\right) + c_1 + c_2^H}{2}$, is the equilibrium price consisting of the marginal cost (c_1) and a mark-up $\left(\frac{t\left(1.5 - b^{H*}\right) - c_1 + c_2^H}{2}\right)$. Thus, the delivered price of firm *A*, $p^* + t\left(\frac{1}{2} - b^{H*}\right) = \frac{t\left(2.5 - 3b^{H*}\right) + c_1 + c_2^H}{2}$, is lower than firm *B*'s marginal cost (c_2^H) due to $(c_2^H - c_1) \geq t\left(2.5 + b^{H*}\right)$, wherever firm *B*'s location is. Obviously, firm *A* will capture the entire market and firm *B* will exit the market. Condition $(c_2^H - c_1) \geq t\left(2.5 + b^{H*}\right)$ is needed, otherwise both firms can earn positive profits by setting product prices larger than their marginal costs. That is because different locations allow the firms to conduct not fierce price competition, hence their profits will strictly increase as their locations approach the market center. Accordingly, it is optimal for both firms to locate at the market center. This will violate the hypothesis of $a + b^H < 1$, and result in no equilibrium.

Proposition 2. *Suppose* $\theta \in (0,1)$ *. If* $(c_2^H - c_1) \geq t\left(2.5 + b^{H*}\right)$ *holds, the Bayesian Nah equilibria are*

$$\left(\left(a^*,\, b^{H*},\, b^{L*}\right),\ \left(p_1^*,\, p_2^{H*},\, p_2^{L*}\right)\right) = \left(\left(\frac{1}{2},\ \left[0,\ \frac{1}{2}\right),\ \frac{1}{2}\right),\ \left(\frac{t\left(1.5 - b^{H*}\right) + c_1 + c_2^H}{2},\ c_2^H,\ \frac{t\left(1.5 - b^{H*}\right) + c_1 + c_2^H}{2}\right)\right).$$

Firms' equilibrium outputs $\left(q_1^*, q_2^{H*}, q_2^{L*}\right) = (0, 0, 1)$ *and equilibrium profits* $\left(\pi_1^*,\ \pi_2^{H*},\ \pi_2^{L*}\right) =$

$\left(0,\ 0,\ \left(\frac{t\left(1.5 - b^{H*}\right) + c_1 + c_2^H - 2c_2^L}{2}\right)\right)$ *will occur with probability* $(1-\theta)$*, and firms' equilibrium outputs*

$\left(q_1^*, q_2^{H*}, q_2^{L*}\right) = (1, 0, 0)$ *and equilibrium profits* $\left(\pi_1^*,\ \pi_2^{H*},\ \pi_2^{L*}\right) = \left(\frac{t\left(1.5 - b^{H*}\right) - c_1 + c_2^H}{2},\ 0,\ 0\right)$ *will occur with probability* θ*.*

Under Hotelling's (1929) set-up with one firm's cost being private information, Proposition 2 displays the existence of BNEs if firm *B* with a large enough c_2^H. At equilibria, firm *A* and firm *B* with c_2^L will still agglomerate at the market center, but firm *B* with c_2^H will choose any location except the market center. Moreover, firm *B* with c_2^H will set the product price equal to its marginal cost, while firm *A* and firm *B* with c_2^L will set the product price lower than c_2^H but higher than c_1. The intuition is as follows. Given firm *A*'s product price p_1, it is optimal for firm *B* with c_2^L to choose the same product price, i.e., $p_2^L = p_1$. That is because it will get zero profit if choosing $p_2^L > p_1$, and obtain non-negative profit $\pi_2^L = \left(p_2^L - c_2^L\right) \geq 0$ if choosing $p_2^L \in \left[c_2^L,\, p_1\right]$. Thus, $p_2^L = p_1$ is the best reply of firm *B* with c_2^L. Then, our incomplete-information game is equivalent to the complete-information one with firm *A* and firm *B* having c_2^H. As shown by Proposition 1, firm *A* will locate at the market center and set the product price higher than c_1, while firm *B* with c_2^H will not locate at the market center and produce zero output. Although firm *B* with c_2^L will choose the same location and product price as firm *A*'s, firm *A* will still earn zero profit because it is less efficient than firm *B* with c_2^L.

In sum, Propositions 1 and 2 show that only the efficient firm will survive in the market at the equilibria of spatial games if firms have asymmetric costs or incomplete information about their rivals' costs. Thus, neither Hotelling equilibrium nor d'Aspremont et al.'s equilibrium is supported by our models.

4. Extensions

In this section, we extend our model by allowing firm A's cost unknown to firm B as well. Let both firms' marginal costs be either c_2^H with probability θ or c_2^L with probability $(1-\theta)$ with $\theta \in (0,1)$. A two-stage game similar to that in Section 2 can be constructed. In the first stage of the game, firms A and B choose respective locations $\left(a^{H*}, a^{L*}\right)$ and $\left(b^{H*}, b^{L*}\right)$ to maximize their expected profits independently and simultaneously. Given the locations, firms A and B choose respective prices $\left(p_1^{H*}, p_1^{L*}\right)$ and $\left(p_2^{H*}, p_2^{L*}\right)$ to maximize their expected profits independently and simultaneously in the second stage of the game. The associated results are presented below and their proofs are available upon request.

