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Asymptotic Properties of Pesaran's CD Test Revisited

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Abstract

In this paper we revisit and derive the asymptotic properties of Pesaran's CD test.

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1. Introduction

In this paper, our main concern is to revisit and derive the asymptotic properties of Pesaran's cross sectional dependence (CD, herafter); a test which is mainly used in applied econometric works (e.g., see Baltagi, 2013; De Hoyos, and Sarafidis 2006; Pesaran, 2002; 2004 and 2006; and Pesaran, Schuermann, and Weiner, 2004; to mention few).

We begin by reviewing the CD test obtained by Pesaran (2002; 2004 and 2006). Some of the results will not be exact but they will be 'good enough'. We then will present techniques for obtaining 'good enough' results, i.e., asymptotic analysis. In general 'good enough' results are sufficient (e.g., see White, 1999). The proofs provided are original and easier compared to those of Pesaran (2002; 2004 and 2006).

The remainder of the paper is organized as follows: Section 2 reviews the Pesaran's CD test. Asymptotic properties of CD test are derived in Section 3. Section 4 concludes the paper.

2. Pesaran's Test of Cross Sectional Dependence

Consider the standard panel-data model

$$y_{it} = \alpha_i + \beta_i' X_{it} + u_{it} \qquad \left(i = 1, \cdots, N; t = 1, \cdots, T\right) \tag{1}$$

where X_{it} is a $k \times 1$ vector of regressors, β_i are defined on a compact set and are allowed to vary across i, and α_i are time-invariant individual nuisance parameters. Under the null hypothesis, u_{it} is assumed to be independent and identically distributed (IID) over periods and across cross-sectional units. Under the alternative, u_{it} may be correlated but the assumption of no serial correlation remains. The hypothesis of interest is,

$$H_0 : \rho_{ij} = \rho_{ji} = \text{corr}\left(u_{it}, u_{jt}\right) = 0 \text{ for } i \neq j \tag{2}$$

vs. $\quad H_1 : \rho_{ij} = \rho_{ji} \neq 0$ for some $i \neq j$.

In the context of seemingly unrelated regression estimation, Breusch and Pagan (1980) proposed an LM statistic, which is valid for fixed N as $T \to \infty$ and is given by,

$$CD_{LM} = T\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{ij}^2$$

where $\hat{\rho}_{ij}$ is the sample estimate of the pair-wise correlation of the residuals,

$$\hat{\rho}_{ij} = \hat{\rho}_{ji} = \frac{\sum_{t=1}^{T} e_{it}e_{jt}}{\left(\sum_{t=1}^{T} e_{it}^2\right)^{\frac{1}{2}}\left(\sum_{t=1}^{T} e_{jt}^2\right)^{\frac{1}{2}}} \tag{3}$$

and e_{it} is the Ordinary Least Squares (OLS) estimate of u_{it} defined by

$$e_{it} = y_{it} - \hat{\alpha}_i - \hat{\beta}_i' X_{it}, \tag{4}$$

with $\hat{\alpha}_i$ and $\hat{\beta}_i$ being the estimates of α_i and β_i computed using the OLS regression of y_{it} on an intercept and X_{it} for each i, separately. CD_{LM} is asymptotically distributed as χ^2 with $N(N-1)/2$ degrees of freedom under the null hypothesis. However, this test is likely to exhibit substantial size distortions when N is large and T is finite. A situation that is commonly met in empirical applications, mainly because the CD_{LM} statistic is not correctly centered for finite T and the bias is likely to get worse with N large.

Pesaran (2004) has proposed the following alternative,

$$CD = \sqrt{\frac{2T}{N(N-1)}}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{ij}\right) \tag{5}$$

and showed that under the null hypothesis of no cross-sectional dependence $CD \Rightarrow N(0,1)$ for T sufficiently large, as $N \to \infty$; where here '$\Rightarrow$' denotes 'converge to'. Unlike the CD_{LM} statistic, the CD statistic has mean at exactly zero for fixed values of T and N, under a wide range of panel-data models, including homogeneous/heterogeneous dynamic models and nonstationary models. Our interest here lies in the asymptotic properties of the above test.

