Economics Bulletin

Volume 37, Issue 1

Variable factor shares and the index number problem: a generalization.

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Abstract

Factor shares vary over time and across countries, so incorporating variable factor shares into growth and development accounting is both warranted and desirable. However, variable factor shares create an index number problem in analyses that rely on our most commonly used production functions. We show that in the presence of competitive factor markets, the problem exists for all workhorse production functions exhibiting constant returns to scale. Therefore, attempts to align empirical growth research with the reality of the factor share data cannot be made using standard techniques. New techniques need to be developed.

Citation: Brad Sturgill and Hernando Zuleta, (2017) "Variable factor shares and the index number problem: a generalization.", *Economics Bulletin*, Volume 37, Issue 1, pages 30-37
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Submitted: September 01, 2016. **Published:** January 13, 2017.

1. Introduction

There is substantial empirical support for the systematic variation of factor shares over time and across countries.[1] Yet the majority of growth and development accounting studies adhere to the constant factor share claim first made by Phelps Brown and Webber (1953) and reiterated by Kaldor (1961) as one of his widely accepted "stylized facts" of macroeconomics.[2] In light of the evidence, why do virtually all empirical growth studies continue to treat factor shares as constant parameters?

A major hurdle to incorporating variable factor shares is the index number problem. This problem stems from the basic properties of our most commonly used production functions. In this letter, we show that variable factor shares, when combined with the assumption of competitive factor markets, create an index number problem in our workhorse constant returns to scale production functions. We illustrate the problem for growth accounting and so focus on the time dimension, but our illustration generalizes to development accounting and cross section analyses.

The index number problem means that incorporating variable factor shares into growth accounting using standard assumptions, techniques and production functions will yield invalid results. New approaches must be created if empirical growth exercises and their resulting conclusions are to reflect the reality of the factor share data.

2. The Index number problem: A simple illustration

Zuleta (2012) argues that changes in factor shares have different effects on output depending on the relative factor abundance of the economy. To illustrate this result, he considers a constant returns to scale Cobb-Douglas production function with two factors: capital (*K*) and labor (*L*). Output per worker $y = Y/L$ can be expressed in terms of capital per worker $k = K/L$ as

$$y_t = A_t k_t^{\alpha_t} \tag{1}$$

where *A* is productivity, α is the elasticity of output with respect to capital and *t* indexes the time period.

Because factor shares are central to this paper, further discussion of α is warranted. The elasticity of output with respect to capital is equivalent to capital's share only if factor markets are competitive. Therefore, implicit in the common reference to α as "capital's share" is the assumption that capital earns its marginal product. Virtually all growth accounting studies measure α as physical capital's share. This means that 1) factor markets are assumed to be competitive in these studies and/or 2) factor shares are interpreted as reasonable estimates of factor elasticities.

Equation (1) reveals that if the share of capital increases, then the effect on output per

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[1]For evidence of the negative time trend in labor's share, see Blanchard (1997), Young (2010), Sachs and Kotlikoff (2012), Elsby, Hobijn, and Sahin (2013), Karabarbounis and Neiman (2013), Rodriguez and Jayadef (2013), and Growiec, McAdam and Muck (2014, 2015), among others. Other authors calculate the income share of reproducible factors (human and physical capital) and non-reproducible factors (natural capital and raw labor) and find that the former is positively correlated with the income level while the latter is negatively correlated with the income level (see Krueger (1999), Acemoglu (2002), Caselli and Feyrer (2007), Zuleta (2008), Sturgill (2012) and Zuleta, Parada, García and Campo (2010)).

[2] See Solow (1957), Klenow and Rodgriguez-Clare (1997), Hall and Jones (1999), Easterly and Levine (2001), Caselli (2005), and Vollrath (2009), among others.

worker depends on the relative abundance of capital,

$$\frac{\partial y}{\partial \alpha} = Ak^{\alpha} \ln k .$$

Specifically, if $k > 1$ the effect is positive, and if $k < 1$ the effect is negative.

