Corporate social responsibility and endogenous competition structure

Toshihiro Matsumura  
_Institute of Social Sciences, University of Tokyo_

Akira Ogawa  
_College of Liberal Arts, International Christian University_

Abstract

We assume that firms care about corporate social responsibility (CSR) and revisit the endogenous choice between price and quantity contracts. We find that a significant (insignificant) asymmetric weight of CSR in their objectives yields Bertrand (Cournot) competition.
1 Introduction

In the last several decades, corporate social responsibility (CSR) has generated a large amount of interest in broad social science literature. Especially, economic researchers have intensively discussed this problem (Kitzmueller and Shimshack, 2012) because many listed firms announce that they are highly concerned with CSR (KPMG, 2013). For example, many big Japanese firms as well as Japanese economic associations such as Japan Association of Corporate Executives, the Japan Business Federation, the Japan Iron and Steel Federation, the Federation of Electric Power Companies of Japan emphasize CSR in their annual reports and websites.

In the literature on CSR, most works assume that competition structure (Bertrand or Cournot) are given exogenously. However, CSR can affect the competition structure and endogenizing the competition structure is important because the implications of CSR crucially depend on the competition structure in the product market (Liu et al, 2015). Furthermore, as Ghosh and Mitra (2014) discussed, Bertrand competition often yields the higher welfare level than the Cournot even when firms are not profit-maximizers. If CSR changes the competition structure from Cournot to Bertrand, CSR results in further welfare improving through the change of competition structure.

Regarding the endogenous competition structure, as Singh and Vives (1984) convincingly pointed out, firms often choose whether to adopt a price contract or quantity contract. According to these studies, the competition structure should not be given exogenously.\(^1\)

In their pioneering work, Singh and Vives (1984) formulated a model in which firms choose a price or quantity contract and endogenize the competition structure (Cournot or Bertrand). They showed that if the goods are substitutes, Cournot competition appears in equilibrium. Tanaka (2001) and Tasnádi (2006) showed that this result is quite robust and holds in various contexts. However, these studies assumed that firms are profit-maximizers. In contrast, in the context of mixed oligopolies, Matsumura and Ogawa (2012) showed that competition structure is changed if one firm is a welfare maximizer and the other firm is a profit maximizer. This suggests the possibility that non-profit maximizing objectives may change the competition structure.\(^2\)

We introduce CSR into the model of Singh and Vives (1984). Following Ghosh and Mitra

\(^1\)Friedman (1988) considered the Cournot (res. Bertrand) model more appropriate for a quantity (res. price) change if it is less flexible or more costly than price (res. quantity) change. However, firms should be able to commit to increasing the costs of price changes. For example, declaring that they will not change their prices frequently, keeping this policy, and establishing this reputation may be profitable for firms because this would deter the consumers from waiting for future price declines. If we consider a model where each firm first chooses whether to make such a commitment and then the firms compete in either price (commitment case) or quantity (noncommitment case), this would correspond to Singh and Vives’s (1984) model. We believe that this could be another rationale for these studies’ model.

\(^2\)The papers on endogenous competition structure and Cournot-Bertrand comparison in mixed oligopolies become rich and diverse. See Chirco and Scrimitore (2013), Chirco et al. (2014), Scrimitore(2013, 2014) and Haraguchi and Matsumura (2016).
(2014), we assume that firms are assumed to maximize the weighted sum of total social surplus and profits.\(^3\) As noted above, many private firms care about social responsibility as well as their own profits, and it is quite natural to assume that firms are not always profit maximizers.\(^4\) We derive a result containing those of Singh and Vives (1984) and Matsumura and Ogawa (2012) as a special case and fill the gap of the discussions on private and mixed oligopolies. We find that Bertrand (Cournot) competition appears in equilibrium with significant (insignificant) asymmetry of the weight of CSR between the firms. This result implies that the asymmetry in objectives rather than non-profit maximizing objective itself plays a crucial role in determining competition structure.