3. Asymptotic Properties

Asymptotic properties of CD test can be derived under the following assumptions:

Assumption 1

For each i, the disturbances, u_{it} are serially independent with zero mean and the variance σ_i^2, such that $0 < \sigma_i < \infty$.

Assumption 2

Under the null hypothesis defined by,

$$H_0 : u_{it} = \sigma_i \varepsilon_{it} \text{, with } \varepsilon_{it} \sim \text{IID}(0,1) \ \forall \text{i and t},$$

The disturbances, ε_{it}, are symmetrically distributed around zero.

Assumption 3

The regressors, X_{it}, are strictly exogenous such that,

$$\text{E}\left(u_{it} \mid X_i\right) = 0 \ , \forall i \text{ and } t,$$

where $X_i = \left(x_{i1}, x_{i2}, \ldots, x_{iT}\right)'$ and $X_i' X_i$ is a positive definite matrix.

Assumption 4

$T > k + 1$ and the OLS residuals, e_{it}, defined by (4), are not all zero.

Theorem 1

Under Assumptions 1-4,

$$\text{E}\left(\hat{\rho}_{ij}\right) = 0 \text{ and } \text{E}\left(CD\right) = 0$$

Proof of Theorem 1: Straightforward. See Pesaran (2004)

Next, we then have the following asymptotic results:

Proposition 1: Consistency

$$p \lim \hat{\rho}_{ij} = \rho_{ij}$$

Proof of Proposition 1:

$$\hat{\rho}_{ij} = \hat{\rho}_{ji} = \frac{\frac{1}{T}\sum_{t=1}^{T} e_{it}e_{jt}}{\left(\frac{\sum_{t=1}^{T} e_{it}^2}{T}\right)^{\frac{1}{2}}\left(\frac{\sum_{t=1}^{T} e_{jt}^2}{T}\right)^{\frac{1}{2}}} \quad \text{or} \quad \hat{\rho}_{ij} = \frac{\frac{1}{T}\left(e_i'e_j\right)}{\left(\frac{e_i'e_i}{T}\right)^{\frac{1}{2}}\left(\frac{e_j'e_j}{T}\right)^{\frac{1}{2}}} \tag{6}$$

e_{it} is the OLS residuals from the individual-specific regressions, defined by (4) and $e_i = \left(e_{i1}, e_{i2}, \ldots, e_{iT}\right)'$.

Also, $$\hat{\rho}_{ij} = \hat{\rho}_{ji} = \frac{\frac{\varepsilon_i' M_i M_j \varepsilon_j}{T}}{\left(\frac{\varepsilon_i' M_i \varepsilon_i}{T}\right)^{\frac{1}{2}}\left(\frac{\varepsilon_j' M_j \varepsilon_j}{T}\right)^{\frac{1}{2}}} \tag{7}$$

where $e_i = \sigma_i M_i \varepsilon_i$; $M_i = I_T - X_i(X_i'X)^{-1}X_i' = I_T - A_i$ and $\varepsilon_i = \left(\varepsilon_{i1}, \varepsilon_{i2}, \ldots, \varepsilon_{iT}\right)'$.

Expanding terms, $$\hat{\rho}_{ij} = \frac{\frac{1}{T}\left[\varepsilon_i'\varepsilon_j - \varepsilon_i'A_i\varepsilon_j - \varepsilon_i'A_j\varepsilon_j + \varepsilon_i'A_iA_j\varepsilon_j\right]}{\left(\frac{\varepsilon_i' M_i \varepsilon_i}{T}\right)^{\frac{1}{2}}\left(\frac{\varepsilon_j' M_j \varepsilon_j}{T}\right)^{\frac{1}{2}}} \tag{8}$$

$$\hat{\rho}_{ij} = \frac{\frac{\varepsilon_i'\varepsilon_j}{T}}{\left(\frac{\varepsilon_i' M_i \varepsilon_i}{T}\right)^{\frac{1}{2}}\left(\frac{\varepsilon_j' M_j \varepsilon_j}{T}\right)^{\frac{1}{2}}} - \frac{\frac{\varepsilon_i'A_i\varepsilon_j - \varepsilon_i'A_j\varepsilon_j + \varepsilon_i'A_iA_j\varepsilon_j}{T}}{\left(\frac{\varepsilon_i' M_i \varepsilon_i}{T}\right)^{\frac{1}{2}}\left(\frac{\varepsilon_j' M_j \varepsilon_j}{T}\right)^{\frac{1}{2}}} \tag{9}$$

