A Revisit to the Grossman Model with Endogenous Health Depreciation

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Abstract

We extend the Grossman health capital model by relaxing the exogenous health depreciation rate to model the direct and indirect channels through which people improve their health through health investment. We confirm that the marginal cost of health supply decreases when the depreciation rate is an endogenous function of health investment and that the marginally reduced cost is greater for people in later life stages. We also find that the indirect channel—depreciation rate reduction—is more effective for people in good health; however, the direct channel—the classical Grossman model—is more powerful for those in poor health. Our findings provide a more comprehensive view of the procedure for optimal health determination.
1. Introduction

Published in 1972, the Grossman health capital model constitutes a major theoretical framework for analyzing the demands of health and health care (Grossman 1972, Zweifel 2012), which has generated substantial empirical and theoretical literature (Hren 2012).

Among the key assumptions of the Grossman model is that the depreciation rate is presumed known, meaning that the model assumes the existence of certainty—i.e., people choose their time of death (McGuire et al. 1988). Several refinements have developed the Grossman model in terms of exogenously determined health depreciation (Hren 2012). Liljas (1998) asserts that the depreciation rate depends on stochastic health status and shows that optimal health capital increases with a higher initial level of health and with an uncertain incidence and degree of illness. Grossman (2000) suggests a method that assumes the depreciation rate to be probability distributed over each life period, thus creating dispersion in time-of-death expectations. Focusing on asset allocation across the life cycle, Yogo (2016) assumes that depreciation is completely stochastic and that people choose their level of health investment after health depreciation is realized in each period. He finds that health expenditure—regarded as an investment in health—decreases with a higher level of health.

The literature focuses on the stochastic process of health depreciation, but it ignores a case that the depreciation rate would endogenously respond to an individual’s health-related decisions during the process to accumulate health capital. Accordingly, we aim to extend the Grossman model by stressing the manipulability of health deterioration; that is, we allow the depreciation rate to be a function of not only age but also investment in health. We are motivated by ample empirical evidence showing that health deterioration is affected by various factors, such as lifestyles and health behaviors. Specifically, Case and Deaton (2005) found that the health of manual laborers declines more rapidly than that of non-manual workers, and Haveman-Nies et al. (2003) showed that the health of people with risky health behaviors (e.g., smoking) deteriorates more rapidly. If, as the research reveals, health depreciation depends on various factors rather than simply on age, a rational individual would seek to slow down her/his health decline by enhancing beneficial health factors. We assume health investment—in a general sense—to be one of those beneficial factors.

Thus, the question becomes whether investment directly benefits health through an iterative process and indirectly benefits health by reducing the deterioration rate. The answer is affirmative and is supported by medical evidence that shows the beneficial influences of physical exercise on diabetes therapy. Exercise has generally been perceived to directly benefit patients with preventable type 2 diabetes through glycemic control (Boulé et al. 2001). Evidence has found that physical exercise also enhances myokine production, which protects against inflammation caused by diabetes (Petersen and Pedersen 2005). Put differently, exercise, as one health investment, is directly beneficial to diabetic patients by lowering their glycemic load and is indirectly beneficial by reducing the risk of health
deterioration due to inflammation, a diabetic complication. Grossman considers the former channel, and we further model the latter channel by endogenizing health depreciation as mentioned above.

In what follows, section 2 revisits the Grossman model with the assumption of endogenous health depreciation. Section 3 provides specifications for the model and introduces corresponding applications. Section 4 provides the conclusions.

2. Grossman Model with Endogenous Health Depreciation

We follow the classical Grossman health capital model (Grossman 1972) but assume that the health depreciation rate, $\delta$, is endogenously determined within the model. In this section, we solve the model and demonstrate the condition of the higher level of optimal health capital level when $\delta$ is a function of investment in health. The utility of a representative individual is as follows:

$$ U = U(TH_0, \ldots, TH_T; Y_0, \ldots, Y_T), $$

where $TH_t = \phi(H_t)$ represents the healthy time in period $t$, which is a function of health capital, $H_t$. Subscript $T$ denotes the individual’s time of death. We assume $dTH_t/dH_t > 0$ and $d^2TH_t/dH_t^2 < 0$. $Y_t$ represents the consumed commodities that contribute to the individual’s utility in period $t$, which is a combination of intermediate goods, $X_t$, and the time used for consumption, $TY_t$; $Y_t = Y(X_t, TY_t; E_t)$. $E_t$ is education level, which stands for technologies that expand the production frontier. The production function of commodities satisfies as a homogeneous function of degree one. The health capital $H_t$ follows an iterated function:

