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Joint aggregation over money and credit card services under risk

William A. Barnett
University of Kansas and Center for Financial Stability

Liting Su
University of Kansas and Center for Financial Stability

Abstract

Modern aggregation theory and index number theory were introduced into monetary economics by Barnett (1980). The widely used Divisia monetary aggregates, provided to the public in monthly releases by the Center for Financial Stability in NY City, are based upon that paper. A key result upon which the rest of the theory depended was Barnett's derivation of the user-cost price of monetary assets. To make that critical part of Barnett's results available prior to publication in the Journal of Econometrics, Barnett (1978) repeated that important proof two years earlier in Economics Letters. The extension of that literature to risk with intertemporally non-separable preferences subsequently appeared in Barnett and Wu (2005). To make that result available prior to publication in the Annals of Finance, the paper's theory without proofs was provided a year earlier by Barnett and Wu (2004) in the Economic Bulletin. The theory was extended by Barnett and Su (2016a) to include the services of credit card transactions volumes under risk. The theory will appear in the proceedings volume of a conference to be held in Rome in June 2017. The proceedings will appear as a special issue of the journal, Macroeconomic Dynamics, in late 2019 at the earliest. We are making available the key results from that paper below, without the proofs. Prior to publication of Barnett and Su (2016a), the proofs will be available in the paper's online working paper version, Barnett and Su (2016b).

The proofs of the theorems in this preliminary article will appear in the full research paper forthcoming in the journal, Macroeconomic Dynamics. We are indebted to Apostolos Serletis for his suggestion of this topic for research. His suggestion is contained in his presentation as discussant of Barnett's Presidential Address at the Inaugural Conference of the Society for Economic Measurement at the University of Chicago, August 18-20,

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Contact: William A. Barnett - williamabarnett@gmail.com, Liting Su - linnasusu@gmail.com.

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1. Introduction

While money is an asset, credit is a liability. In accounting conventions, assets and liabilities cannot be added together. But aggregation theory and economic index number theory are based on microeconomic theory, not accounting conventions. Economic aggregates measure service flows. To the degree that money and some forms of credit produce joint services, those services can be aggregated. A particularly conspicuous example is credit card services, which are directly involved in transactions and contribute to the economy's liquidity in ways not dissimilar to those of money.[1]

Barnett, Chauvet, Leiva-Leon, and Su (2016) derived the aggregation and index number theory needed to aggregate jointly over the services of money and credit cards. The derivation assumes perfect certainty or risk neutrality. Barnett and Su (2016) extend the theory by removing the assumption of risk neutrality. The derivation is thereby altered by replacing the perfect certainty first order conditions with the relevant Euler equations. We provide the theoretical results without the proofs. Prior to publication of Barnett and Su (2016a), the proofs will be available in the paper's online working paper version, Barnett and Su (2016b).

An extensive literature exists on policy relevance of the existing Divisia monetary aggregates, which do not include the services of credit cards.[2] The Bank of England provides them officially for the UK. The central bank of Poland and the Bank of England also provide them for their countries. The European Central Bank provides them to its Governing Council at its policy meetings, but does not provide them to the public. The Bank of Japan has them, but does not provide them to the public. For the United States, the Center for Financial Stability (CFS) provides the Divisia monetary aggregates through formal monthly releases, received by thousands of subscribers throughout the world and also made available to Bloomberg terminal users.[3] In the near future, the CFS plans to begin making the Divisia monetary aggregates available for Europe, China, and India. As a result of the new developments by Barnett, Chauvet, Leiva-Leon, and Su (2016) and by Barnett and Su (2016a), the CFS is preparing to begin releasing Divisia monetary aggregates augmented to include credit card services. Because of the high volatility and level of credit card interest rates, the adjustment for risk using the results below are likely to be of much more importance for the new augmented Divisia monetary aggregates than risk adjustment for the existing Divisia monetary aggregates, excluding the transactions services of credit cards.[4]

2. Flow of funds budget constraint

[1] A long literature exists on the defects of monetary aggregates that do not include credit card services and the inability to solve that problem by accounting means. See, e.g., Bernanke and Blinder (1988), Duca and Whitesell (1995), and Telyukova and Wright (2008).
[2] See, e. g., Barnett (2012), Belongia and Ireland (2006; 2014; 2015a,b; 2016), Barnett and Chauvet (2011), Serletis and Rahman (2013), Barnett and Serletis (2000), and Serletis and Gogas (2014).
[3] The CFS also keeps the information from the monthly releases online as a permanent historical database. See http://www.centerforfinancialstability.org/amfm.php.
At last count, CFS receives visitors from over 187 of the 195 countries in the world. Divisia monetary aggregates are available from nongovernmental sources for over 40 countries throughout the world. See http://www.centerforfinancialstability.org/amfm_int.php. Also see http://www.centerforfinancialstability.org/WBarnett.php.
[4] Regarding risk adjustment without inclusion of credit card services, see Poterba and Rotemberg (1987).

