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Extreme Risk Value and Dependence Structure of the China Securities Index 300

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Abstract

A time-varying copulas–conditional value at risk (CVaR) model is estimated to analyze the extreme risk value and dependence structure of the China Securities Index 300 (CSI 300) and index futures portfolios. The goodness-of-fit test as well as the in-sample and out-of-sample tests show that time-varying copulas outperform constant copulas. Specifically, the Student's t, normal, Plackett, and the rotated Gumbel copulas outperform the rotated Clayton copulas.

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1. Introduction

Time-varying copulas have been widely applied in recent years. For example, Chollete et al. (2009) develop a regime-switching copula (RSC). Creal et al. (2011) propose the generalized autoregressive score (GAS) time-varying copulas. Time-varying copula models also include conditional normal, conditional Gumbel, and conditional symmetrized Joe-Clayton copulas. In this paper, a time-varying copulas-conditional value at risk (CVaR) model is applied to analyze the extreme risk value and dependence structure of the China Securities Index 300 (CSI 300) and index futures portfolios. An AR-GARCH (1, 1)-t model is estimated. Nine constants and two time-varying copula models are compared; the goodness-of-fit test as well as the in-sample and out-of-sample tests show that time-varying copulas outperform constant copula models.

2. Methodology and Data

Time-varying copulas allow the copula parameters to change over time. Consider a bivariate time series, with $H(x_{1t}, x_{2t})$ being the joint distribution. Let $F_1(x_{1t})$ and $F_2(x_{2t})$ be the marginal distributions. There exists a copula C with the following joint distribution function:

$$H(x_{1t}, x_{2t}) = C(F_1(x_{1t}), F_2(x_{2t})) \quad (1)$$

The copula function $C(x_t)$ is shown as (1) if either one of $F_1(x_{1t})$ and $F_2(x_{2t})$ is continuous.

This paper considers two time-varying copulas: the elliptical copula (the Student's t GAS) and the Archimedean copula (the rotated Gumbel GAS). The inference function for margins (IFM) method of Joe (1997), which considers the estimation error from marginal distributions, is used for parameter estimation.

The daily return of the CSI 300 and the consecutive price of index futures (IFLX0) of the current month are used to characterize specific features of the stock markets and index futures markets of China. The sample is drawn during the period from April 16, 2010 (the start date of the IFLX0) to September 7, 2012, which covers 585 trading days. Table 1 shows the summary statistics, where the daily return series of both indices clearly indicate fat-tail distributions. The kurtosis of IFLX0 is 5.257, which is slightly higher than the 4.456 of CSI 300, indicating that the index futures are more volatile.

Table 1: Summary statistics

	IFLX0	CSI 300
	Summary Statistics	
Mean	-0.001	-0.001
Std dev	0.015	0.014
Skewness	0.019	-0.170
Kurtosis	5.257	4.456
Correl (lin/rnk)	0.944	0.928

3. Estimation of the Marginal Distribution

Using the Schwarz criterion, the AR (2)-GJR (1, 1)-t model and AR (0) - GJR (1, 1) -t model are selected to fit the marginal distributions of the IFLX0 and CSI 300 Index, respectively. The models are

$$R_t = \phi_0 + \phi_1 R_{t-1} + \phi_2 R_{t-2} + \varepsilon_t$$
$$\varepsilon_t = \eta_t \sqrt{\sigma_t^{\ 2}} \tag{2}$$
$$\sigma_t^{\ 2} = \omega + \delta\ \sigma_{t-1}^2 + \alpha \varepsilon_{t-1}^{\ \ 2} + \beta\ \varepsilon_{t-1}^2 I(-\varepsilon_{t-1})$$

where

$\eta_t \xrightarrow{iid}$ Skewed $t_{(\lambda,\nu)}$, $\lambda \in (-1,1)$ is the skewness degree, and $\nu \in (2,\infty]$ is the degree of fat tail, R_t is the daily return, and ε_t is the new information.