$$ H_{t+1} = (1 - \delta_t)H_t + pI_t. $$

$H_{t+1}$, the health capital in period $t + 1$, is an accumulation of health investment, $I_t$ (e.g., regular exercise, good eating habits), with a direct productivity parameter $p \in [0,1]$ and the health capital $H_t$ depreciated at $\delta_t$. The time of death comes when $H_T = H_{min}$—that is, when the health capital binds to the minimum required level for surviving. In equation (2), $I_t$ is produced with a combination of medical care, $M_t$, and the corresponding time used for health production, $TI_t$: $I_t = l(M_t, TI_t; E_t)$. $E_t$ is identical to that in the commodity production function, and the health-investment production function also satisfies as a homogeneous function of degree one. The new investment channel to benefit health is through the depreciation rate, $\delta_t$:

$$ \delta_t = \delta(I_t, A_t; \theta_t), $$

where $A_t$ denotes age such that $\partial\delta_t/\partial A_t < 0$, and $\theta_t$ stands for a depreciation-efficiency parameter that is affected by various demographic and socio-economic factors (e.g., gender and marital status). Equations (2) and (3) illustrate the direct and indirect benefits, respectively, of current health investment, both of which contribute to a higher level of health.
capital in the next period. Our assumption in equation (3), in line with that of Grossman, indicates the time-limited health benefits of investment, regardless of the channels. It would be reasonable to expect that physical exercise in the current year would bring better health in the following year, but it would be too extreme to state that the benefits would last for two decades. Grossman lets health increase proportionally with \( \alpha = 3 \) to the investment in it, while we assume
\[
\alpha \delta_t \frac{\delta_t}{\delta_t + \gamma_t} < 0, \quad \alpha > 0
\]
to model an intuitive diminishing return on investment to improve health—e.g., having supplements may be beneficial to health, but the effects will attenuate after a certain amount of consumption.

The budget constraint, equation (4), and the time constraint, equation (5), are as follows:
\[
\sum_{t=0}^{T} \left( v_t M_t + p_t X_t \right) \frac{(1 + r)^t}{(1 + r)^t} = \sum_{t=0}^{T} w_t T U H_t + V_0, \tag{4}
\]
\[
\Omega - T U H_t = T H_t = T I_t + T Y_t + T W_t, \tag{5}
\]
where \( v_t \) and \( p_t \) are the prices of factor inputs \( M_t \) and \( X_t \) for health investment production and commodity consumption. \( w_t \), the wage rate, is the price of the time used for various purposes in equation (5). The individual’s total time, \( \Omega \), is divided into healthy time, \( T H_t \), and unhealthy time, \( T U H_t \). The healthy time is further stratified into time for health investment, \( T I_t \), time for consuming commodities, \( T Y_t \), and time for working, \( T W_t \). In addition, \( r \) denotes the discount rate, and \( V_0 \) refers to the initial assets. Nonetheless, really only one constraint exists: equation (4) is not independent of equation (5). Therefore, substituting \( T W_t \) in (5) for \( T W_t \) in (4) gives a simple constraint:
\[
\sum_{t=0}^{T} \left( C_t^I + C_t^Y + w_t T U H_t \right) \frac{(1 + r)^t}{(1 + r)^t} = R, \tag{6}
\]
where \( C_t^I = v_t M_t + w_t T W_t \) and \( C_t^Y = p_t X_t + w_t T Y_t \) are the costs of health investment and commodity consumption, respectively. \( R = \sum_{t=0}^{T} \Omega/(1 + r)^t + V_0 \) is the discounted endowment of time and assets. By utility optimization, the Optimal Health Capital (OHC) in period \( t \) is determined by the equilibrium condition (see appendix):
\[
\phi_t'(H_t) \left[ w_t + \frac{U_{TH_t}}{\lambda} (1 + r)^t \right] = \frac{\pi_t^I (r - \hat{\pi}_t + \delta_t)}{p - \frac{\partial \delta_t}{\partial I_{t-1}} H_t}. \tag{7}
\]

The \( \phi_t'(H_t) \) represents health capital’s marginal production of healthy time; \( U_{TH_t} \) is the marginal utility of healthy time; \( \lambda \) denotes the shadow price of the initial assets; \( \pi_t^I = \frac{dC_t^I}{dI_{t-1}} \) is the marginal cost of health investment; and \( \hat{\pi}_t^I = (\pi_t^I - \pi_t^I - 1) / \pi_t^I - 1 \) is the percentage change in the marginal cost during one period. The left and right hand sides of equation (7) imply the marginal benefit (\( MB \)) and marginal cost (\( MC \)) of health capital, respectively, in period \( t \). The endogenous depreciation rate that decreases with investment
(i.e., \( \frac{\partial \delta_{t-1}}{\partial I_{t-1}} < 0 \)) results in a lower level of \( MC \) than that in the classical Grossman model, which, in turn, contributes to a higher level of optimal health capital.\(^1\)

### 3. Specifications and Applications

In this section, we specify the model in section 2 and provide several applications. First, we specify the depreciation rate and investigate an age profile of health investment to reduce the depreciation rate. Second, we discuss the differences in the two channels through which an individual invests to improve his/her health. Third, we derive the optimal health capital and illustrate how the endogenous depreciation rate increases longevity.