We begin by defining the variables for the representative consumer:

$\mathbf{x}_s$ = vector of per capita (planned) consumptions of N goods and services (including those of durables) during period s.
$\mathbf{p}_s$ = vector of goods and services expected prices, and of durable goods expected rental prices during period s.
m_{is} = planned per capita real balances of monetary asset i during period s $(i = 1,2,\ldots,n)$.
c_{js} = planned per capita real expenditure with credit card type j for transactions during period s $(j = 1,2,\ldots,k)$. In the jargon of the credit card industry, those contemporaneous expenditures are called "volumes."
z_{js} = planned per capita rotating real balances in credit card type j during period s from transactions in previous periods $(j = 1,2,\ldots,k)$.
$y_{js} = c_{js} + z_{js}$= planned per capita total balances in credit type j during period s $(j = 1,2,\ldots,k)$.
r_{is} = expected nominal holding period yield (including capital gains and losses) on monetary asset i during period s $(i = 1,2,\ldots,n)$.
e_{js} = expected interest rate on c_{js}.
$\overline{e}_{js}$ = expected interest rate on z_{js}.
A_s = planned per capita real holdings of the benchmark asset during period s.
R_s = expected (one-period holding) yield on the benchmark asset during period s.
L_s = per capita labor supply during period s.
w_s = expected wage rate during period s.
$p_s^* = p_s^*(\mathbf{p}_s)$ is the true cost of living index, as defined in Barnett (1978,1980).

The benchmark asset is defined to provide no services other than its expected yield, R_s, which motivates holding of the asset solely as a means of accumulating wealth. As a result, R_s is the maximum expected holding period yield available to consumers in the economy in period s from holding a secured asset. The benchmark asset is held to transfer wealth by consumers between multiperiod planning horizons, rather than to provide liquidity or other services. In contrast, $\overline{e}_{js}$ is not the interest rate on an asset and is not secured. It is the interest rate on an unsecured liability, subject to substantial default and fraud risk. Hence, $\overline{e}_{js}$ can be higher than the benchmark asset rate, and historically has always been much higher than the benchmark asset rate.

The decision problem we model is not of a single economic agent, but rather of the "representative consumer," aggregated over all consumers. All quantities are therefore averaged over all consumers. This modeling assumption is particularly important in understand the credit card quantities and interest rates used in our research. About 20% of credit card holders in the United States do not pay explicit interest on credit card balances, since those credit card transactions are paid off by the end of the period. But the 80% who do pay interest pay very high interest rates. The Federal Reserve provides two interest rate series for credit card debt. One, $\overline{e}_{js}$, includes interest only on accounts that do pay interest to the credit card issuing banks, while the other series, e_{js}, includes the approximately 20% that do not pay interest. The latter interest rate is thereby lower, since it is averaged over interest paid on both categories of accounts. Although e_{js} is less than $\overline{e}_{js}$, e_{js} has nevertheless always been higher than the benchmark rate.

Barnett, Chauvet, Leiva-Leon, and Su (2016) use the latter interest rate, e_{js}, in their augmented Divisia monetary aggregates formula, since the contemporaneous per capita transactions volumes in the model are averaged over both categories of credit card holders. They do not include rotating balances used for transactions in prior periods, since to do so would involve double counting of transactions services.

The resulting flow of funds identity for each period s is:

$$\begin{aligned}\mathbf{p}_s'\mathbf{x}_s = w_sL_s &+ \sum_{i=1}^{n}\left[(1+r_{i,s-1})p_{s-1}^*m_{i,s-1} - p_s^*m_{is}\right] \\ &+ \sum_{j=1}^{k}\left[p_s^*c_{js} - (1+e_{j,s-1})p_{s-1}^*c_{j,s-1}\right] \\ &+ \sum_{j=1}^{k}\left[p_s^*z_{js} - \left(1+\overline{e}_{j,s-1}\right)p_{s-1}^*z_{j,s-1}\right] + [(1+R_{s-1})p_{s-1}^*A_{s-1} \\ &- p_s^*A_s].\end{aligned} \tag{1}$$

Planned per capita total balances in credit type *j* during period *s* are then $y_{js} = c_{js} + z_{js}$.