The key element among different GARCH models depends on the form of the conditional covariance. In the GJR-GARCH conditional variance equation,

$$I(-\varepsilon_{t-1}) = \begin{cases} 0, & \varepsilon_t \geq 0 \\ 1, & \varepsilon_t < 0 \end{cases}.$$

$\varepsilon_t > 0$ demonstrates positive information or positive impact, while $\varepsilon_t < 0$ indicates bad news or negative impact that affects conditional variance. The impact of good news on conditional variance is α, whereas the impact of bad news is $\alpha + \beta$. Therefore, investors are more responsive to bad news if $\beta > 0$, and more sensitive to good news if $\beta < 0$.

Table 2: Marginal Distribution Parameter Estimation

	IFLX0	CSI 300
	Conditional Mean	
ϕ_0	0.000	-0.001
ϕ_1	-0.054	-
ϕ_2	0.015	-
	Conditional Variance	
ω	0.000	0.000
α	0.029	0.014
δ	0.019	0.009
β	0.946	0.965
	Skew t Density	
λ	4.332	6.028
υ	0.058	0.026
	GoF Tests	
Cross-equation Effect p-value	0.0860	0.5687
KS p-value	0.257	0.053

Table 2 shows the estimation results of the marginal distribution parameter. Both series have $\beta > 0$ (0.946 for IFLX0 and 0.965 for CSI 300), which means investors are more responsive to bad news. A cross-equation effect is shown in Table 2, which suggests that there is no cross-equation effect for both series. The Kolmogorov-Smirnov test shows that the AR-GJR (1, 1)-t distribution is well-specified for the marginal distributions of both series.

4. Estimation of the Time-Varying Copula Parameter

Multi-stage maximum likelihood (MSML) based on the estimation of marginal distributions is applied to estimate time-varying parameters of copulas. Several constant copulas are also estimated for comparison: Table 3 shows that Student's t, normal, Plackett, and rotated Gumbel copulas outperform the rotated Clayton copula in terms of the log-likelihood values.

Table 3: Constant Copula Model Parameter Estimates

	Parametric		
	Est. Param		log L
Normal	0.9429		**642.4**
Clayton	4.6731		532.9
Rotated Clayton	4.5163		520.3
Plackett	86.338		**637.5**
Frank	9.0000		506.6
Gumbel	4.3520		629.4
Rotated Gumbel	4.3912		**633.3**
Sym Joe–Clayton(τ^{L}, τ^{U})	0.7343,	0.889	Inf
Student's t(ρ, ν^{-1})	0.9000,	0.4762	**647.4**

Tail dependence is estimated using the rotated Gumbel (lower tail) and Student's t models (upper and lower tail). Results for the rotated Clayton copula are omitted because it only features the upper tail. Figure 1 illustrates the results. The Gumbel copula is higher than the Student's t copula during the sample period, which indicates greater dependence between the two indices in a bear market as compared to a bull market. Tail dependence is low during the market downturn from July 2010 to February 2012. This result indicates that investment in both markets can better diversify market risk in a crisis, while a short hedge may not, as the two indices are likely to fall together during a market crash.