2 Model

We adopt a standard differentiated duopoly with linear demand (Dixit, 1979). The utility function of the representative consumer is \(U(q_1, q_2) = \alpha(q_1 + q_2) - \beta(q_1^2 + 2\delta q_1 q_2 + q_2^2)/2 + y\), where \(y\) is the consumption of an outside good that is provided competitively with a unit price. Parameters \(\alpha\) and \(\beta\) are positive constants, and \(\delta \in (0, 1)\) represents the degree of product differentiation: a smaller \(\delta\) indicates a larger degree of product differentiation.

Firms 1 and 2 produce differentiated commodities for which the inverse demand function is given by \(p_i = \alpha - \beta q_i - \beta \delta q_j\) \((i = 1, 2, i \neq j)\), where \(p_i\) and \(q_i\) are firm \(i\)’s price and quantity, respectively. The marginal production costs are constant. Let \(m\) denote each firm’s marginal cost. We assume that \(\alpha > m\), otherwise no firm produces.

Firm \(i\)’s payoff is \(V_i = \theta_i SW + (1 - \theta_i)\pi_i\), where \(\theta_i \in [0, 1]\), \(SW\) is the total social surplus (i.e., the sum of the firms’ profits and consumer surplus), and \(\pi_i\) is firm \(i\)’s profit. \(\theta_i\) \((i = 1, 2)\) indicates the weight of social responsibility in the payoff of each firm. \(\pi_i\) \((i = 1, 2)\) and \(SW\) are given by

\[
\pi_i = (p_i - m)q_i = (\alpha - \beta q_i - \beta \delta q_j - m)q_i,
\]

\[
SW = \pi_1 + \pi_2 + \left[ \alpha(q_1 + q_2) - \frac{\beta(q_1^2 + 2\delta q_1 q_2 + q_2^2)}{2} - p_1 q_1 - p_2 q_2 \right] = \pi_1 + \pi_2 + \frac{\beta}{2}(q_1^2 + 2\delta q_1 q_2 + q_2^2).
\]

The game runs as follows. In the first stage, each firm chooses whether it adopts a price contract or a quantity one. In the second stage, after observing the rival’s choice in the first stage, each firm simultaneously chooses \(p\) or \(q\), according to the decision in the first stage.

\(^3\)This approach is widely adopted in the literature on mixed oligopolies where a partially privatized firm cares about both its own profits and social welfare (see Matsumura, 1998, Lee, 1999). They generalized this partial privatization approach and considered the situation where all firms care about both social welfare and their own profits. See also Matsumura and Ogawa (2014).

\(^4\)We obtain the same qualitative results if firms maximize the weighted sum of consumer surplus and profits.
3 Second-stage games

First, we discuss four possible subgames: both choose quantities \((q-q\text{ game})\), both choose prices \((p-p\text{ game})\), only firm 1 chooses quantity \((q-p\text{ game})\), and only firm 1 chooses price \((p-q\text{ game})\). Let \(a := \alpha - m\).

### 3.1 \(q-q\) game

Consider the situation where both firms choose quantities. The first-order condition for firm \(i (i = 1, 2)\) is given by

\[
\frac{\partial V_i}{\partial q_i} = a - 2q_i \beta - q_i \beta \delta + q_i \beta \theta_i = 0 \ (i \neq j).
\]

The second-order condition is satisfied. From the first-order condition, we obtain the following reaction function of firm \(i\).

\[
R_{qq}^i(q_j) = \frac{a - \beta \delta q_j}{\beta(2 - \theta_i)} = 0 \ (i \neq j).
\]

These lead to the following equilibrium quantities, and the resulting value of objective functions.

\[
q_{1qq} = \frac{(2 - \theta_2 - \delta)a}{\beta(\theta_1 \theta_2 - 2 \theta_2 - 2 \theta_1 - \delta^2 + 4)},
\]

\[
q_{2qq} = \frac{(2 - \theta_1 - \delta)a}{\beta(\theta_1 \theta_2 - 2 \theta_2 - 2 \theta_1 - \delta^2 + 4)},
\]

\[
V_{1qq} = \frac{a^2 B_1}{2 \beta(\theta_1 \theta_2 - 2 \theta_2 - 2 \theta_1 - \delta^2 + 4)^2},
\]

\[
V_{2qq} = \frac{a^2 B_2}{2 \beta(\theta_1 \theta_2 - 2 \theta_2 - 2 \theta_1 - \delta^2 + 4)^2},
\]

where the constant \(B_1, B_2\), and other constants are reported in the Appendix.