3. Risk adjustment

3.1 The decision

Define Y to be the consumer's survival set, assumed to be compact. The consumption possibility set, $S(s)$, for period s is the set of survivable points, $(\mathbf{m}_s, \mathbf{c}_s, \mathbf{x}_s, A_s)$ satisfying equation (1).

The benchmark asset A_s provides no services other than its yield, R_s. As a result, the benchmark asset does not enter the consumer's contemporaneous utility function. The asset is held only as a means of accumulating wealth. The consumer's subjective rate of time preference, ξ, is assumed to be constant. The single-period utility function, $u(\mathbf{m}_t, \mathbf{c}_t, \mathbf{x}_t)$, is assumed to be increasing and strictly quasi-concave.

The consumer's decision problem is the following.

Problem 1. Choose the deterministic point $(\mathbf{m}_t, \mathbf{c}_t, \mathbf{x}_t, A_t)$ and the stochastic process $(\mathbf{m}_s, \mathbf{c}_s, \mathbf{x}_s, A_s)$, $s = t+1, \ldots, \infty$, to maximize

$$u(\mathbf{m}_t, \mathbf{c}_t, \mathbf{x}_t) + E_t[\sum_{s=t+1}^{\infty}\left(\frac{1}{1+\xi}\right)^{s-t} u(\mathbf{m}_s, \mathbf{c}_s, \mathbf{x}_s)], \tag{2}$$

subject to $(\mathbf{m}_s, \mathbf{c}_s, \mathbf{x}_s, A_s) \in S(s)$ for $s = t$, t+1, … , ∞, and also subject to the transversality condition

$$\lim_{s\to\infty} E_t \left(\frac{1}{1+\xi}\right)^{s-t} A_s = 0. \tag{3}$$

3.2 Existence of an augmented monetary aggregate for the representative consumer

We assume that the utility function, u, is blockwise weakly separable in $(\mathbf{m}_s, \mathbf{c}_s)$ and in $\boldsymbol{x}_s$. Hence, there exists an augmented monetary aggregator function, $\mathcal{M}$, consumer goods aggregator function, X, and utility functions, F and H, such that

$$u(\mathbf{m}_s, \mathbf{c}_s, \mathbf{x}_s) = F[\mathcal{M}(\mathbf{m}_s, \mathbf{c}_s), X(\mathbf{x}_s)]. \tag{4}$$

We define the utility function V by $V(\mathbf{m}_s, \mathbf{c}_s, X_s) = F[\mathcal{M}(\mathbf{m}_s, \mathbf{c}_s), X_s]$, where aggregate consumption of goods is defined by $X_s = X(\boldsymbol{x}_s)$. It follows that the exact augmented monetary aggregate is

$$\mathcal{M}_s = \mathcal{M}(\mathbf{m}_s, \mathbf{c}_s). \tag{5}$$

The Euler equations that will be of the most use to us below are those for monetary assets and credit card services. Those Euler equations are

$$E_s\left[\frac{\partial V}{\partial m_{is}} - \rho \frac{p_s^*(R_s - r_{is})}{p_{s+1}^*}\frac{\partial V}{\partial X_{s+1}}\right] = 0 \tag{6a}$$

and

$$E_s\left[\frac{\partial V}{\partial c_{js}} - \rho \frac{p_s^*\left(e_{js} - R_s\right)}{p_{s+1}^*}\frac{\partial V}{\partial X_{s+1}}\right] = 0 \tag{6b}$$

for all $s \geq t$, $i = 1, \dots, n$, and $j = 1, \dots, k$, where $\rho = 1/(1+\xi)$ and where p_s^* is the exact price aggregate that is dual to the consumer goods quantity aggregate X_s.

Similarly, we can acquire the Euler equation for the consumer goods aggregate, X_s, rather than for each of its components. The resulting Euler equation for X_s is

$$E_s\left[\frac{\partial V}{\partial X_s} - \rho \frac{p_s^*(1 + R_s)}{p_{s+1}^*}\frac{\partial V}{\partial X_{s+1}}\right] = 0. \tag{6c}$$

3.3 User cost under risk aversion

We now find the formula for the user costs of monetary services and credit card services under risk.