Figure 1: Conditional tail dependence

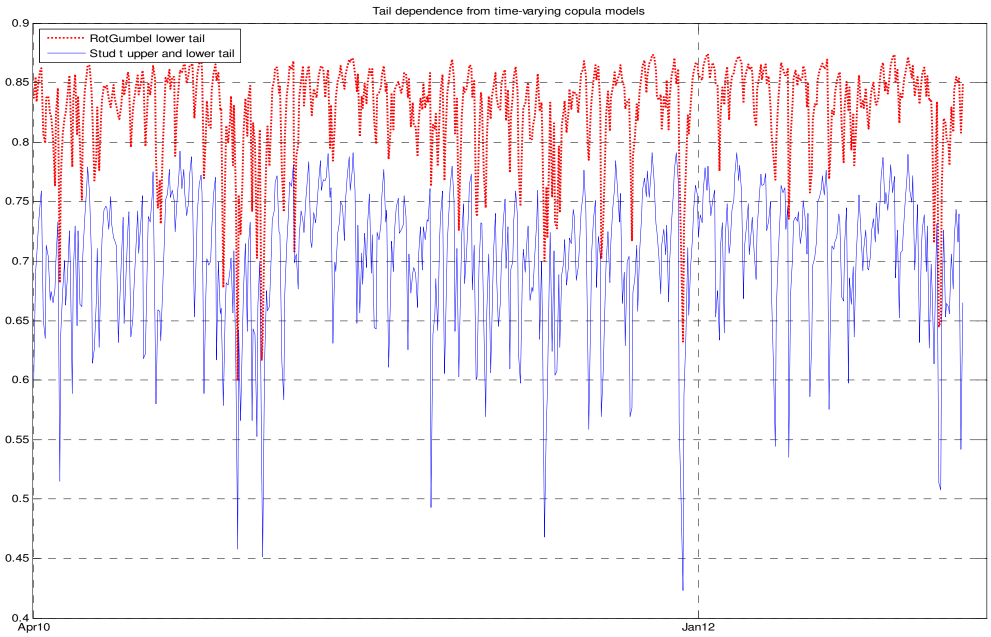


Table 4 presents the standard errors of bootstrap estimates. Time-varying copulas outperform constant copulas because the log-likelihood of the Student's t GAS copula is the highest among all six estimated copulas. For the in-sample test, the pairwise comparison test of Rivers and Vuong (2002) is applied. The Giacomini and White (2006) test is applied in the out-of-sample test. The out-of-sample forecasting models are considered based on a fixed window estimation using data from April 16, 2010 to November 23, 2011, which is two-thirds of the sample period. The estimated model is evaluated based on the remaining data from November 24, 2011 to September 7, 2012, and the out-of-sample log-likelihood values of the models are compared. Tables 5 and 6 show the results of the model selection test. A t-value greater than 2 suggests that the left copula outperforms the top one. The results show that time-varying copulas outperform constant models.

Table 4: Standard Errors of Estimated Copulas

		Naive	MSML	Boot	Sim
Constant					
Normal	$\hat{\rho}$		0.9429		
	s.e.	0.0032	0.0075	0.0071	0.0056
	$\log L$		642.4		
Clayton	$\hat{\kappa}$		4.6731		
	s.e.	0.2111	0.5266	0.4087	0.3807
	$\log L$		532.9		
Rotated Gumbel	$\hat{\kappa}$		4.3912		
	s.e.	0.1528	0.3401	0.2520	0.2211
	$\log L$		633.3		
Student's t	$\hat{\rho}$		0.9000		
	s.e.	0.0000	0.0404	0.0000	0.0074
	$\hat{\upsilon}^{-1}$		0.4762		
	s.e.	0.0000	0.2785	0.0126	0.0517
	$\log L$		647.4		

		Naive	MSML	Boot
Time-varying				
Rotated Gumbel GAS	$\hat{\omega}$		0.2875	
	s.e.	0.0834	62.356	0.2869
	$\hat{\alpha}$		0.2521	
	s.e.	0.0389	2.5404	0.0789
	$\hat{\beta}$		0.7655	
	s.e.	0.0636	46.774	0.2330
	$\log L$		649.3	
Student's t GAS	$\hat{\omega}$		1.0211	
	s.e.	0.2042	0.0000	0.6468
	$\hat{\alpha}$		0.4026	
	s.e.	0.0572	0.0000	0.0878
	$\hat{\beta}$		0.7078	
	s.e.	0.4722	66.948	0.1888
	$\hat{\upsilon}^{-1}$		0.3255	
	s.e.	0.0857	64.440	0.0603
	$\log L$		**676.5**	

Note: NAÏVE means naïve standard errors, where the estimation error from the earlier stages of estimation (AR, GARCH and marginal distributions) is ignored.