Proposition 3. *In a location-then-price game with both firms having incomplete information about their rivals' marginal costs, there exists no Bayesian Nash equilibrium.*

The intuition of Proposition 3 is simple. Under the current set-up, firm A will regard firm B's marginal cost as $E(c) = \theta c_2^H + (1-\theta)c_2^L$, and so will firm B regard firm A's. Then, our game is equivalent to a typical Hotelling model with two firms having the same marginal cost. Thus, the arguments of d'Aspremont et al. (1979) apply and no equilibrium exists. However, if firm A's marginal cost is c_1^H or c_1^L and firm B's marginal cost equals c_2^H or c_2^L with unequal expectations, i.e., $E_A(c) \equiv \theta c_1^H + (1-\theta)c_1^L \neq E_B(c) \equiv \theta c_2^H + (1-\theta)c_2^L$, then some equilibria as shown in Proposition 2 may exist.

5. Conclusions

This paper extends Hotelling's model by letting firms have asymmetric costs or incomplete information on their rivals' costs. Our results support neither Hotelling (1929) equilibrium nor d'Aspremont et al.'s (1979) equilibrium.

Appendix

Proof of Proposition 1: If $\theta = 1$, our game has complete information with firm *A* having marginal cost c_1 and firm *B* having c_2^H. According to whether $a + b^H = 1$, there are two cases as follows.

(i) If $a + b^H = 1$, then the firms' profit functions are respectively

$$\pi_1 = \begin{cases} (p_1 - c_1) \times 0 & \text{if } p_1 > p_2^H \\ (p_1 - c_1) \times 1 & \text{if } p_1 \le p_2^H \end{cases} \quad \text{and} \quad \pi_2^H = \begin{cases} (p_2^H - c_2^H) \times 1 & \text{if } p_2^H < p_1 \\ (p_2^H - c_2^H) \times 0 & \text{if } p_2^H \ge p_1 \end{cases}$$

by (1), and (9)-(10). Since both firms locate at the same point, they will conduct a traditional Bertrand competition with equilibrium prices $\left(p_1^*, p_2^{H*}\right) = \left(c_2^H, c_2^H\right)$ and equilibrium profits $\left(\pi_1^*, \pi_2^{H*}\right) = \left(\left(c_2^H - c_1\right) > 0,\ 0\right)$.

However, given $a^* = 1/2$ and $p_1^* = c_2^H$, it is better for firm *B* with c_2^H to locate at any point $b^H \in [0,\ 1/2)$, to set product price $p_2^H = c_2^H + \varepsilon > c_2^H$ with $\varepsilon \in \left(0,\ \frac{t\left(1+2b^H\right)}{2}\right)$, and to earn positive profit $\pi_2^H\left(a^* + b^H < 1\right) = \varepsilon \cdot \frac{t\left(1+2b^H\right) - 2\varepsilon}{4t} > \pi_2^{H*}\left(a^* + b^{H*} = 1\right) = 0$.[2] Thus, no SPNE exists in this case.

(ii) If $a + b^H < 1$, then the firms' profit functions are respectively

$$\pi_1 = \frac{(p_1 - c_1) \times \left[t\left(1 + a - b^H\right) - p_1 + p_2^H\right]}{2t} \quad \text{and} \tag{A1}$$

$$\pi_2^H = \frac{\left(p_2^H - c_2^H\right) \times \left[t\left(1 - a + b^H\right) + p_1 - p_2^H\right]}{2t} \tag{A2}$$

by (4)-(5) and (9)-(10). Let $L_1 = \frac{(p_1 - c_1) \times \left[t\left(1 + a - b^H\right) - p_1 + p_2^H\right]}{2t} - \lambda_1 (c_1 - p_1)$ be the Lagrange function of problem (12) with π_1 defined in (A1), where λ_1 is the associated Lagrange multiplier. Then, the first-order conditions are

$$\frac{\partial L_1}{\partial p_1} = \frac{t\left(1 + a - b^H\right) + c_1 - 2p_1 + p_2^H}{2t} + \lambda_1 = 0 \quad \text{and} \tag{A3}$$

$$\frac{\partial L_1}{\partial \lambda_1} = p_1 - c_1 \ge 0,\ \frac{\partial L_1}{\partial \lambda_1} \cdot \lambda_1 = 0,\ \lambda_1 \ge 0. \tag{A4}$$

Let $L_2^H = \frac{\left(p_2^H - c_2^H\right) \times \left[t\left(1 - a + b^H\right) + p_1 - p_2^H\right]}{2t} - \lambda_2^H \left(c_2^H - p_2^H\right)$ be the Lagrange function of the problem (13) with π_2^H defined in (A2), where λ_2^H is the associated Lagrange multiplier. Then, the first-order

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[2] Substituting $a^* = 1/2$, $b^H \in [0, 1/2)$, $p_1^* = c_2^H$, and $p_2^H = c_2^H + \varepsilon$ into (5) yields $q_2^H\left(a^* + b^H < 1\right) = \left[t\left(1 + 2b^H\right) - 2\varepsilon\right]/4t > 0$ and $\pi_2^H\left(a^* + b^H < 1\right) = \varepsilon \times \left[t\left(1 + 2b^H\right) - 2\varepsilon\right]/4t > 0$ by $\varepsilon < t\left(1 + 2b^H\right)/2$.

conditions are

$$\frac{\partial L_2^H}{\partial p_2^H}=\frac{t\left(1-a+b^H\right)+c_2^H+p_1-2p_2^H}{2t}+\lambda_2^H=0 \text{ and} \tag{A5}$$

$$\frac{\partial L_2^H}{\partial \lambda_2^H}=p_2^H-c_2^H\geq 0,\ \frac{\partial L_2^H}{\partial \lambda_2^H}\cdot\lambda_2^H=0,\ \lambda_2^H\geq 0. \tag{A6}$$

Based on whether the constraints in (A4) and (A6) bind or not, there are four possible product-price pairs, which are grouped into three sub-cases below.