Definition 1. *The contemporaneous risk-adjusted real user cost price of the services of* m_{it}^a *is* $\wp_{it}^a$*, defined such that*

$$\wp_{it}^{a} = \frac{\frac{\partial V}{\partial m_{it}^{a}}}{\frac{\partial V}{\partial X_t}}, i = 1,2, \dots, n+k.$$

The above definition for the contemporaneous user cost states that the real user cost price of an augmented monetary asset is the marginal rate of substitution between that asset and consumer goods.

For notational convenience, we convert the nominal rates of return, r_{it}, e_{jt} and R_t, to real total rates, $1 + r_{it}^*$, $1 + e_{jt}^*$ and $1 + R_t^*$ such that

$$1 + r_{it}^* = \frac{p_t^*(1 + r_{it})}{p_{t+1}^*}, \tag{7a}$$

$$1 + e_{jt}^* = \frac{p_t^*(1 + e_{jt})}{p_{t+1}^*}, \tag{7b}$$

$$1 + R_t^* = \frac{p_t^*(1 + R_t)}{p_{t+1}^*}, \tag{7c}$$

where r_{it}^*, e_{jt}^*, and R_t^* are called the real rates of excess return. Under this change of variables and observing that current-period marginal utilities are known with certainty, Euler equations (6a), (6b), and (6c) become

$$\frac{\partial V}{\partial m_{it}} - \rho E_t\left[(R_t^* - r_{it}^*)\frac{\partial V}{\partial X_{t+1}}\right] = 0, \tag{8}$$

$$\frac{\partial V}{\partial c_{jt}} - \rho E_t\left[\left(e_{jt}^* - R_t^*\right)\frac{\partial V}{\partial X_{t+1}}\right] = 0, \tag{9}$$

and

$$\frac{\partial V}{\partial X_t} - \rho E_t\left[(1 + R_t^*)\frac{\partial V}{\partial X_{t+1}}\right] = 0. \tag{10}$$

We now can provide our user cost theorem under risk.

Theorem 1 (a). *The risk adjusted real user cost of the services of monetary asset i under risk is $\wp_{it}^{m} = \pi_{it} + \psi_{it}$, where*

$$\pi_{it} = \frac{E_t R_t^* - E_t r_{it}^*}{1 + E_t R_t} \tag{11}$$

and

$$\psi_{it} = \rho(1-\pi_{it})\frac{Cov\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho\frac{Cov\left(r_{it}^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}}. \quad (12)$$

(b). *The risk adjusted real user cost of the services of credit card type j under risk is* $\mathcal{p}_{jt}^c = \tilde{\pi}_{jt} + \tilde{\psi}_{jt}$, *where*

$$\tilde{\pi}_{jt} = \frac{E_t e_{jt}^* - E_t R_t^*}{1+E_t R_t} \quad (13)$$

and

$$\tilde{\psi}_{jt} = \rho\frac{Cov\left(e_{jt}^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}} - \rho\left(1+\tilde{\pi}_{jt}\right)\frac{Cov\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right)}{\frac{\partial V}{\partial X_t}}. \quad (14)$$

3.4 Generalized augmented Divisia index under risk aversion

In the case of risk aversion, the first-order conditions are Euler equations. We now use those Euler equations to derive a generalized Divisia index, as follows.

Theorem 2. *In the share equations,* $\omega_{it} = \pi_{it}^a m_{it}^a / \boldsymbol{\pi}_t^{a\prime}\mathbf{m}_t^a$, *in Barnett, Chauvet, Leiva-Leon, and Su (2016), we replace the user costs,* $\boldsymbol{\pi}_t^a = (\boldsymbol{\pi}_t', \tilde{\boldsymbol{\pi}}_t')'$ *by the risk-adjusted user costs,* $\mathcal{p}_{it}^a$, *defined by Definition 1, to produce the risk adjusted shares,* $\mathcal{s}_{it} = \mathcal{p}_{it}^a m_{it}^a / \sum_{j=1}^{n+k} \mathcal{p}_{jt}^a m_{jt}^a$. *Under our weak-separability assumption,* $V(\mathbf{m}_s, \mathbf{c}_s, X_s) = F[\mathcal{M}(\mathbf{m}_s, \mathbf{c}_s), X_s]$, *and our assumption that the monetary aggregator function,* $\mathcal{M}$, *is linearly homogeneous, the following generalized augmented Divisia index is true under risk:*

$$d\log \mathcal{M}_t = \sum_{i=1}^{n+k} \mathcal{s}_{it} d\log m_{it}^a. \quad (15)$$