Table 5: In-Sample Model Comparison

	Normal	Clayton	Rotated Gumbel	Student's t
Normal	-	-	-	-
Clayton	**-4.9706**	-	-	-
Rotated Gumbel	-0.6674	**10.105**	-	-
Student's t	**1.7096**	**7.1378**	1.4469	-
Log L	642.4	532.9	633.3	647.4
Rank	2	4	3	1

Table 6: Out-of-Sample Model Comparison

	Normal	Clayton	Rotated Gumbel	Stud t	RGum-GAS	Stud t-GAS
Normal	-	-	-	-	-	-
Clayton	-1.02	-	-	-	-	-
Rotated Gumbel	1.18	**3.60**	-	-	-	-
Stud t	0.98	**3.03**	-0.10	-	-	-
RGum-GAS	1.56	**4.01**	**1.88**	1.08	-	-
Stud t-GAS	**2.05**	**3.81**	**2.32**	**2.93**	1.49	-
Log L	195.8	179.9	206.6	206.3	211.8	219.5
Rank	5	6	3	4	2	1

5. VaR and CVaR under Copulas and Optimal Portfolio Selection

We conduct the following Monte Carlo simulation to calculate the time-varying VaR and CVaR values:

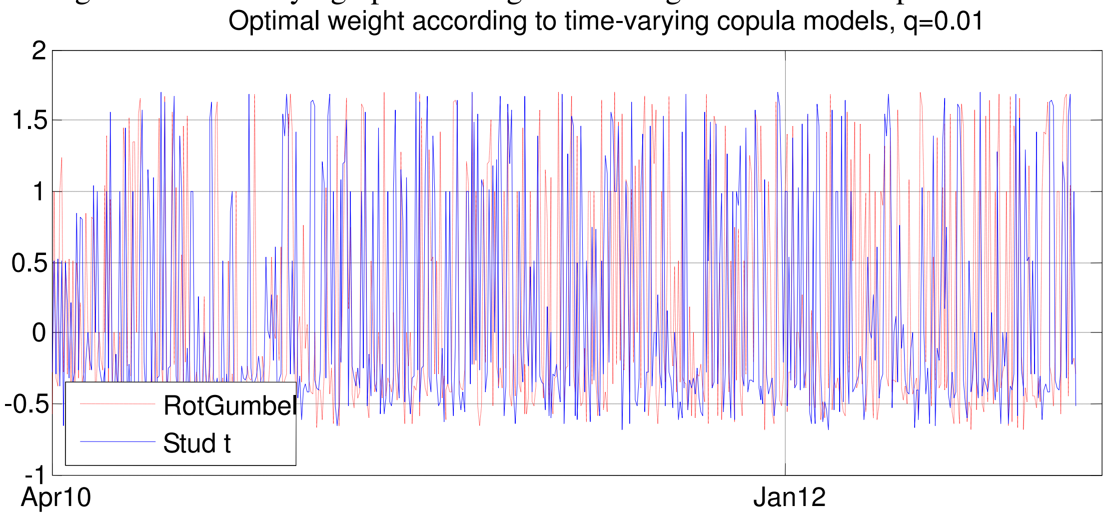
Step 1: Generate a pair of uniformly distributed random numbers $(u_{j,x}, u_{j,y}) \sim C(.)$.

Step 2: Calculate $\hat{\eta}_j = (\hat{\eta}_{j,x}, \hat{\eta}_{j,y}) = (\hat{F}_1^{-1}(u_{j,x}), \hat{F}_2^{-1}(u_{j,y}))$, where $F_1(x)$ and $F_2(x)$ are the skewed t-distributions.