Then, $$\operatorname{plim}\hat{\rho}_{ij} = p\lim\left(\frac{\frac{\varepsilon_i'\varepsilon_j}{T}}{\left(\frac{\varepsilon_i' M_i \varepsilon_i}{T}\right)^{\frac{1}{2}}\left(\frac{\varepsilon_j' M_j \varepsilon_j}{T}\right)^{\frac{1}{2}}} - \frac{\frac{\varepsilon_i'A_i\varepsilon_j - \varepsilon_i'A_j\varepsilon_j + \varepsilon_i'A_iA_j\varepsilon_j}{T}}{\left(\frac{\varepsilon_i' M_i \varepsilon_i}{T}\right)^{\frac{1}{2}}\left(\frac{\varepsilon_j' M_j \varepsilon_j}{T}\right)^{\frac{1}{2}}}\right) \tag{10}$$

And $$\text{p}\lim \hat{\rho}_{ij} = p\lim\left(\frac{\frac{\varepsilon_i'\varepsilon_j}{T}}{\left(\frac{\varepsilon_i' M_i \varepsilon_i}{T}\right)^{\frac{1}{2}}\left(\frac{\varepsilon_j' M_j \varepsilon_j}{T}\right)^{\frac{1}{2}}}\right) - p\lim\left(\frac{\frac{\varepsilon_i' A_i \varepsilon_j - \varepsilon_i' A_j \varepsilon_j + \varepsilon_i' A_i A_j \varepsilon_j}{T}}{\left(\frac{\varepsilon_i' M_i \varepsilon_i}{T}\right)^{\frac{1}{2}}\left(\frac{\varepsilon_j' M_j \varepsilon_j}{T}\right)^{\frac{1}{2}}}\right)$$

(11)

$p\lim\frac{\varepsilon_i' A_i \varepsilon_i}{T} = 0$ because $p\lim\frac{X_i'\varepsilon_i}{T} = 0$. But, $\frac{\varepsilon_i' A_i \varepsilon_i}{T} = o_p(1)$ (12)

where, $$\frac{\varepsilon_i' A_i \varepsilon_i}{T} = \left(\frac{\varepsilon_i' X_i}{T}\right)\left(\frac{X_i' X_i}{T}\right)^{-1}\left(\frac{X_i'\varepsilon_i}{T}\right) = o_p(1), \quad (13)$$

$$\frac{\varepsilon_i' A_j \varepsilon_j}{T} = \left(\frac{\varepsilon_i' X_j}{T}\right)\left(\frac{X_j' X_j}{T}\right)\left(\frac{X_j'\varepsilon_j}{T}\right) = o_p(1), \quad (14)$$

$$\frac{\varepsilon_i' A_i A_j \varepsilon_j}{T} = \left(\frac{\varepsilon_i' X_i}{T}\right)\left(\frac{X_i' X_i}{T}\right)^{-1}\left(\frac{X_i' X_j}{T}\right)\left(\frac{X_j' X_j}{T}\right)^{-1}\left(\frac{X_j'\varepsilon_j}{T}\right) = o_p(1), \quad (15)$$

and $\frac{\varepsilon_i' M_i \varepsilon_i}{T} = 1 + o_p(1)$ (16)