### 3.2 \(p-p\) game

Consider the situation where both firms choose prices. The demand function is given by

\[
q_i = \frac{\alpha - \alpha \delta - p_i + \delta p_j}{\beta(1 + \delta)(1 - \delta)} \ (i \neq j).
\]

The first-order condition for firms \(i\) is given by,

\[
\frac{\partial V_i}{\partial p_i} = \frac{m - 2p_i + \alpha + p_j \delta - \delta \alpha + p_i \theta_i - \alpha \theta_i - m \delta \theta_i + \delta \alpha \theta_i}{\beta(1 - \delta)(1 + \delta)} = 0 \ (i \neq j).
\]

The second-order condition is satisfied. From the first-order condition, we obtain the following reaction function of firm \(i\).

\[
R_{pp}^i(p_j) = \frac{m + \alpha + p_j \delta - \delta \alpha - \alpha \theta_i + \delta \theta_i a}{2 - \theta_i}.
\]
These lead to the following equilibrium prices and resulting values of objective functions.

\[
\begin{align*}
p_1^{pp} & = \frac{\alpha(1 - \theta_1) + m + \delta \theta_1 a}{\theta_1 \theta_2 - 2 \theta_2 - 2 \theta_1 - \delta^2 + 4}, \\
p_2^{pp} & = \frac{\alpha(1 - \theta_2) + m + \delta \theta_2 a}{\theta_1 \theta_2 - 2 \theta_2 - 2 \theta_1 - \delta^2 + 4}, \\
V_1^{pp} & = \frac{a^2 B_3}{\beta(\delta + 1)(1 - \delta)(\theta_1 \theta_2 - 2 \theta_2 - 2 \theta_1 - \delta^2 + 4)^2}, \\
V_2^{pp} & = \frac{a^2 B_4}{\beta(\delta + 1)(1 - \delta)(\theta_1 \theta_2 - 2 \theta_2 - 2 \theta_1 - \delta^2 + 4)^2}.
\end{align*}
\]

3.3 \textit{p – q game}

Consider the situation where firm 1 chooses price and firm 2 chooses quantity. The first-order conditions for firms 1 and 2 are respectively,

\[
\begin{align*}
\frac{\partial V_1}{\partial p_1} & = \frac{m - 2 p_1 + \alpha + p_1 \theta_1 - \alpha \theta_1 - q_2 \beta \delta + q_2 \beta \delta \theta_1}{\beta} = 0, \\
\frac{\partial V_2}{\partial q_2} & = \alpha - m + p_1 \delta - 2 q_2 \beta - \delta \alpha + m \delta \theta_2 - p_1 \delta \theta_2 + q_2 \beta \theta_2 + 2 q_2 \beta \delta^2 - q_2 \beta \delta^2 \theta_2 = 0.
\end{align*}
\]

The second-order conditions are satisfied. From the first-order conditions, we obtain the following reaction functions of firms 1 and 2.

\[
\begin{align*}
R_1^{pq}(q_2) & = \frac{\alpha(1 - \theta_1) + m - \beta \theta(1 - \theta_1)}{2 - \theta_1}, \\
R_2^{pq}(p_1) & = \frac{\alpha(1 - \delta) - m + \delta p_1 + m \delta \theta_2 + \delta \theta_2 p_1}{\beta(1 - \delta)(1 + \delta)(2 - \theta_2)}.
\end{align*}
\]

These lead to the following equilibrium choice of each firm and resulting values of objective functions.