The problem with this result is that the magnitude of k can be manipulated simply by altering the units used to measure the inputs. Moreover, the aforementioned partial derivative is not insensitive to the choice of measurement units. It should not matter if we measure physical capital in dollars or in thousands of dollars nor should it matter if we measure workers in number of people or thousands of people. Because it does matter, stating that "k is greater than 1 at time t" is a meaningless statement. In turn, stating that "$\frac{\partial y}{\partial \alpha}$ is greater than 1 at time t" is also meaningless. This is the classic index number problem. We show that it exists not just for the simple Cobb-Douglas function but for any constant elasticity of substitution (CES) or variable elasticity of substitution (VES) production function exhibiting constant returns to scale.

3. The Index Number Problem: Formal Proofs

Proposition 1: For any constant returns to scale CES or VES production function, the impact of a change in factor shares on output per worker depends on the magnitude of factor specific productivities, which are scaling parameters that are unobserved and dependent on the choice of measurement units.

3.1 Proof 1

For any production function with constant returns to scale (CRS) and two factors of production $F(K_t, L_t)$, the assumption of competitive factor markets implies

$$F_K(.) = \alpha_t \frac{F(\phi_{K,t}K_t,\ \phi_{L,t}L_t)}{K_t} \text{ and } F_L(.) = (1-\alpha_t)\frac{F(\phi_{K,t}K_t,\ \phi_{L,t}L_t)}{L_t}$$

where K_t is physical capital, L_t is labor, α_t is the elasticity of output with respect to capital, $1-\alpha_t$ is the elasticity of output with respect to labor, and $\phi_{K,t}$ and $\phi_{L,t}$ are physical capital and labor specific productivities, respectively. The ϕ parameters depend on the measurement units associated with each factor and can be thought of as "scaling" parameters. So $\phi_{K,t}K_t$ is "effective" physical capital and $\phi_{L,t}L_t$ is "effective" labor.

Given the assumption of CRS, the growth rate of output for a given value of α_t is

$$\frac{\Delta Y_t}{Y_t} = \alpha_t\left(\frac{\Delta K_t}{K_t} + \frac{\Delta\phi_{K,t}}{\phi_{K,t}}\right) + (1-\alpha_t)\left(\frac{\Delta L_t}{L_t} + \frac{\Delta\phi_{L,t}}{\phi_{L,t}}\right). \quad (2)$$

In per worker terms,

$$\frac{\Delta y_t}{y_t} = \alpha_t\left(\frac{\Delta k_t}{k_t} + \frac{\Delta\phi_{K,t}}{\phi_{K,t}}\right) + (1-\alpha_t)\left(\frac{\Delta\phi_{L,t}}{\phi_{L,t}}\right) . \quad (3)$$

This is equivalent to:

$$\ln y_{t+1} - \ln y_t = \alpha_t\left(\ln k_{t+1} - \ln k_t + \ln\phi_{K,t+1} - \ln\phi_{K,t}\right) + (1-\alpha_t)\left(\ln\phi_{L,t+1} - \ln\phi_{L,t}\right) \quad (4)$$

Now, suppose there is a change in α_t which affects y_{t+1} but not y_t then

$$\frac{\partial(ln y_{t+1} - ln y_t)}{\partial\alpha_t} = \left(ln\left(\frac{\emptyset_{K,t+1}}{\emptyset_{L,t+1}}\right) + ln k_{t+1}\right) . \quad (5)$$

Therefore, the effect of a change in physical capital's share depends on the scaling parameters, $\frac{\phi_{K,t+1}}{\phi_{L,t+1}}$.

The CES and the VES Production Functions.

Now apply equations (2) - (5) to the CES and VES forms. Consider the following production function,

$$Y_t = \left(\gamma\left(\phi_{K,t}K_t\right)^{\lambda} + (1-\gamma)\left(\phi_{L,t}L_t\right)^{\lambda}\right)^{\frac{1}{\lambda}} \quad . \tag{6}$$

The λ parameter can take on values between $-\infty$ and 1 and determines the degree of substitutability between capital and labor. Specifically, $\frac{1}{1-\lambda}$ is the elasticity of substitution between capital and labor. The distribution parameter $\gamma \in (0,1)$ helps define the factor elasticities. Assuming competitive factor markets,

$$F_K(.) = \alpha_t \frac{F(K_t,L_t)}{K_t} \text{ and } F_L(.) = (1-\alpha_t)\frac{F(K_t,L_t)}{L_t}$$

where

$$\alpha_t = \frac{\frac{\gamma}{1-\gamma}\left(\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}\right)^{\lambda}}{\left(\frac{\gamma}{1-\gamma}\left(\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}\right)^{\lambda}+1\right)} \text{ and } 1-\alpha_t = \frac{1}{\left(\frac{\gamma}{1-\gamma}\left(\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}\right)^{\lambda}+1\right)} \quad . \tag{7}$$