Case 1: Suppose $p_1>c_1$ and $p_2^H>c_2^H$. Then we have $\lambda_1=\lambda_2^H=0$, and (A3) and (A5) become

$$\frac{\partial L_1}{\partial p_1}=\frac{t\left(1+a-b^H\right)+c_1-2p_1+p_2^H}{2t}=0 \text{ and } \frac{\partial L_2^H}{\partial p_2^H}=\frac{t\left(1-a+b^H\right)+c_2^H+p_1-2p_2^H}{2t}=0.$$

Solving these two equations yields equilibrium prices $p_1^*=\frac{t\left(3+a-b^H\right)+2c_1+c_2^H}{3}>c_1$ and $p_2^{H*}=\frac{t\left(3-a+b^H\right)+c_1+2c_2^H}{3}>c_2^H$. Substituting $\left(p_1^*,\ p_2^{H*}\right)$ into (A1) and (A2) yields firms' equilibrium profits $\left(\pi_1^*,\ \pi_2^{H*}\right)=\left(\frac{\left[t\left(3+a-b^H\right)-c_1+c_2^H\right]^2}{18t},\ \frac{\left[t\left(3-a+b^H\right)+c_1-c_2^H\right]^2}{18t}\right)$.

Since the equilibrium profits of firm *A* and firm *B* with c_2^H are strictly increasing functions of a and b^H, respectively, the optimal locations should be $a^*=b^{H*}=1/2$, which violates the hypothesis of $a+b^H=1$. Thus, no SPNE exists in this case.

Case 2: Suppose $p_1>c_1$ and $p_2^H=c_2^H$. Then we have $\lambda_1=0$ and $\lambda_2^H\geq 0$, and (A3) and (A5) become

$$\frac{\partial L_1}{\partial p_1}=\frac{t\left(1+a-b^H\right)+c_1-2p_1+c_2^H}{2t}=0 \text{ and } \frac{\partial L_2^H}{\partial p_2^H}=\frac{t\left(1-a+b^H\right)-c_2^H+p_1}{2t}+\lambda_2^H=0.$$

Solving these two equations yields equilibrium prices $\left(p_1^*,\ p_2^{H*}\right)=\left(\frac{t\left(1+a-b^H\right)+c_1+c_2^H}{2},\ c_2^H\right)$ and $\lambda_2^{H*}=-\frac{t\left(3-a+b^H\right)+c_1-c_2^H}{4t}\geq 0$. To make $\lambda_2^{H*}\geq 0$ hold, condition

$$c_2^H-c_1\geq t\left(3-a+b^H\right) \tag{A7}$$

is needed. Note that (A7) also guarantees the output of firm *B* with c_2^H being zero and firm *A*'s being one. Substituting $\left(p_1^*,\ p_2^{H*}\right)$ into (A1) and (A2) yields firms' equilibrium profits $\left(\pi_1^*,\ \pi_2^{H*}\right)$

$=\left(\frac{t\left(1+a-b^{H}\right)-c_1+c_2^H}{2},\ 0\right).$

Since π_1^* is strictly increasing with a and π_2^{H*} is independent of b^H, $\left(a^*,\ b^{H*}\right)$ $=\left(1/2,\left[0,1/2\right)\right)$ are firms' optimal locations. Here $b^{H*}=1/2$ is ruled out to meet the requirement of $a^*+b^{H*}<1$. Substituting $\left(a^*,\ b^{H*}\right)$ into equilibrium prices and (A7) generates Proposition 1.

Case 3: Because the proofs for $\left(p_1=c_1,\ p_2^H>c_2^H\right)$ and $\left(p_1=c_1,\ p_2^H=c_2^H\right)$ are similar, we demonstrate the former. Then, we have $\lambda_1\geq 0$ and $\lambda_2^H=0$. Accordingly, equations (A3) and (A5) become

$$\frac{\partial L_1}{\partial p_1}=\frac{t\left(1+a-b^H\right)-c_1+p_2^H}{2t}+\lambda_1=0 \text{ and } \frac{\partial L_2^H}{\partial p_2^H}=\frac{t\left(1-a+b^H\right)+c_2^H+c_1-2p_2^H}{2t}=0.$$

Since $t\left(1+a-b^H\right)>0$ by $t>0$ and $a,\ b^H\in\left[0,\ 1/2\right]$, $p_2^H-c_1>0$ by $p_2^H>c_2^H>c_1$, and $\lambda_1\geq 0$; we must have $\partial L_1/\partial p_1>0$, which contradicts $\partial L_1/\partial p_1=0$. Thus, no SPNE exists in this case.

We can apply the same arguments to case $\theta=0$, and obtain the equilibria in Proposition 1.

Proof of Proposition 2: If $\theta\in(0,1)$, our game has incomplete information with firms *A* and *B*. According to whether $a+b^H=1$ and $a+b^L=1$, there are four cases as follows.