\[
\begin{align*}
p_1^{pq} & = \frac{(\alpha + m)(2 - \theta_2) - \delta a - 2 \alpha \theta_1 + \delta \theta_1 a + \alpha \theta_1 \theta_2 - 2 m \delta^2 - \delta^3 \alpha(1 - \theta_1 - \theta_2) - \delta^2 \theta_1 \theta_2 a}{\theta_1 \theta_2 - 2 \theta_2 - 2 \theta_1 - 3 \delta^2 + \delta^2 \theta_1 + \delta^2 \theta_2 + 4}, \\
q_2^{pq} & = \frac{a(-\delta - \theta_1 - \delta \theta_2 + \delta \theta_1 \theta_2 + 2)}{\beta(\theta_1 \theta_2 - 2 \theta_2 - 2 \theta_1 - 3 \delta^2 + \delta^2 \theta_1 + \delta^2 \theta_2 + 4)}, \\
V_1^{pq} & = \frac{a^2 B_5}{2 \beta(-2 \theta_1 - 2 \theta_2 + \theta_1 \theta_2 - 3 \delta^2 + \delta^2 \theta_1 + \delta^2 \theta_2 + 4)^2}, \\
V_2^{pq} & = \frac{a^2 B_6}{2 \beta(-2 \theta_1 - 2 \theta_2 + \theta_1 \theta_2 - 3 \delta^2 + \delta^2 \theta_1 + \delta^2 \theta_2 + 4)^2}.
\end{align*}
\]
3.4 $q - p$ game

Consider the situation where firm 1 chooses quantity and firm 2 chooses price. The first-order conditions for firms 1 and 2 are respectively,

$$\frac{\partial V_1}{\partial q_1} = \alpha - m + p_2 \delta - 2q_1 \beta - \delta \alpha + m \delta \theta_1 - p_2 \delta \theta_1 + q_1 \beta \theta_1 + 2q_1 \beta \delta^2 - q_1 \beta \delta^2 \theta_1 = 0,$$

$$\frac{\partial V_2}{\partial p_2} = m - 2p_2 + \alpha + p_2 \theta_2 - \alpha \theta_2 - q_1 \beta \delta + q_1 \beta \delta \theta_2 = 0.$$

The second-order conditions are satisfied. From the first-order conditions, we obtain the following reaction functions of firms 1 and 2.

$$R_{q}^{bp}(p_2) = \frac{\alpha(1 - \delta) - m + \delta p_2 + m \delta \theta_1 + \delta \theta_1 p_2}{\beta(1 - \delta)(1 + \delta)(2 - \theta_1)}.$$

$$R_{q}^{bp}(q_1) = \frac{\alpha(1 - \theta_2) + m - \beta \delta(1 - \theta_2)q_1}{2 - \theta_2}.$$

These lead to the following equilibrium choice of each firm and resulting value of objective functions.

$$q_1^{bp} = \frac{a(-\delta - \theta_2 - \delta \theta_1 + \delta \theta_2 \theta_1 + 2)}{\beta(2 \theta_1 - 2 \theta_2 - 3 \delta^2 + \delta^2 \theta_2 + \delta^2 \theta_1 + 4)},$$

$$p_2^{bp} = \frac{(\alpha + m)(2 - \theta_1) - \delta a - 2a \theta_2 + \delta \theta_2 a + \alpha \theta_2 \theta_1 - 2m \delta^2 - \delta^2 \alpha(1 - \theta_2 - \theta_1) - \delta^2 \theta_2 \theta_1 a}{\theta_2 \theta_1 - 2 \theta_1 - 2 \theta_2 - 3 \delta^2 + \delta^2 \theta_2 + \delta^2 \theta_1 + 4},$$

$$V_1^{bp} = \frac{2a^2 B_7}{2(2 \theta_1 - 2 \theta_2 + \theta_1 \theta_2 - 3 \delta^2 + \delta^2 \theta_1 + \delta^2 \theta_2 + 4)^2},$$

$$V_2^{bp} = \frac{a^2 B_8}{2(2 \theta_1 - 2 \theta_2 + \theta_1 \theta_2 - 3 \delta^2 + \delta^2 \theta_1 + \delta^2 \theta_2 + 4)^2}.$$

4 Result

We discuss the choice at the first stage. Table 1 summarizes this game.