Step 3: Calculate $(\hat{R}_{x,t}, \hat{R}_{y,t}) = (\hat{\mu}_{x,t} + \hat{\eta}_{j,x}\sqrt{\hat{\sigma}_{x,t}^{\;2}}, \hat{\mu}_{y,t} + \hat{\eta}_{j,y}\sqrt{\hat{\sigma}_{y,t}^{\;2}})$ by using the result from the GARCH (1, 1) model.

Step 4: Simulate the portfolio return $R_t = w_1\hat{R}_{x,t} + w_2\hat{R}_{y,t}$. The above process is repeated 5000 times to calculate the optimal weight on index futures determined by optimal portfolio selection. It is based on the estimated mean-variance under CVaR constraints. The confidence level α of the optimal VaR and CVaR is 99%.

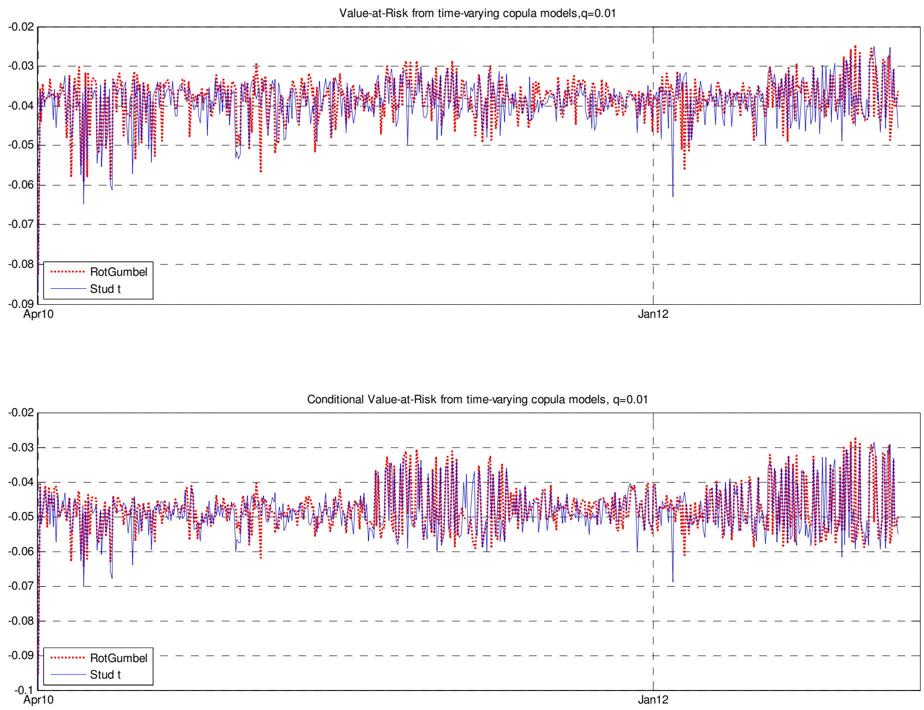
Figure 2: Time-varying optimal weight according to Student's t copula



Note: q =0.01 means significant at the 1% level.

Figure 2 shows that time-varying optimal weights on index futures are mostly positive. A larger weight on index futures than on the CSI 300 lends to better risk hedging features of index futures. Figure 3 shows that the VaR and CVaR are less than -6% from July 2010 to February 2012. This is consistent with the increase of risks during a crisis. CVaR is lower than VaR in most of our sample period because CVaR considers tail risk beyond the VaR and thus is a better risk measure.