Since $p\lim\left(\frac{\frac{\varepsilon_i' A_i \varepsilon_j - \varepsilon_i' A_j \varepsilon_j + \varepsilon_i' A_i A_j \varepsilon_j}{T}}{\left(\frac{\varepsilon_i' M_i \varepsilon_i}{T}\right)^{\frac{1}{2}}\left(\frac{\varepsilon_j' M_j \varepsilon_j}{T}\right)^{\frac{1}{2}}}\right) = 0$, we then have,

$$\text{p}\lim \hat{\rho}_{ij} = \left(\frac{\frac{\varepsilon_i'\varepsilon_j}{T}}{\left(\frac{\varepsilon_i'\varepsilon_i}{T} - p\lim\left(\frac{\varepsilon_i' X_i \left(X_i' X_i\right)^{-1} X_i'\varepsilon_i}{T}\right)\right)^{\frac{1}{2}}\left(\frac{\varepsilon_j'\varepsilon_j}{T} - p\lim\left(\frac{\varepsilon_j' X_j \left(X_j' X_j\right)^{-1} X_j'\varepsilon_j}{T}\right)\right)^{\frac{1}{2}}}\right)$$

(17)

Under (6), $p\lim \frac{\varepsilon_i' X_i \left(X_i' X_i\right)^{-1} X_i' \varepsilon_i}{T} = 0$, and $\text{p}\lim \hat{\rho}_{ij} = \left(\frac{\varepsilon_i' \varepsilon_j}{\left(\varepsilon_i' \varepsilon_i\right)^{\frac{1}{2}} \left(\varepsilon_j' \varepsilon_j\right)^{\frac{1}{2}}} \right)$. (18)

Now let show that $\left(\frac{\varepsilon_i' \varepsilon_j}{\left(\varepsilon_i' \varepsilon_i\right)^{\frac{1}{2}} \left(\varepsilon_j' \varepsilon_j\right)^{\frac{1}{2}}} \right) = \frac{\frac{1}{T}\sum_{t=1}^{T}\left(u_{it}u_{jt}\right)}{\left(\frac{\sum_{t=1}^{T} u_{it}^2}{T}\right)^{\frac{1}{2}} \left(\frac{\sum_{t=1}^{T} u_{jt}^2}{T}\right)^{\frac{1}{2}}} = \rho_{ij}$

(19)

Since, $\rho_{ij} = \frac{\frac{1}{T}\sum_{t=1}^{T}\left(u_{it}u_{jt}\right)}{\left(\frac{\sum_{t=1}^{T} u_{it}^2}{T}\right)^{\frac{1}{2}} \left(\frac{\sum_{t=1}^{T} u_{jt}^2}{T}\right)^{\frac{1}{2}}} = \frac{\left(u_i' u_j\right)}{\left(u_i' u_i\right)^{\frac{1}{2}} \left(u_j' u_j\right)^{\frac{1}{2}}}$

(20)

with $u_i = \left(u_{i1}, u_{i2}, \ldots, u_{iT}\right)'$; $u_i' u_j = \varepsilon_i' \sigma_i \sigma_j \varepsilon_j$ and $u_i' u_i = \varepsilon_i' \sigma_i \sigma_i \varepsilon_i$ (21)

$$\rho_{ij} = \left(\frac{\varepsilon_i' \sigma_i \sigma_j \varepsilon_j}{\left(\varepsilon_i' \sigma_i^2 \varepsilon_i\right)^{\frac{1}{2}} \left(\varepsilon_j' \sigma_j^2 \varepsilon_j\right)^{\frac{1}{2}}} \right), 0 < \sigma_i < \infty \ ; \ \rho_{ij} = \left(\frac{\varepsilon_i' \varepsilon_j}{\left(\varepsilon_i' \varepsilon_i\right)^{\frac{1}{2}} \left(\varepsilon_j' \varepsilon_j\right)^{\frac{1}{2}}} \right) \quad (22)$$

Finally, $p\lim \hat{\rho}_{ij} = \rho_{ij}$ ■

Proposition 2: Asymptotic Normality

Under Assumptions 1 - 4

$$z_N = \sqrt{\frac{2}{N\left(N-1\right)}} \left(\sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \hat{\rho}_{ij} \right) \Rightarrow N\left(0,1\right) \quad \text{as } N \to \infty .$$

Proof of Proposition 2:

The proof here follows results in Cameron and Trivedi (2005) and White (1999).