(i) Suppose $a+b^H=1$ and $a+b^L=1$. Then firms' (expected) profit functions are

$$\mathrm{E}\pi_1=\left(p_1-c_1\right)\left[\theta\cdot q_1^H\left(a+b^H=1\right)+\left(1-\theta\right)\cdot q_1^L\left(a+b^L=1\right)\right]$$
$$=\begin{cases}\left(p_1-c_1\right)\times 0 \text{ if } p_1>p_2^H \text{ and } p_1\geq p_2^L\\ \left(p_1-c_1\right)\times\theta \text{ if } p_1\leq p_2^H \text{ and } p_1\geq p_2^L\\ \left(p_1-c_1\right)\times\left(1-\theta\right) \text{ if } p_1>p_2^H \text{ and } p_1<p_2^L\\ \left(p_1-c_1\right)\times 1 \text{ if } p_1\leq p_2^H \text{ and } p_1<p_2^L,\end{cases} \qquad \text{(A8)}$$

$$\pi_2^H=\left(p_2^H-c_2^H\right)\cdot q_2^H\left(a+b^H=1\right)=\begin{cases}\left(p_2^H-c_2^H\right)\times 1 \text{ if } p_2^H<p_1\\ \left(p_2^H-c_2^H\right)\times 0 \text{ if } p_2^H\geq p_1\end{cases} \text{ and} \qquad \text{(A9)}$$

$$\pi_2^L=\left(p_2^L-c_2^L\right)\cdot q_2^L\left(a+b^L=1\right)=\begin{cases}\left(p_2^L-c_2^L\right)\times 1 \text{ if } p_2^L\leq p_1\\ \left(p_2^L-c_2^L\right)\times 0 \text{ if } p_2^L>p_1\end{cases} \qquad \text{(A10)}$$

by (1), (6) and (9)-(11). We first show the following lemma.

Lemma A: *Given firm A's product price* p_1, *the best reply of firm B with* c_2^L *is to choose* p_1 *as well.*

Proof. Given p_1, we have $0 \le \pi_2^L = (p_2^L - c_2^L) \le (p_1 - c_2^L)$ for all $p_2^L \le p_1$ by $p_2^L \ge c_2^L$ and (A10). By contrast, for $p_2^L > p_1$, we have $0 = \pi_2^L = \left(p_2^L - c_2^L\right) < \left(p_1 - c_2^L\right)$ by (A10) again. The two inequalities above suggest that the best strategy of firm *B* with c_2^L is to choose $p_2^L = p_1$.

Based on Lemma A and $c_2^L < c_1 < c_2^H$, we have unique price equilibrium $\left(p_1^*, p_2^{H*}, p_2^{L*}\right) = \left(c_2^H, c_2^H, c_2^H\right)$ in the second stage. Suppose $p_1^* = c_2^H > c_1$. By (A9), firm *B* with c_2^H will get negative profit by choosing $p_2^H < p_1 = c_2^H$, and get zero profit by choosing $p_2^H \ge p_1 = c_2^H$. Thus, we have $p_2^{H*} \ge c_2^H$, and firm *A* will get higher profit by choosing $p_1 \in \left(c_2^H, p_2^H\right]$ than choosing $p_1 = c_2^H$. This contradicts hypothesis $p_1^* = c_2^H$. By contrast, if $p_2^{H*} = c_2^H$, firm *A* will get $(p_1 - c_1)\theta$ by choosing $p_1 \in (c_1, c_2^H]$, and get zero profit if choosing $p_1 > c_2^H$ by (A8) and Lemma A. Thus, we have $p_1^* = c_2^H$. Then the equilibrium price pair is $\left(p_1^*, p_2^{H*}, p_2^{L*}\right) = \left(c_2^H, c_2^H, c_2^H\right)$ in the second stage. Moreover, firms' (expected) equilibrium profits are $\left(\mathrm{E}\pi_1^*, \pi_2^{H*}, \pi_2^{L*}\right) = \left(\left(c_2^H - c_1\right)\theta > 0,\ 0,\ \left(c_2^H - c_2^L\right) > 0\right)$.

However, given $\left(p_1^*, p_2^{H*}, p_2^{L*}\right) = \left(c_2^H, c_2^H, c_2^H\right)$, it is better for firm *B* with c_2^H to locate at any point $b^H \in [0, 1/2)$, to set product price $p_2^H = c_2^H + \varepsilon > c_2^H$ with $\varepsilon \in \left(0, \frac{t\left(1+2b^H\right)}{2}\right)$, and to earn positive profit $\pi_2^H\left(a^* + b^H < 1\right) = \varepsilon \cdot \frac{t\left(1+2b^H\right) - 2\varepsilon}{4t} > \pi_2^{H*}\left(a^* + b^{H*} = 1\right) = 0$ as argued in footnote 2. Thus, no BNE exists in this case.