<table>
<thead>
<tr>
<th>1 \ 2</th>
<th>quantity</th>
<th>price</th>
</tr>
</thead>
<tbody>
<tr>
<td>quantity</td>
<td>$(V_1^{bp}, V_2^{bp})$</td>
<td>$(V_1^{bp}, V_2^{bp})$</td>
</tr>
<tr>
<td>price</td>
<td>$(V_1^{bp}, V_2^{bp})$</td>
<td>$(V_1^{bp}, V_2^{bp})$</td>
</tr>
</tbody>
</table>

Table 1: First Stage Game

If both firms are welfare-maximizers, the first-best outcome is achieved regardless of the first stage choices, and thus they do not matter for welfare-maximizers. Henceforth, we restrict our attention to the cases where $(\theta_1, \theta_2) \neq (1, 1)$.

Although we cannot solve the game explicitly in the case of general asymmetric objective functions, we can show this tendency using the numerical results for four values of $\delta$. Figure 1
shows the conditions for Bertrand and Cournot outcomes to appear in equilibrium, implying that the Bertrand outcome is more likely to be in equilibrium when the difference between $\theta_1$ and $\theta_2$ is large.

We explain the intuition behind why a Bertrand outcome appears in equilibrium when the payoff asymmetry is significant.

First, we explain why choosing a price contract is more profitable for firm 2 that is much closer to being a profit-maximizer than firm 1. As Singh and Vives (1984) suggested, the demand elasticity of both firms increases when firm 2 chooses $p$ than when it chooses $q$. Because firm 1 is highly concerned with welfare, firm 1 does not want to reduce the output of firm 2, and thus it becomes less aggressive when firm 2 chooses $p$ than when it chooses $q$. In order to make the rival less aggressive, firm 2 chooses $p$.

Second, we explain why choosing a price contract is better for firm, which is much closer to being a welfare-maximizer than firm 1. The demand elasticity of both firms becomes larger when firm 1 chooses $p$ than when it chooses $q$. Because firm 2 is closer to being a profit-maximizer, firm 1 becomes more aggressive when firm 1 chooses $p$ than when it chooses $q$. This aggressive behavior improves firm 1’s payoff because it is more concerned with welfare. Thus, firm 1 also chooses $p$.

Finally, we explain the intuition behind why a Cournot outcome appears in equilibrium when the payoff asymmetry is insignificant.\(^5\)

Suppose that both $\theta_1$ and $\theta_2$ are small ($\theta_1, \theta_2 < \delta$). Suppose that firm 2 chooses the quantity contract. Each firm prefers a smaller output of the rival because it increases its demand and thus its profit. When firm 1 chooses the price contract, an increase in the firm 2’s output reduces firm 1’s output. When firm 1 chooses the quantity contract, this effect disappears because the firm 1’s output is determined before observing the firm 2’s output. Thus, firm 2 chooses a smaller output when firm 1 also chooses the quantity contract. To make the rival less aggressive, firm 1 chooses the quantity contract.

Suppose that both $\theta_1$ and $\theta_2$ are large ($\theta_1, \theta_2 > \delta$). Suppose that firm 2 chooses the quantity contract. Each firm prefers a larger output of the rival because it enhances welfare. When firm 1 chooses the price contract, an increase in the firm 2’s output reduces firm 1’s output. When firm 1 chooses the quantity contract, this effect disappears. Thus, firm 2 chooses a larger output when firm 1 also chooses the quantity contract. To make the rival more aggressive, firm 1 chooses the quantity contract.