Step 1:

First note that under Assumptions 1-4, $\hat{\rho}_{ij}$ and $\hat{\rho}_{is}$ are cross sectionally independent for $i \neq j \neq s$. In particular,

$$\mathrm{E}\left(\hat{\rho}_{ij}\hat{\rho}_{is}\right) = \sum_{t=1}^{T}\sum_{t'=1}^{T}\mathrm{E}\left(\xi_{it}\xi_{jt}\xi_{it'}\xi_{st'}\right) = \sum_{t=1}^{T}\sum_{t'=1}^{T}\mathrm{E}\left(\xi_{it}\xi_{it'}\right)\mathrm{E}\left(\xi_{jt}\right)\mathrm{E}\left(\xi_{st'}\right) = 0 \tag{23}$$

And $\mathrm{var}\left(\hat{\rho}_{ij}\right) = \mathrm{E}\left(\rho_{ij}^2\right) \leq 1$. Therefore, based on the standard Central Limit Theorem (CLT), for any fixed $T > k+1$ and as $N \to \infty$

$$z_N \Rightarrow N\left(0, v_z^2\right), \tag{24}$$

where $v_z^2 = \lim_{N\to\infty}\left[\frac{2}{N\left(N-1\right)}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\mathrm{E}\left(\hat{\rho}_{ij}^2\right)\right)\right] \leq 1$ (25)

Hence for a fixed $T > k+1$ and as $N \to \infty$ we have,

$$\frac{z_N}{v_z} = \frac{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{ij}}{\sqrt{\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\mathrm{E}\left(\hat{\rho}_{ij}^2\right)}} \Rightarrow N\left(0,1\right). \tag{26}$$

z_N is asymptotically normally distributed.

When $T \to \infty$, to see the asymptotic distribution we note that

$$\sqrt{T}\hat{\rho}_{ij} \Rightarrow N\left(0,1\right) \tag{27}$$

Step 2:

$$\sqrt{T}\hat{\rho}_{ij} = \frac{\frac{1}{\sqrt{T}}\left(\varepsilon_i' M_i M_j \varepsilon_j\right)}{\left(\frac{\varepsilon_i' M_i \varepsilon_i}{T}\right)^{\frac{1}{2}}\left(\frac{\varepsilon_j' M_j \varepsilon_j}{T}\right)^{\frac{1}{2}}} \tag{28}$$

Using (12), (13), (14), (15) and (16), we have

$$\sqrt{T}\,\hat{\rho}_{ij} = T^{\frac{-1}{2}}\varepsilon_i'\varepsilon_j + o_p(1) \tag{29}$$

$$\sqrt{T}\,\hat{\rho}_{ij} = \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\varepsilon_{it}\varepsilon_{jt} + o_p(1) \tag{30}$$

Under H_0, ε_{it} and ε_{jt} are independently distributed and serially uncorrelated with mean zero and a unit variance. Therefore, for each $\mathrm{i} \neq \mathrm{j}$, as $T \to \infty$

$$\sqrt{T}\,\hat{\rho}_{ij} \Rightarrow N(0,1) \tag{31}$$

Hence, $\hat{\rho}_{ij}$ is asymptotically normally distributed as $T \to \infty$.■

Proposition 3: Asymptotic Efficiency

$$\lim_{N\to\infty} \operatorname{var}(z_N) \to 0 \text{ and } \lim_{T\to\infty} \operatorname{var}\left(\frac{\sqrt{T}}{T}\hat{\rho}_{ij}\right) \to 0$$

Proof of Proposition 3:

The proof here is based on results in Greene (2012) and White (1999).

Step 1:

$$\operatorname{var}(z_N) = \operatorname{var}\left[\sqrt{\frac{2}{N(N-1)}}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{ij}\right)\right] = \frac{2}{N(N-1)}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\operatorname{var}\left(\hat{\rho}_{ij}\right)\right) \tag{32}$$

Therefore, $\lim_{N\to\infty} \operatorname{var}(z_N) \to 0$.