(ii) Suppose $a + b^H < 1$ and $a + b^L < 1$. Then, firms' (expected) profit functions are

$$\mathrm{E}\pi_1 = \left(p_1 - c_1\right)\left[\theta \cdot q_1^H\left(a + b^H < 1\right) + (1-\theta) \cdot q_1^L\left(a + b^L < 1\right)\right]$$

$$= \frac{\left(p_1 - c_1\right)\left\{t\left[1 + a - \theta b^H - (1-\theta) b^L\right] - p_1 + \theta p_2^H + (1-\theta) p_2^L\right\}}{2t}, \tag{A11}$$

$$\pi_2^H = \left(p_2^H - c_2^H\right) \cdot q_2^H\left(a + b^H < 1\right) = \frac{\left(p_2^H - c_2^H\right)\left[t\left(1 - a + b^H\right) + p_1 - p_2^H\right]}{2t} \quad \text{and} \tag{A12}$$

$$\pi_2^L=\left(p_2^L-c_2^L\right)\cdot q_2^L\left(a+b^L<1\right)=\frac{\left(p_2^L-c_2^L\right)\left[t\left(1-a+b^L\right)+p_1-p_2^L\right]}{2t} \tag{A13}$$

by (4)-(5), (7)-(8), and (9)-(11). Let $L_1=\frac{(p_1-c_1)\left\{t\left[1+a-\theta b^H-(1-\theta)b^L\right]-p_1+\theta p_2^H+(1-\theta)p_2^L\right\}}{2t}-\lambda_1(c_1-p_1)$ be the Lagrange function of problem (12) with $\mathrm{E}\pi_1$ defined in (A11), where λ_1 is the associated Lagrange multiplier. Then, the first-order conditions are

$$\frac{\partial L_1}{\partial p_1}=\frac{t\left[1+a-\theta b^H-(1-\theta)b^L\right]+c_1-2p_1+\theta p_2^H+(1-\theta)p_2^L}{2t}+\lambda_1=0 \text{ and} \tag{A14}$$

$$\frac{\partial L_1}{\partial \lambda_1}=p_1-c_1\geq 0,\ \frac{\partial L_1}{\partial \lambda_1}\cdot\lambda_1=0,\ \lambda_1\geq 0. \tag{A15}$$

Let $L_2^H=\frac{\left(p_2^H-c_2^H\right)\left[t\left(1-a+b^H\right)+p_1-p_2^H\right]}{2t}-\lambda_2^H\left(c_2^H-p_2^H\right)$ be the Lagrange function of problem (13) with π_2^H defined in (A12), where λ_2^H is the associated Lagrange multiplier. Then, the first-order conditions are

$$\frac{\partial L_2^H}{\partial p_2^H}=\frac{t\left(1-a+b^H\right)+c_2^H+p_1-2p_2^H}{2t}+\lambda_2^H=0 \text{ and} \tag{A16}$$

$$\frac{\partial L_2^H}{\partial \lambda_2^H}=p_2^H-c_2^H\geq 0,\ \frac{\partial L_2^H}{\partial \lambda_2^H}\cdot\lambda_2^H=0,\ \lambda_2^H\geq 0. \tag{A17}$$

Let $L_2^L=\frac{\left(p_2^L-c_2^L\right)\left[t\left(1-a+b^L\right)+p_1-p_2^L\right]}{2t}-\lambda_2^L\left(c_2^L-p_2^L\right)$ be the Lagrange function of problem (14) with π_2^L defined in (A13), where λ_2^L is the associated Lagrange multiplier. Then, the first-order conditions are

$$\frac{\partial L_2^L}{\partial p_2^L}=\frac{t\left(1-a+b^L\right)+c_2^L+p_1-2p_2^L}{2t}+\lambda_2^L=0 \text{ and} \tag{A18}$$

$$\frac{\partial L_2^L}{\partial \lambda_2^L}=p_2^L-c_2^L\geq 0,\ \frac{\partial L_2^L}{\partial \lambda_2^L}\cdot\lambda_2^L=0,\ \lambda_2^L\geq 0. \tag{A19}$$

Based on whether the constraints in (A15), (A17) and (A19) bind or not, there are eight possible product-price pairs, which are grouped into three sub-cases below.

Case (iia): Because the proofs for $\left(p_1>c_1,\ p_2^H>c_2^H,\ p_2^L>c_2^L\right)$ and $\left(p_1>c_1,\ p_2^H=c_2^H,\ p_2^L>c_2^L\right)$ are similar, we demonstrate the former. Then, we have $\lambda_1 = \lambda_2^H = \lambda_2^L = 0$, and (A14), (A16) and (A18) can be reduced to

$$\frac{\partial L_1}{\partial p_1} = \frac{t\left[1 + a - \theta b^H - (1-\theta)b^L\right] + c_1 - 2p_1 + \theta p_2^H + (1-\theta)p_2^L}{2t} = 0,$$

$$\frac{\partial L_2^H}{\partial p_2^H} = \frac{t\left(1 - a + b^H\right) + c_2^H + p_1 - 2p_2^H}{2t} = 0 \quad \text{and}$$

$$\frac{\partial L_2^L}{\partial p_2^L} = \frac{t\left(1 - a + b^L\right) + c_2^L + p_1 - 2p_2^L}{2t} = 0, \text{ respectively.}$$