#### 3.1 Endogenous Depreciation, Health Investment, and Age

Health depreciation takes the following form:

\[
\delta_t = \delta(I_t, A_t; \theta_t) = \gamma_1 I_t^{\gamma_2} A_t^{\gamma_3},
\]

where \( \gamma_1 > 1 \) if the efficiency parameter \( \theta_t \) is affected by factors that damage health (e.g., air and water pollution), which, in turn, increases the depreciation rate; \( \gamma_1 \in (0,1) \) if the factors benefit health (e.g., medical technology innovation) and reduce the depreciation rate. \( \gamma_2 \in (-1,0) \) indicates a diminishing return on health investment to reduce the depreciation rate, and \( \gamma_3 > 1 \) shows that health depreciates at an increasing rate with age. Figure 1 displays equation (8) in two scenarios regarding the investment return \( \gamma_2 \), where \( \gamma_2 = -0.5 \) in panel (a) and \( \gamma_2 = -0.7 \) in panel (b).\(^2\)

![Figure 1. Health depreciation rate with age and health investment.](image)

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1. The marginal cost in the classical model is \( MC = \frac{r - \pi t_{t-1} + \delta_t}{p} \), where Grossman assumes the productivity of investment \( p = 1 \).
2. \( \gamma_1 = 0.1 \) and \( \gamma_3 = 2 \) in Figure 1. The age range is [20, 80], and the health investment range is normalized at (0, 100).
In accordance with Grossman, the depreciation rate increases with age. Our specification further suggests that health deteriorates at an accelerating rate with age (i.e., a convex curve with a positive slope in the depreciation-age sectional view). By contrast, health investment contributes to a lower depreciation rate (i.e., a convex curve with a negative slope in the depreciation-investment sectional view). Comparing panels (a) and (b), we confirm a lower depreciation rate with a higher return on investment—i.e., a larger absolute value of $\gamma_2$, ceteris paribus.

Another important point to discuss is the age profile of health investment to reduce the depreciation rate. In Figure 2, the marginal depreciation rate with respect to health investment, $\partial \delta_t / \partial I_t$, is negative in each life stage. Furthermore, as $\partial^2 \delta_t / \partial I_t \partial A_t$ is negative in our specification, the return on investment to reduce the depreciation rate is greater for people in a later life stage (e.g., at all investment levels, we can confirm a greater reduced depreciation rate at age 80 than at age 20).

Figure 2. Marginal health depreciation rate with age and health investment.

An older man’s health deteriorates more rapidly (i.e., health decreases at a higher depreciation rate) than that of a young man, but high-speed health deterioration tends to be much more sensitive to the same level of health investment. For instance, daily jogging may not make much difference in the deterioration of a 20-year-old man, but it may lead to a significant difference in the deterioration of a 60-year-old man.

3.2 Direct and Indirect Benefits from Health Investment

According to equation (8), we specify the health-iterated function (recall equation (2)) as

$$H_{t+1} = (1 - \gamma_1 I_t^2 A_t^2) H_t + pI_t$$

Here, to distinguish the different implications of the two channels through which a person influences

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3 $\gamma_1 = 0.1$, $\gamma_2 = -0.5$, and $\gamma_3 = 2$ in Figure 2. The age and health investment ranges are identical to those in Figure 1.
the future trajectory of her/his health, we discuss two extreme cases: (a) when health investment affects health only through the direct channel (i.e., the Grossman model) and (b) when health investment affects health only through the indirect channel—the depreciation rate. The corresponding \( MC \) in cases (a) and (b) are as follows:

\[
MC_a = \frac{\pi_t I_{t-1} (r - \hat{\pi}_{t-1} + \delta_t)}{p}, \tag{9a}
\]

\[
MC_b = \frac{\pi_t I_{t-1} (r - \hat{\pi}_{t-1} + \delta_t)}{-\gamma_1 y_2 I_{t-1}^{y_2-1} A_{t-1}^{y_3} H_{t-1}}, \tag{9b}
\]

Both \( MC_a \) and \( MC_b \) are larger than \( MC \) in equation (7), which is intuitive, as the summed influences from the direct and indirect channels will be stronger than those from either channel. Digging deeper, \( MC_a \) and \( MC_b \) are binding if \( p = -\gamma_1 y_2 I_{t-1}^{y_2-1} A_{t-1}^{y_3} H_{t-1} \), indicating that the direct channel is more powerful than the indirect one (i.e., \( MC_a < MC_b \)) when \( H_{t-1} < -\gamma_1 y_2 I_{t-1}^{y_2-1} A_{t-1}^{y_3} / p \), and vice versa. This simple method models the greater effectiveness of the direct/indirect channel of investment in improving the health of people with poor/good health. Specifically, anti-inflammatory medicine for a patient with severe diabetic complications and daily jogging for a non-serious diabetic patient may correspond to effective ways of maintaining or improve their health, while jogging may not be an effective cure for patients with complications, and anti-inflammatory medicine may be too much for a non-serious patient.