3.5 CCAPM Special Case

We now consider a special case, based on the usual assumptions in CAPM theory of either quadratic utility or Gaussian stochastic processes. Consider first the following case of utility that is quadratic in consumption of goods, conditionally on the level of monetary asset and credit card services.

Assumption 1. Let V have the form

$$V(\mathbf{m}_t, \mathbf{c}_t, X_t) = F[\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t), X_t] = A[\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)]X_t - \frac{1}{2}B[\mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)]X_t^2, \quad (16)$$

where A is a positive, increasing, concave function and B is a nonnegative, decreasing, convex function.

The alternative assumption is Gaussianity, as follows:

Assumption 2. Let $\left(r_{it}^*, e_{jt}^*, X_{t+1}\right)$ be a trivariate Gaussian process for each asset $i = 1, \dots, n,$ and credit card service, $j = 1, \dots, k$.

We also make the following conventional CAPM assumption:

Assumption 3. The benchmark rate process is deterministic or already risk-adjusted, so that R_t^* is the risk-free rate.

Under this assumption, it follows that

$$Cov\left(R_t^*, \frac{\partial V}{\partial X_{t+1}}\right) = 0.$$

We define $H_{t+1} = H(\mathcal{M}_{t+1}, X_{t+1})$ to be the well-known Arrow-Pratt measure of absolute risk aversion,

$$H(\mathcal{M}_{t+1}, X_{t+1}) = -\frac{E_t[V'']}{E_t[V']}, \tag{17}$$

where $V' = \partial V(\mathbf{m}_{t+1}^a, X_{t+1})/\partial X_{t+1}$ and $V'' = \partial^2 V(\mathbf{m}_{t+1}^a, X_{t+1})/\partial X_{t+1}^2$. In this definition, risk aversion is measured relative to consumption risk, conditionally upon the level of augmented monetary services produced by $\mathcal{M}_{t+1} = \mathcal{M}(\mathbf{m}_t, \mathbf{c}_t)$.

The following theorem identifies the effect of the risk adjustment on the expected own interest rates in the user cost formulas.

Theorem 3. *Let $\hat{H}_t = H_{t+1}X_t$. Under the assumptions of Lemma 2, we have the following for each asset $i = 1, \dots, n$, and credit card service, $j = 1, \dots, k$.*

$$\mathcal{p}_{it}^m = \frac{E_t R_t^* - (E_t r_{it}^* - \phi_{it})}{1 + E_t R_t^*}, \tag{18}$$

where

$$\phi_{it} = \hat{H}_t \, Cov\left(r_{it}^*, \frac{X_{t+1}}{X_t}\right), \tag{19}$$

and

$$\mathcal{p}_{jt}^c = \frac{(E_t e_{jt}^* - \tilde{\phi}_{jt}) - E_t R_t^*}{1 + E_t R_t^*}, \tag{20}$$

where

$$\tilde{\phi}_{jt} = \hat{H}_t \, Cov\left(e_{jt}^*, \frac{X_{t+1}}{X_t}\right). \tag{21}$$

Theorem 3 shows that the risk adjustment on the own interest rate for a monetary asset or credit card service depends upon relative risk aversion, $\hat{H}_t$, and the covariance between the consumption growth path, X_{t+1}/X_t, and the real rate of excess return earned on a monetary asset, r_{it}^*, or paid on a credit card service, e_{jt}^*.

4. Conclusions

Since credit card interest rates are high and volatile, risk adjustment of the credit-card-augmented Divisia monetary aggregates, originated by Barnett, Chauvet, Leiva-Leon, and Su (2016), could be significant. The extension to risk aversion is provided in this paper, with the proofs to become available in the forthcoming major article, Barnett and Su (2016a), and until then available in the working paper version at Barnett and Su (2016b).

Empirical application of this theory remains a topic for future research. A more demanding approach would remove the CCAPM assumption of intertemporal separability, in accordance with Barnett and Wu (2004, 2005).

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