Step 2:

$$p\lim \hat{\rho}_{ij} = \rho_{ij}$$

and $\lim_{T\to\infty} \operatorname{var}\left(\frac{\sqrt{T}}{T}\hat{\rho}_{ij}\right) = \lim_{T\to\infty}\frac{T}{T^2}\operatorname{var}\left(\hat{\rho}_{ij}\right) = \lim_{T\to\infty}\frac{1}{T}\operatorname{var}\left(\hat{\rho}_{ij}\right)$; with $\operatorname{var}\left(\hat{\rho}_{ij}\right) = 1$. We can then easily show that, $\lim_{T\to\infty} \operatorname{var}\left(\frac{\sqrt{T}}{T}\hat{\rho}_{ij}\right) \to 0$.

In both cases (as $N \to \infty$ and as $T \to \infty$), z_N and $\frac{\sqrt{T}}{T}\hat{\rho}_{ij}$ have the smallest asymptotic variance.■

CD being consistent, asymptotic normally distributed with smallest asymptotic variance is called asymptotically efficient.

Proposition 4: Invariance

If $\hat{\rho}_{ij}$ is a consistent estimator of ρ_{ij} and if h is a one-to-one function, then $h\left(\hat{\rho}_{ij}\right)$ is a consistent estimator of $h\left(\rho_{ij}\right)$.

Proof of Proposition 4:

The invariance principle is thoroughly investigated in White (1999).

If h is one-to-one function, then

$$\rho_{ij} = h^{-1}\left(h\left(\rho_{ij}\right)\right) \tag{33}$$

since $\hat{\rho}_{ij}$ is the estimate of ρ_{ij}, so

$$\hat{\rho}_{ij} = h^{-1}\left(h\left(\rho_{ij}\right)\right) \text{or } h\left(\hat{\rho}_{ij}\right) = h\left(\rho_{ij}\right). \tag{34}$$

But $\hat{\rho}_{ij}$ being a consistent estimator of ρ_{ij}, $h\left(\hat{\rho}_{ij}\right) = h\left(\rho_{ij}\right)$.

As an illustration,

$h\left[\sqrt{\frac{2T}{N(N-1)}}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\hat{\rho}_{ij}\right)\right]$ is also a consistent estimator of $h\left[\sqrt{\frac{2T}{N(N-1)}}\left(\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\rho_{ij}\right)\right]$. An interesting example is that $\ln\left(\hat{\rho}_{ij}\right)$ is a consistent estimator of $\ln\left(\rho_{ij}\right)$ ■

4. Final Remarks

There are various available tests for cross-sectional dependency analysis in panel data. The Pesaran's CD test has good asymptotic properties and should be recommended in empirical analysis.

References

Baltagi, BH (2013) Econometric Analysis of Panel Data. Fifth Edition. John Wiley and Sons

Cameron, CA and Trivedi, KP (2005) Microeconometrics Methods And Applications, Cambridge University Press.

De Hoyos, ER, and Sarafidis V (2006) 'Testing for Cross-Sectional Dependence in Panel -Data Models', *The Stata Journal 6*, *number 4, pp. 482-496*.

Greene, WH (2012) Econometrics Analysis. Seventh Edition, Prentice Hall, NJ, USA.

Pesaran, MH (2002) 'Estimation and Inference in Large Heterogeneous Panels with Cross Section Dependence', University of Cambridge DAE Working Paper, No.0305 and CESifo Working Paper Series No 869, October 2002.

Pesaran, MH (2004) 'General Diagnostic Tests for Cross Section Dependence in Panels', *Cambridge Working Paper in Economics No.1240.*

Pesaran, MH (2006) 'Estimation and Inference in Large Heterogeneous Panels with a Multi Error Structure', *Econometrica*, 74, 967-1012.

Pesaran, MH, Schuermann, T, and Weiner, SM (2004) 'Modeling Regional Interdependencies using a Global Error-Correcting Macroeconomic Model', *Journal Of Business Economics and Statistics* (with Discussions and a Rejoinder), 22, 129 -181.

White, A (1999) Asymptotic Theory for Econometricians, Academic Press.