The criterion for measuring the effectiveness of the two channels, \(-\gamma_1 y_2 I_{t-1}^{y_2-1} A_{t-1}^{y_3} / p\), decreases with the amount of health investment and increases with age, ceteris paribus. In other words, the indirect channel is more effective for an individual who invests generously in her/his health, and the direct channel becomes increasingly important as people age.

3.3 Optimal Health Capital

In what follows, we further specify the \( MB \) in equation (7) to derive the optimal health capital with the endogenous depreciation rate. Healthy time often takes the following form:

\[
TH_t = \phi(H_t) = \alpha_1 (H_t - H_{\min})^{\alpha_2}, \tag{10}
\]

where \( \alpha_1 > 0, \alpha_2 \in (0,1) \), and \( H_{\min} \) is a constant that indicates the minimum health capital required to survive. Health investment takes the following form:

\[
I_t = I(M_t, TI_t, E_t, \beta_1 M_t^{\beta_2} T I_{t-1}^{1-\beta_2}), \tag{11}
\]

where \( \beta_1 > 1, \beta_2 \in (0,1) \). To simplify the analysis, utility is specified to be perfectly separable in terms of healthy time and commodities:

\[
U_t = \rho_1 TH_t + h(Y_t), \tag{12}
\]
where $\rho_t > 0$ shows that healthy time linearly increases utility, and $h(\cdot)$ is a function that shows how commodities contribute to utility. According to the aforementioned specifications, the $MB$ is specified to take the following form:

$$MB = \alpha_1 \alpha_2 (w_t + \rho_1)(H_t - H_{\min})^{\alpha_2 - 1}.$$  \hfill (13)

In Figure 3, we first compare the optimal health capital when the depreciation rate is endogenous to that when the depreciation rate is exogenous. To simplify the discussion, we show the corresponding marginal cost ($MC_{en}$ and $MC_{ex}$) only in two time periods, $t_0$ and $t_1$, where $t_0 < t_1$. In either time period, as shown in section 2, the endogenous depreciation rate provides a lower $MC$ (e.g., $MC_{en,t0} < MC_{ex,t0}$), which, in turn, results in a higher level of optimal health capital (e.g., $H_{en,t0} > H_{ex,t0}$).

Figure 3. Optimal health capital by exogenous and endogenous depreciation rates.

As the depreciation rate is more sensitive to health investment in later life stages, the gap between $MC_{en}$ and $MC_{ex}$ is larger in period $t_1$ than in period $t_0$. However, this wider $MC$ gap does not produce a larger optimal health difference. In fact, the $MB$ in the later life period $t_1$ becomes less elastic to the cost of health, such that the optimal health difference in $t_1$ shrinks drastically compared with that in $t_0$ (i.e., $H_{en,t1} - H_{ex,t1} < H_{en,t0} - H_{ex,t0}$).

Although the optimal health gap decreases with age, it does reflect a process through which the endogenous health depreciation extends longevity. Specifically, if the optimal health in period $t_1$ with an exogenous depreciation rate ($H_{ex,t1}$) binds to $H_{\min}$—that is, the $MC_{ex,t1}$ is too high for an individual to survive—he/she will die in $t_1$. By contrast, when investment benefits health both directly and indirectly, the $MC_{en,t1}$ in period $t_1$ will be affordable to help this person survive, and death will occur in some period $t_k$, where $k > 1$.

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$^4$ We assume $(1 + r)^t = \lambda$ in equation (7).
4. Conclusion

The Grossman health capital model is regarded as a major breakthrough in health economics. Grossman insightfully provides a framework to show the determination of health capital. We have extended this model by relaxing the exogenous health depreciation rate to model the direct and indirect channels through which people improve their health through health investment. We confirm that the marginal cost of health supply decreases when the depreciation rate is a function of health investment and that the marginally reduced cost is greater for people in later life stages. We also find that the indirect channel—depreciation rate reduction—is more effective for people in good health; however, the direct channel—the classical Grossman model—is more powerful for those in poor health. Our findings provide a more comprehensive view of the procedure for optimal health determination.

References