Solving these equations yields equilibrium prices $p_1^* = \frac{t\left[3+a-\theta b^H-(1-\theta)b^L\right]+2c_1+\theta c_2^H+(1-\theta)c_2^L}{3} > c_1$, $p_2^{H*} = \frac{t\left[6-2a+(3-\theta)b^H-(1-\theta)b^L\right]+2c_1+(3+\theta)c_2^H+(1-\theta)c_2^L}{6} > c_2^H$, and $p_2^{L*} = \frac{t\left[6-2a-\theta b^H+(2+\theta)b^L\right]+2c_1+\theta c_2^H+(4-\theta)c_2^L}{6} > c_2^L$. Substituting $\left(p_1^*, p_2^{H*}, p_2^{L*}\right)$ into (A11)-(A13) yields firms' (expected) equilibrium profits $\mathrm{E}\pi_1^* = \frac{\left\{t\left[3+a-\theta b^H-(1-\theta)b^L\right]-c_1+\theta c_2^H+(1-\theta)c_2^L\right\}^2}{18t} > 0$, $\pi_2^{H*} = \frac{\left\{t\left[6-2a+(3-\theta)b^H-(1-\theta)b^L\right]+2c_1-(3-\theta)c_2^H+(1-\theta)c_2^L\right\}^2}{72t} > 0$ and $\pi_2^{L*} = \frac{\left\{t\left[6-2a-\theta b^H+(2+\theta)b^L\right]+2c_1+\theta c_2^H-(2+\theta)c_2^L\right\}^2}{72t} > 0$. It is easy to see that $\mathrm{E}\pi_1^*$, π_2^{H*} and π_2^{L*} are strictly increasing functions of a, b^H, and b^L, respectively. These imply that all firms will locate at the market center, which contradicts the hypotheses of $a + b^H < 1$ and $a + b^L < 1$. Thus, no BNE exists in this case.

Case (iib): Because the proofs for $\left(p_1 = c_1, p_2^H > c_2^H, p_2^L > c_2^L\right)$ and $\left(p_1 = c_1, p_2^H = c_2^H, p_2^L > c_2^L\right)$ are similar, we demonstrate the former. Then, we have $\lambda_1 \geq 0$, $\lambda_2^H = 0$ and $\lambda_2^L = 0$. Accordingly, equations (A14), (A16) and (A18) become

$$\frac{\partial L_1}{\partial p_1} = \frac{t\left[1 + a - \theta b^H - (1-\theta)b^L\right] + c_1 - 2c_1 + \theta p_2^H + (1-\theta)p_2^L}{2t} + \lambda_1 = 0,$$

$$\frac{\partial L_2^H}{\partial p_2^H} = \frac{t\left(1 - a + b^H\right) + c_2^H + c_1 - 2p_2^H}{2t} = 0 \quad \text{and}$$

$$\frac{\partial L_2^L}{\partial p_2^L} = \frac{t\left(1 - a + b^L\right) + c_2^L + c_1 - 2p_2^L}{2t} = 0, \text{ respectively.}$$

Solving these equations yields equilibrium prices $\left(p_1^*, p_2^{H*}, p_2^{L*}\right)=\left(c_1, \frac{t\left(1-a+b^H\right)+c_1+c_2^H}{2}, \frac{t\left(1-a+b^L\right)+c_1+c_2^L}{2}\right)$ and $\lambda_1^*=-\frac{t\left[3+a-\theta b^H-(1-\theta)b^L\right]-c_1+\theta c_2^H+(1-\theta)c_2^L}{4t}$. Substituting $\left(p_1^*, p_2^{H*}, p_2^{L*}\right)$ into (4)-(5) and (7)-(8) yields equilibrium outputs $\left(q_1^{H*}, q_1^{L*}, q_2^{H*}, q_2^{L*}\right)=$ $\left(\frac{t\left(3+a-b^H\right)-c_1+c_2^H}{4t}>0, \max\left\{0, \frac{t\left(3+a-b^L\right)-c_1+c_2^L}{4t}\right\}\geq 0,\right.$ $\left.\frac{t\left(1-a+b^H\right)+c_1-c_2^H}{4t}, \frac{t\left(1-a+b^L\right)+c_1-c_2^L}{4t}>0\right)$. To make $\lambda_1^*\geq 0$ and $p_2^{H*}>c_2^H$ hold, the following conditions are needed.

$$\frac{\theta[t(3+a-b^H)-c_1+c_2^H]+(1-\theta)[t(3+a-b^L)-c_1+c_2^L]}{4t}\leq 0 \text{ and}$$

$$(c_2^H-c_1)<t\left(1-a+b^H\right). \tag{A20}$$

The second condition in (A20) guarantees $q_2^{H*}>0$. However, the first condition in (A20) will not hold because $\frac{\theta\left[t\left(3+a-b^H\right)-c_1+c_2^H\right]}{4t}+\frac{(1-\theta)\left[t\left(3+a-b^L\right)-c_1+c_2^L\right]}{4t}=\theta q_1^{H*}+\left(1-\theta\right)q_1^{L*}>0$ by $q_1^{L*}\geq 0$, $q_1^{H*}>0$, and $\theta\in\left(0,1\right)$. Accordingly, no BNE exists in this case.