\(^5\)Ghosh and Mitra (2014) showed that Cournot competition appears in equilibrium when $\theta_1 = \theta_2$. 
Appendix

\[ B_1 = 8\theta_1 - 8\delta - 8\theta_2 + 4\delta\theta_2 - 4\theta_1\theta_2 + 2\delta\theta_1\theta_2 + 2\delta^2 - 12\theta_1^3 + 3\theta_1^3 + 2\theta_2^2 + 2\delta\theta_1^2 - 6\delta\theta_1 - 2\delta^3\theta_1 - \theta_1\theta_2^2 + 8\theta_1\theta_2 - 2\theta_1^3\theta_2 - 2\delta\theta_1^2\theta_2 + 2\delta^2\theta_1^2 + 8, \]

\[ B_2 = 8\theta_2 - 8\theta_1 - 8\delta + 4\delta\theta_1 - 4\theta_1\theta_2 + 2\delta\theta_1\theta_2 + 2\delta^2 + 2\theta_1^2 - 12\theta_2^2 + 3\theta_2^3 + 2\delta\theta_2^2 - 6\delta\theta_2 + 2\delta^3\theta_2 - 2\theta_1\theta_2^2 - 2\delta\theta_1^2\theta_2 + 2\delta^2\theta_2^2 + 8. \]

\[ B_3 = 8\theta_1 - 8\theta_2 + 8\delta\theta_1 - 4\delta\theta_2 - 4\theta_1\theta_2 - 2\delta\theta_1\theta_2 - 2\delta^2 - 2\theta_1^3 - 3\theta_1^3 + 2\theta_2^2 - 10\delta\theta_1^2 - 2\delta^2\theta_1 + 3\delta\theta_1^2 + 2\delta\theta_2^2 + 8\delta^2\theta_2^2 + 2\delta^2\theta_1 + 4\delta\theta_1^2 + 2\delta^4\theta_1 - \theta_1\theta_2^2 + 8\theta_1\theta_2 - 2\theta_1^3\theta_2 - 12\delta\theta_1\theta_2 - 12\delta\theta_1^2\theta_2 - 4\delta\theta_1^3\theta_2 - 2\delta^3\theta_1\theta_2^2 + \delta\theta_1^2 - 2\delta^3\theta_2^2 - 2\delta^3\theta_1^2 - \delta^3\theta_2^2 - 6\delta\theta_1\theta_2^2 + 5\delta^3\theta_1^2\theta_2^2 + 8\delta^2\theta_2^2 + 2\delta\theta_1\theta_2^2 - 2\delta^2\theta_1\theta_2^2 + 8, \]

\[ B_4 = 8\theta_2 - 8\theta_1 - 4\delta\theta_1 + 8\delta\theta_2 - 4\theta_1\theta_2 - 2\delta\theta_1\theta_2 - 2\delta^2 - 2\theta_1^2 - 12\theta_2^2 + 3\theta_1^3 + 2\theta_2^2 + 2\delta^2\theta_1 + 3\theta_1^3 + 2\theta_2^2 + 2\delta^2\theta_1^2 + 8\delta^2\theta_2^2 + 2\delta^2\theta_1 + 4\delta\theta_1^2 + 2\delta^4\theta_1 - \theta_1\theta_2^2 + 8\theta_1\theta_2 - 2\theta_1^3\theta_2 - 12\delta\theta_1\theta_2 - 12\delta\theta_1^2\theta_2 - 4\delta\theta_1^3\theta_2 - 2\delta^3\theta_1\theta_2^2 + \delta\theta_1^2 - 2\delta^3\theta_2^2 - 2\delta^3\theta_1^2 - \delta^3\theta_2^2 - 6\delta\theta_1\theta_2^2 + 5\delta^3\theta_1^2\theta_2^2 + 8\delta^2\theta_2^2 + 2\delta\theta_1\theta_2^2 - 2\delta^2\theta_1\theta_2^2 + 8, \]