So, $\frac{\alpha_t}{1-\alpha_t} = \frac{\gamma}{1-\gamma}\left(\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}\right)^{\lambda}$. Changes in α_t can occur because of changes in $\gamma, \frac{\phi_{K,t}}{\phi_{L,t}}, \frac{(K_t)}{(L_t)}$ or λ.

Now, consider the production function in per worker terms

$$y_t = \frac{Y_t}{L_t} = \phi_{L,t}(1-\gamma)\left(\frac{\gamma}{1-\gamma}\left(\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}\right)^{\lambda}+1\right)^{\frac{1}{\lambda}}. \tag{8}$$

Therefore,

$$ln(y_t) = ln\left((1-\gamma)\phi_{L,t}\right) + \frac{1}{\lambda} ln\left(\frac{\gamma}{1-\gamma}\left(\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}\right)^{\lambda}+1\right) \tag{9}$$

and

$$\frac{\partial ln(y_t)}{\partial \lambda} = \frac{1}{\lambda^2}\left[\frac{\lambda\frac{\gamma}{1-\gamma}\left(\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}\right)^{\lambda} ln\left(\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}\right) - ln\left(\frac{\gamma}{1-\gamma}\left(\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}\right)^{\lambda}+1\right)}{\frac{\gamma}{1-\gamma}\left(\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}\right)^{\lambda}+1}\right] \quad . \tag{10}$$

Therefore

$$\left(\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}\right)^{\lambda}\frac{ln\left(\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}\right)}{ln\left(\frac{\gamma}{1-\gamma}\left(\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}\right)^{\lambda}+1\right)} > \frac{1}{\lambda}\frac{1-\gamma}{\gamma} \Leftrightarrow \frac{\partial ln(y_t)}{\partial \lambda} > 0 \ . \tag{11}$$

First, note that the left hand side of the first inequality is strictly increasing in $\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}$ while the right hand side is constant. Second, note that if $\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t} > 1$ then the left hand side is strictly increasing in λ while the right hand side is decreasing. Finally, if $\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t} \leq 1$ then $\frac{\partial ln(y_t)}{\partial \lambda} < 0$. Therefore, it is clear that in order to identify the effect of a change in the elasticity of substitution on output per worker we need to properly identify $\frac{\phi_{K,t}K_t}{\phi_{L,t}L_t}$, which varies with the choice of measurement units.[3]

3.2 Proof 2

Suppose we do not think of factor specific productivities (ϕ_K and ϕ_L) as being an element of the initial production function. Can it still be shown that the response of output per worker to a change in factor shares depends on a "scaling" parameter and the choice of measurement units? Consider the following.

Let the aggregate production function be given by

$$Y_t = A_t f\left(K_t, L_t, \alpha_t, \beta_t, \lambda_t\right) \tag{12}$$

where *Y* is output, *K* is physical capital, *L* is labor, α is physical capital's share, β is labor's share, and λ determines the degree of substitutability between capital and labor. If we assume constant returns to scale and competitive factor markets (i.e. $\beta = 1 - \alpha$), then the per worker form of equation (12) can be written as

$$y_t = A_t f\left(k_t, \alpha_t, \lambda_t\right) \tag{13}$$

where *y* is output per worker and *k* is physical capital per worker. Taking natural logs, we have

$$\ln y_t = \ln A_t + \ln f\left(k_t, \alpha_t, \lambda_t\right) . \tag{14}$$

Taking the derivative with respect to time yields:

$$\frac{1}{y_t}\frac{dy_t}{dt} = \underbrace{\frac{1}{A_t}\frac{dA_t}{dt}}_{\substack{Residual \\ Growth\ (RG)}} + \underbrace{\frac{1}{f(\cdot)}\left[f_k(\cdot)\frac{dk_t}{dt} + f_\alpha(\cdot)\frac{d\alpha_t}{dt} + f_\lambda(\cdot)\frac{d\lambda_t}{dt}\right]}_{Observable\, Growth\ \ (OG)} \quad . \tag{15}$$

The *OG* term is what matters. Growth in *y* is not impacted by a change in measurement units, and the growth in *A*, because it is a residual, will be impacted if *OG* is impacted. Given that we are interested in the case of variable factor shares, consider the following scenarios:

[3] Though we analyze a scenario where there are only two inputs, physical capital and labor, this conclusion generalizes to any number of inputs.