<u>Case (iic)</u>: Because the proofs for price pairs $\left(p_1>c_1, p_2^H>c_2^H, p_2^L=c_2^L\right)$, $\left(p_1>c_1, p_2^H=c_2^H, p_2^L=c_2^L\right)$, $\left(p_1=c_1, p_2^H>c_2^H, p_2^L=c_2^L\right)$ and $\left(p_1=c_1, p_2^H=c_2^H, p_2^L=c_2^L\right)$ are similar, we demonstrate the first pair. Then, we have $\lambda_1=0$, $\lambda_2^H=0$ and $\lambda_2^L\geq 0$, and (A14), (A16) and (A18) become

$$\frac{\partial L_1}{\partial p_1}=\frac{t\left[1+a-\theta b^H-(1-\theta)b^L\right]+c_1-2p_1+\theta p_2^H+(1-\theta)c_2^L}{2t}=0,$$

$$\frac{\partial L_2^H}{\partial p_2^H}=\frac{t\left(1-a+b^H\right)+c_2^H+p_1-2p_2^H}{2t}=0 \text{ and}$$

$$\frac{\partial L_2^L}{\partial p_2^L}=\frac{t\left(1-a+b^L\right)-c_2^L+p_1}{2t}+\lambda_2^L=0, \text{ respectively.}$$

Since $t\left(1-a+b^L\right)>0$ by $t>0$ and $a,b^L\in\left[0,1/2\right]$, $(p_1-c_2^L)>0$ by $p_1>c_1>c_2^L$, and $\lambda_2^L\geq 0$; we must have $\partial L_2^L/\partial p_2^L>0$, which contradicts $\partial L_2^L/\partial p_2^L=0$. Thus, no BNE exists in this case.

(iii) Suppose $a+b^H<1$ and $a+b^L=1$. Then, by (4)-(6) and (9)-(11), firms' (expected) profit functions are

$$\mathrm{E}\pi_1 = (p_1 - c_1)\left[\theta \cdot q_1^H \left(a + b^H < 1\right) + (1-\theta)\cdot q_1^L \left(a + b^L = 1\right)\right]$$

$$= \begin{cases} (p_1 - c_1)\left[\theta \cdot \dfrac{t\left(1.5 - b^H\right) - p_1 + p_2^H}{2t}\right] & \text{if } p_1 \geq p_2^L \\ (p_1 - c_1)\left[\theta \cdot \dfrac{t\left(1.5 - b^H\right) - p_1 + p_2^H}{2t} + (1-\theta)\right] & \text{if } p_1 < p_2^L, \end{cases} \quad \text{(A21)}$$

$$\pi_2^H = \left(p_2^H - c_2^H\right)\cdot q_2^H\left(a + b^H < 1\right) = \left(p_2^H - c_2^H\right)\left[\frac{t\left(0.5 + b^H\right) + p_1 - p_2^H}{2t}\right] \text{ and} \quad \text{(A22)}$$

$$\pi_2^L = \left(p_2^L - c_2^L\right)\cdot q_2^L\left(a + b^L = 1\right) = \begin{cases} \left(p_2^L - c_2^L\right)\times 1 & \text{if } p_2^L \leq p_1 \\ \left(p_2^L - c_2^L\right)\times 0 & \text{if } p_2^L > p_1. \end{cases} \quad \text{(A23)}$$

Let $L_1 = (p_1 - c_1)\left[\theta \cdot \frac{t\left(1.5 - b^H\right) - p_1 + p_2^H}{2t}\right] - \lambda_1 (c_1 - p_1)$ be the Lagrange function of problem (12) with $\mathrm{E}\pi_1$ defined in (A21), where λ_1 is the associated Lagrange multiplier.[3] Then, the first-order conditions are

$$\frac{\partial L_1}{\partial p_1} = \theta \cdot \frac{t\left(1.5 - b^H\right) + c_1 - 2p_1 + p_2^H}{2t} + \lambda_1 = 0 \text{ and} \quad \text{(A24)}$$

$$\frac{\partial L_1}{\partial \lambda_1} = p_1 - c_1 \geq 0,\ \frac{\partial L_1}{\partial \lambda_1}\cdot \lambda_1 = 0,\ \lambda_1 \geq 0. \quad \text{(A25)}$$

Let $L_2^H = \left(p_2^H - c_2^H\right)\left[\frac{t\left(0.5 + b^H\right) + p_1 - p_2^H}{2t}\right] - \lambda_2^H\left(c_2^H - p_2^H\right)$ be the Lagrange function of problem (13) with π_2^H defined in (A22), where λ_2^H is the associated Lagrange multiplier. Then, the first-order conditions are

$$\frac{\partial L_2^H}{\partial p_2^H} = \frac{t\left(0.5 + b^H\right) + c_2^H + p_1 - 2p_2^H}{2t} + \lambda_2^H = 0 \text{ and} \quad \text{(A26)}$$

$$\frac{\partial L_2^H}{\partial \lambda_2^H} = p_2^H - c_2^H \geq 0,\ \frac{\partial L_2^H}{\partial \lambda_2^H}\cdot \lambda_2^H = 0,\ \lambda_2^H \geq 0. \quad \text{(A27)}$$

Since the profit of firm *B* with c_2^L in (A23) is the same as that in (A10), firm *B* with c_2^L will choose $p_2^L = p_1$ given firm *A*'s price p_1 by Lemma A. Then, according to whether the constraints

[3] Since $p_2^L = p_1$ by Lemma A, firm *A*'s expected profit function is the first part of (A21).

in (A25) and (A27) bind or not, there are four possible product-price pairs, which are grouped into three sub-cases below.