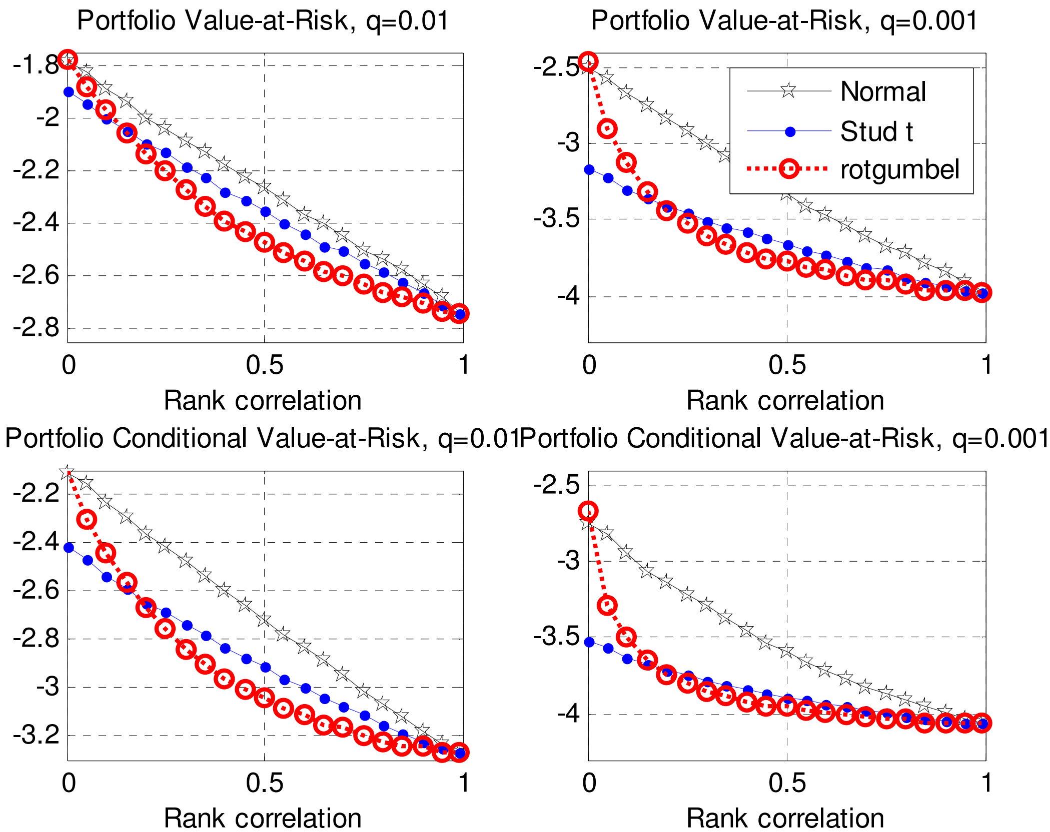
Figure 3: Value-at-risk and conditional value-at-risk



Note: q =0.01 means significant at the 1% level.

Figure 4 shows Patton's (2012) concluding conversions of three copulas at the rank correlation of 1. CVaR values are clearly lower than the VaR values between the rank correlations of 0.2 and 0.6. The CVaR model during a crisis considers the frictions of information transmission between these two markets. Meanwhile, the correlation between the two markets drops to less than 0.6. Thus, CVaR is lower than VaR when the rank correlation is lower than 0.6. In contrast, the correlation in Patton (2012) is between 0.3 and 0.7, which is more balanced around 0.5.

Figure 4: Value-at-risk and conditional value-at-risk and rank correlation



Note: q =0.01 means significant at the 1% level and q=0.001 means significant at the 0.1% level.

6. Conclusion

In this paper, a time-varying copulas-CVaR model is employed to analyze the extreme risk value and dependence structure between CSI 300 Index and index futures. Nine constant and two time-varying copula models are tested. It is found that the Student's t, normal, Plackett, and rotated Gumbel copulas outperform the rotated Clayton copulas, and that the time-varying copulas outperform all constant copulas. The value of the Gumbel copula is higher than that of the Student's t copula during the sample period, which indicates higher dependence between the two indices in a bear market than in a bull market. Tail dependence is low during the market downturn from July 2010 to February 2012, which indicates that investment in both markets can better diversify market risk during crises. The tendency toward lower tail correlation reflects the underdevelopment of investment instruments in China and the relatively high risk aversion of Chinese investors.

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