<u>Scenario 1</u>
Assumptions: $\lambda = 0$ and α <u>varies</u> over time
(i.e. Cobb-Douglas with variable factor shares)

Assuming that we have competitive factor markets and constant returns to scale, factors earn their marginal products and we can define $\alpha = \frac{f_k(\cdot)k}{f(\cdot)}$. Thus, under Scenario 1, we can write the *OG* term from equation (15) as

$$\alpha_t \frac{dk_t}{dt}\frac{1}{k_t} + \frac{1}{f(\cdot)} f_\alpha(\cdot)\frac{d\alpha_t}{d_t} \quad . \tag{16}$$

The impact of a change in measurement units on the *OG* term will depend on the specific functional form of $f(\cdot)$. Let us assume that the unit elasticity assumption is encompassed by a Cobb Douglas technology of the form $f(\cdot) = k_t^{\alpha_t}$. The expression in (16) can now be written as

$$\alpha_t \frac{dk_t}{dt}\frac{1}{k_t} + \frac{1}{k_t^{\alpha_t}} k_t^{\alpha_t} \ln k_t \frac{d\alpha_t}{d_t} = \alpha_t \frac{dk_t}{dt}\frac{1}{k_t} + \ln k_t \frac{d\alpha_t}{d_t} . \tag{17}$$

Suppose now that there is a change in units used to measure *k*. Perhaps the original data are in dollars but are then multiplied by the constant $\theta = 1/1000$ to convert the data into thousands of dollars.[4] Expression (17) now becomes

$$\begin{aligned} &\alpha_t \frac{d(\theta k_t)}{dt}\frac{1}{\theta k_t} + \ln(\theta k_t)\frac{d\alpha_t}{d_t} \\ &= \alpha_t \frac{\theta dk_t}{dt}\frac{1}{\theta k_t} + (\ln\theta + \ln k_t)\frac{d\alpha_t}{d_t} \quad . \\ &= \alpha_t \frac{dk_t}{dt}\frac{1}{k_t} + (\ln\theta + \ln k_t)\frac{d\alpha_t}{d_t} \end{aligned} \tag{18}$$

Because $\alpha_t \frac{dk_t}{dt}\frac{1}{k_t} + (\ln\theta + \ln k_t)\frac{d\alpha_t}{d_t} \neq \alpha_t \frac{dk_t}{dt}\frac{1}{k_t} + \ln k_t \frac{d\alpha_t}{d_t}$, the *OG* term is affected by θ, and we have a units of measurement issue. Note that if alpha were constant, the value of θ has no impact on the *OG* term and the index number problem goes away.

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[4] Of course, θ can be any constant; the specific value does not matter.

Scenario 2
Assumptions: λ is constant but non-zero, and α varies over time
(i.e. CES with variable shares)

Assuming competitive factor markets and a per worker constant returns to scale CES functional form of $f(\cdot)=\left[\alpha_t k_t^{\lambda}+(1-\alpha_t)\right]^{\frac{1}{\lambda}}$ we can write the *OG* term from equation (15) as

$$\begin{aligned}&\alpha_t \frac{dk_t}{dt}\frac{1}{k_t}+\frac{1}{\left[\alpha_t k_t^{\lambda}+(1-\alpha_t)\right]^{\frac{1}{\lambda}}}\frac{1}{\lambda}\left[\alpha_t k_t^{\lambda}+(1-\alpha_t)\right]^{\frac{1}{\lambda}-1}\left[k_t^{\lambda}-1\right]\frac{d\alpha_t}{dt}\\&=\alpha_t \frac{dk_t}{dt}\frac{1}{k_t}+\frac{1}{\lambda}\frac{\left[k_t^{\lambda}-1\right]}{\alpha_t k_t^{\lambda}+(1-\alpha_t)}\frac{d\alpha_t}{dt}\end{aligned} \quad . \qquad (19)$$