Case (iiia): Suppose $p_1 > c_1$, $p_2^H > c_2^H$ and $p_2^L = p_1$. Then we have $\lambda_1 = \lambda_2^H = 0$, and (A24) and (A26) become

$$\frac{\partial L_1}{\partial p_1} = \theta \cdot \frac{t\left(1.5 - b^H\right) + c_1 - 2p_1 + p_2^H}{2t} = 0 \text{ and}$$

$$\frac{\partial L_2^H}{\partial p_2^H} = \frac{t\left(0.5 + b^H\right) + c_2^H + p_1 - 2p_2^H}{2t} = 0, \text{ respectively.}$$

Solving these two equations yields equilibrium prices $\left(p_1^*, p_2^{H*}, p_2^{L*}\right)$ $= \left(\frac{t\left(3.5-b^H\right)+2c_1+c_2^H}{3}, \frac{t\left(2.5+b^H\right)+c_1+2c_2^H}{3}, \frac{t\left(3.5-b^H\right)+2c_1+c_2^H}{3}\right)$. Substituting $\left(p_1^*, p_2^{H*}, p_2^{L*}\right)$ into (A21)-(A23) yields firms' (expected) equilibrium profits $\left(\mathrm{E}\pi_1^*, \pi_2^{H*}, \pi_2^{L*}\right) =$ $\left(\frac{\theta\left[t\left(3.5-b^H\right)-c_1+c_2^H\right]^2}{18t}, \frac{\left[t\left(2.5+b^H\right)+c_1-c_2^H\right]^2}{18t}, \frac{t\left(3.5-b^H\right)+2c_1+c_2^H-3c_2^L}{3}\right)$.

Since π_2^{H*} is a strictly increasing function of b^H, we have $b^{H*} = 1/2$, which violates the hypothesis of $b^H \in [0, 1/2)$. Thus, no BNE exists in this case.

Case (iiib): Suppose $p_1 > c_1$, $p_2^H = c_2^H$ and $p_2^L = p_1$. Then, we have $\lambda_1 = 0$ and $\lambda_2^H \geq 0$, and (A24) and (A26) become

$$\frac{\partial L_1}{\partial p_1} = \theta \cdot \frac{t\left(1.5 - b^H\right) + c_1 - 2p_1 + c_2^H}{2t} = 0 \text{ and}$$

$$\frac{\partial L_2^H}{\partial p_2^H} = \frac{t\left(0.5 + b^H\right) + c_2^H + p_1 - 2c_2^H}{2t} + \lambda_2^H = 0, \text{ respectively.}$$

Solving these two equations yields equilibrium prices $\left(p_1^*, p_2^{H*}, p_2^{L*}\right)$ $= \left(\frac{t\left(1.5-b^H\right)+c_1+c_2^H}{2}, c_2^H, \frac{t\left(1.5-b^H\right)+c_1+c_2^H}{2}\right)$ and $\lambda_2^H = -\frac{t\left(2.5+b^H\right)+c_1-c_2^H}{4t} \geq 0$. To make $\lambda_2^{H*} \geq 0$ hold, the following should hold.

$$(c_2^H - c_1) \geq t\left(2.5 + b^H\right) \qquad \text{(A28)}$$

Condition (A28) also guarantees the output of firm B with c_2^H being zero. Substituting $\left(p_1^*,\, p_2^{H*},\, p_2^{L*}\right)$ into (A21)-(A23) yields firms' (expected) equilibrium profits $\left(\mathrm{E}\pi_1^*,\, \pi_2^{H*},\, \pi_2^{L*}\right) =$ $\left(\frac{\theta\left[t\left(1.5-b^H\right)-c_1+c_2^H\right]}{2},\, 0,\, \frac{t\left(1.5-b^H\right)+c_1+c_2^H-2c_2^L}{2}\right)$.

Since the equilibrium profits of firm A and firm B with c_2^L are independent of their locations, $a^* = b^{L*} = 1/2$ can be optimal location. Similarly, $b^{H*} \in [0, 1/2)$ can be optimal location because $\pi_2^{H*} = 0$ is independent of b^H. However, we should rule out $b^{H*} = 1/2$ to make $a^* + b^{H*} < 1$ hold. Thus, firms' optimal locations are $\left(a^*,\, b^{H*},\, b^{L*}\right) = \left(1/2,\, [0,\, 1/2),\, 1/2\right)$. Substituting $\left(a^*,\, b^{H*},\, b^{L*}\right)$ into equilibrium prices and (A28) generates Proposition 2.

Case (iiic): Because the proofs for $\left(p_1 = c_1,\, p_2^H > c_2^H,\, p_2^L = p_1\right)$ and $\left(p_1 = c_1,\, p_2^H = c_2^H,\, p_2^L = p_1\right)$ are similar, we demonstrate the former. Then, we have $\lambda_1 \ge 0$ and $\lambda_2^H = 0$, and (A24) and (A26) can be reduced to

$$\frac{\partial L_1}{\partial p_1} = \theta \cdot \frac{t\left(1.5-b^H\right)-c_1+p_2^H}{2t} + \lambda_1 = 0 \text{ and}$$

$$\frac{\partial L_2^H}{\partial p_2^H} = \frac{t\left(0.5+b^H\right)+c_2^H+c_1-2p_2^H}{2t} = 0, \text{ respectively.}$$

Since $b^H \in [0,\, 1/2)$, $t > 0$, $p_2^H > c_2^H > c_1$, $\theta \in (0,\, 1)$ and $\lambda_1 \ge 0$, we have $\partial L_1/\partial p_1 > 0$, which contradicts $\partial L_1/\partial p_1 = 0$. Thus, no BNE exists in this case.

(iv) Suppose $a + b^H = 1$ and $a + b^L < 1$. The proofs are similar to those in Cases (iiia) and (iiic). Thus, they are omitted and available upon request.

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