\[ B_5 = 8\theta_1 - 8\delta - 8\theta_2 + 4\delta\theta_2 - 4\theta_1\theta_2 - 2\delta\theta_1\theta_2 - 2\delta^3 - 12\theta_1^2 + 3\theta_1^3 + 2\theta_2^2 + 2\delta^2\theta_1 + 10\delta\theta_1^2 + 12\delta^2\theta_1 + 4\delta^3\theta_1 - 4\delta\theta_1\theta_2 - \delta^3\theta_1 - 4\delta^4\theta_1 + 4\delta^4\theta_2 - \theta_1\theta_2^2 + 8\theta_1\theta_2 - 2\delta\theta_1\theta_2 - 2\delta^3\theta_1\theta_2 - 2\delta^3\theta_1\theta_2 + 2\delta^3\theta_1\theta_2 - 12\delta\theta_1\theta_2 + 6\delta^3\theta_1\theta_2 + 2\delta^3\theta_1\theta_2 + 10\delta^2\theta_1^2 - 2\delta^2\theta_1^2 + 2\delta^2\theta_1^2 + 2\delta^3\theta_1^2 - 6\delta\theta_1\theta_2^2 + 5\delta^3\theta_1\theta_2^2 + 8\delta^2\theta_2^2 + 2\delta\theta_1\theta_2^2 - 2\delta^2\theta_1\theta_2^2 + 8, \]
\[ B_6 = 8\theta_2 - 8\theta_1 - 8\delta + 4\delta\theta_1 - 4\theta_1\theta_2 + 2\delta\theta_1\theta_2 - 6\delta^2 + 8\delta^3 - 2\delta^4 + 2\theta_1^2 - 12\theta_2^2 + 3\theta_2^3 + 8\frac{\delta^2\theta_1}{\theta_1} + 2\delta\theta_2^2 - 14\delta^2\theta_2 - 4\delta^3\theta_1 + 2\delta^3\theta_2 + 5\delta^4\theta_2 + 8\theta_1\theta_2^2 - \theta_1^2\theta_2 - 2\theta_1\theta_2^3
\]

\[ B_7 = 8\theta_1 - 8\theta_2 - 8\delta + 4\delta\theta_2 - 4\theta_2\theta_1 + 2\delta\theta_2\theta_1 - 6\delta^2 + 8\delta^3 - 2\delta^4 + 2\theta_2^2 - 12\theta_1^2 + 3\theta_1^3 + 8\frac{\delta^2\theta_2}{\theta_2} + 2\delta\theta_1^2 - 14\delta^2\theta_1 - 4\delta^3\theta_2 + 2\delta^3\theta_1 + 5\delta^4\theta_1 + 8\theta_2\theta_1^2 - \theta_2^2\theta_1 - 2\theta_2\theta_1^3
\]

\[ B_8 = 8\theta_2 - 8\delta - 8\theta_1 + 4\delta\theta_1 - 4\theta_1\theta_2 - 2\delta\theta_2\theta_1 - 6\delta^2 + 4\delta^3 + 2\delta^4 - 12\theta_2^2 + 3\theta_2^3 + 2\theta_2^4 + 2\delta\theta_2^2 + 10\delta^2\theta_2 + 12\delta^2\theta_1 + 4\delta^3\theta_2 - 4\delta^3\theta_1 - \delta^4\theta_2 - 4\delta^4\theta_1 + \theta_2^2\theta_1 + 8\theta_2^2\theta_1 + 2\delta\theta_1^2 + 4\delta\theta_2^2\theta_1 - 4\delta^2\theta_2\theta_1 - 2\delta^2\theta_2\theta_1 + 6\delta^3\theta_2\theta_1 + 2\delta^3\theta_2\theta_1 + 10\delta^2\theta_2^2
\]

\[ -2\delta\theta_2^3 + 4\delta^2\theta_1^2 - 2\delta\theta_1^2 - 2\delta\theta_1^2 + 2\delta\theta_2^2 - 6\delta\theta_2^2\theta_1 + \delta^2\theta_2\theta_1 + 2\delta\theta_2\theta_1 - 2\delta^3\theta_2\theta_1
\]

\[ -8\delta\theta_2^2\theta_1 + 2\delta^3\theta_2\theta_1 - \delta^4\theta_2\theta_1 + 2\delta^2\theta_2^2\theta_1 - \delta^2\theta_2\theta_1 + 2\delta^3\theta_2\theta_1 + 8. \]
References


Figure 1: Equilibrium under asymmetric thetas

(delta=0.2)

(delta=0.4)

(delta=0.6)

(delta=0.8)

(Note) "asymmetry" means the equilibrium in which one firm chooses the price and the other firm chooses the quantity.