Suppose now that there is a change in units so that k is multiplied by θ. Expression (19) now becomes

$$\begin{aligned}&\alpha_t \frac{d(\theta k_t)}{dt}\frac{1}{\theta k_t}+\frac{1}{\lambda}\frac{(\theta k_t)^{\lambda}-1}{\alpha_t(\theta k_t)^{\lambda}+(1-\alpha_t)}\frac{d\alpha_t}{dt}\\&=\alpha_t \frac{dk_t}{dt}\frac{1}{k_t}+\frac{1}{\lambda}\frac{\theta^{\lambda}k_t^{\lambda}-1}{\alpha_t\theta^{\lambda}k^{\lambda}+(1-\alpha_t)}\frac{d\alpha_t}{dt}\end{aligned} \quad . \qquad (20)$$

Expression (19) is not equal to expression (20), so the *OG* term is dependent on the value of θ, and we again have an index number problem.

Scenario 3
Assumptions: λ and α both vary over time
(i.e. VES with variable shares)

The λ parameter now has a time subscript and we can write the *OG* term from equation (15) as

$$\begin{aligned}&\left\{\begin{array}{l}\alpha_t \frac{dk_t}{dt}\frac{1}{k_t}+\frac{1}{\lambda_t}\frac{\left[k_t^{\lambda_t}-1\right]}{\alpha_t k_t^{\lambda_t}+(1-\alpha_t)}\frac{d\alpha_t}{dt}+\\ \frac{1}{\left[\alpha_t k_t^{\lambda_t}+(1-\alpha_t)\right]^{\frac{1}{\lambda_t}}}\left[\alpha_t k_t^{\lambda}+(1-\alpha_t)\right]^{\frac{1}{\lambda_t}}\ln\left[\alpha_t k_t^{\lambda_t}+(1-\alpha_t)\right]\left(-\lambda_t\right)^{-2}\lambda_t\alpha_t\end{array}\right\}k_t^{\lambda_t-1}\frac{d\lambda_t}{d_t}\\&=\left\{\begin{array}{l}\alpha_t \frac{dk_t}{dt}\frac{1}{k_t}+\frac{1}{\lambda_t}\frac{\left[k_t^{\lambda_t}-1\right]}{\alpha_t k_t^{\lambda_t}+(1-\alpha_t)}\frac{d\alpha_t}{dt}\\ +\ln\left[\alpha_t k_t^{\lambda_t}+(1-\alpha_t)\right]\left(-\lambda_t^{-1}\alpha_t k_t^{\lambda_t-1}\right)\frac{d\lambda_t}{d_t}\end{array}\right\}\end{aligned} \quad . \qquad (21)$$

Suppose now that there is a change in units so that k is multiplied by θ. Expression (21) now becomes

$$\alpha_t \frac{dk_t}{dt}\frac{1}{k_t} + \frac{1}{\lambda_t}\frac{\theta^{\lambda_t} k_t^{\lambda_t} - 1}{\alpha_t \theta^{\lambda} k_t^{\lambda_t} + (1-\alpha_t)}\frac{d\alpha_t}{dt} + \ln\left[\alpha_t (\theta k_t)^{\lambda_t} + (1-\alpha_t)\right]\left(-\lambda_t^{-1}\alpha_t (\theta k_t)^{\lambda_t - 1}\right)\frac{d\lambda_t}{d_t} \ . \tag{22}$$

Expression (21) is not equal to expression (22), so the *OG* term is dependent on the value of θ, and we have an index number problem.

4. Conclusion

We show that existing growth accounting techniques are plagued by an index number problem when factor shares vary over time. Finding a solution to the index number problem would be very useful because the current literary frontier is not in line with the reality of the factor share data. Sturgill (2014) incorporates variable factor shares into a development accounting framework. By considering variance decompositions of translog multilateral indices, Sturgill is able to eliminate the sensitivity of the results to a change in measurement units. However, Sturgill only circumvents the index number problem. He ignores the means of the production function variables and does not consider the response of the output level to a change in factor shares, which is where the problem resides.

One potential solution may be to move away from constant returns to scale production functions. Another possibility would be to find a way to estimate and properly identify ϕ_K and ϕ_L, the scaling parameters.

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