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On the power of the simulation-based ADF test in bounded time series

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Abstract

Cavaliere and Xu developed in 2014 simulation-based versions of existing unit root tests (among which the ADF), valid in case the time series under test is bounded. In this note, we present a Monte Carlo study to investigate, in particular, the power of the simulation-based ADF test against bounded near unit root and bounded fractionally integrated alternative processes. Results show a rather good performance of the simulation-based ADF test, particularly for near unit root alternatives.

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1. Introduction

In recent years, interest has been growing in testing for unit root in the presence of bounds. Granger (2010) defined a bounded (limited) process as "one that has bounds either below or above or both". From an economic point of view, bounded time series can emerge as a result of two reasons. On the one hand, bounded time series can be observed when agents regulate the dynamic of an economic variable only in case its value overtakes a certain threshold (bound). For instance, when a monetary authority intervenes to maintain the exchange rate inside a target zone, the dynamics of the exchange rate can be represented by a bounded process. Phenomena of this kind have recently received some attention in the theoretical literature (among others Hommes (2006), Bauer et al. (2009), DeGrauwe and Grimaldi (2006)); in addition, there have been previous attempts to empirically analyse them via rather *ad hoc* models as in Sarno and Taylor (2002). On the other hand, bounded time series emerge also in case of by-construction limited variables. Expenditure shares, unemployment rate are only some of the possible examples of real data time series that take values only in a restricted interval.

One feature that makes the analysis of bounded time series very interesting is that in actual facts they often exhibit strong persistence or even nonstationarity: overlooking the specificity of the bounded series and, hence, failing to adjust the empirical methodology for its analysis and treating them as integrated in the usual sense is possibly leading to wrong inferential conclusions. More specifically, Cavaliere (2005), developing a framework where bounded time series can properly be paired with the concept of $I(1)$ process, shows that the test statistics of the PP test for unit root may have quite a different behaviour in case of bounds as the latter are nuisance parameters on which the limiting distribution depends. The effects in the asymptotic null distribution are not negligible since ignoring the bounds can actually lead to overrejecton of the unit root null hypothesis.

To circumvent the problem, Cavaliere (2005) then proposes a two-stage procedure that firstly estimates consistently the nuisance parameters related to the bounds and then derives proper bound-robust asymptotic critical values for the PP unit root test by Phillips and Perron (1988). However, this approach suffers from finite sample size problems. With respect to this, an improvement is proposed in a subsequent paper by Cavaliere and Xu (2014) who develop a more robust approach, built on the the ADF test (Dickey and Fuller (1979), Said and Dickey (1984)) and the autocorrelation-robust 'M' unit root test by Perron and Ng (1996), Stock (1999) and Ng and Perron (2001).

A specific feature of the new approach is that it resorts to direct simulation methods to obtain approximate p-values from the asymptotic distributions of the standard unit root statistics. Through a Monte Carlo experiment, Cavaliere and Xu (2014) study the size of the simulation-based ADF and 'M' unit root tests, revealing quite a satisfactory behaviour. However, no finite sample findings are provided relatively to the power of the tests, which, in general, is quite a serious issue for unit root tests, known to have potentially low power levels for a variety of data generating processes.

Here, we intend to fill this gap by conducting a Monte Carlo experiment whose aim is to study the power of the new simulation based procedure proposed by Cavaliere and Xu (2014). In particular, we concentrate on the ADF version of the procedure and compare it with that of the traditional ADF test for several data generating processes. More specifically, we consider bounded near unit root processes and bounded (a.k.a. regulated) fractionally integrated processes, introduced by Trokic (2013), and the results show that, compared to the traditional ADF test, the simulation-based ADF test has no loss in terms of finite sample power performance, being sometimes even able to outperform it.

The structure of the paper is as follows. In the second section the simulation-based ADF test by Cavaliere and Xu (2014) is recalled. The third section is devoted to the Monte Carlo experiment. The fourth section concludes.

2. The simulation-based ADF test for bounded time series

In general, let a bounded time series X_t, with fixed bounds $\underline{b}$ and $\overline{b}$ ($\underline{b} < \overline{b}$), be the finite realization of a stochastic process such that $X_t \in \left[\underline{b}, \overline{b}\right]$ almost surely for all t. It is then possible to give the definition of the bounded integrated, $BI(1)$, process as in Cavaliere (2005):

$$\begin{aligned} X_t &= \theta + Y_t \\ Y_t &= Y_{t-1} + u_t \end{aligned} \tag{1}$$

the term u_t being defined as:

$$u_t = \epsilon_t + \underline{\xi_t} + \overline{\xi_t} \tag{2}$$

where ϵ_t is a stationary unbounded process with zero mean and $\underline{\xi_t}$ and $\overline{\xi_t}$ are non-negative processes such that $\underline{\xi_t} > 0$ if and only if $Y_{t-1} + \epsilon_t < \underline{b} - \theta$ and $\overline{\xi_t} > 0$ if and only if $Y_{t-1} + \epsilon_t > \overline{b} - \theta$. Consequently, $BI(1)$ behaves similarly to a traditional unit root process when it is away from the bounds; however, when approaching the bounds its behaviour is mean-reverting since the terms $\underline{\xi_t}$ and $\overline{\xi_t}$, also called regulators, force the process between $\underline{b}$ and $\overline{b}$.

Testing for unit root is one of the issues related to the definition of this class of processes. Cavaliere (2005) shows that the asymptotic distributions of the Phillips-Perron (PP) unit root test statistics by Phillips and Perron (1988) are strongly affected by the presence of bounds. More precisely, limiting distributions depend on nuisance parameters that are, in turn, a function of the bounds. As a consequence, the distribution of the unit root statistics is increasingly shifted to the left, the tighter are the bounds. This leads to an overrejection of the unit root null hypothesis since the bounds, which regulate the dynamic of the series, can somewhat induce mean reversion. It is only when the bounds are sufficiently far from each other that inference based on the usual critical values of the unit root test is valid.

To avoid this, Cavaliere (2005) proposes a two-stage procedure that firstly estimates consistently the nuisance parameters related to the bounds and then derives proper bound-robust asymptotic critical values for the PP unit root test. In spite of being asymptotically valid, this version of the PP test still has a poor finite sample size performance.[1] For this reason Cavaliere and Xu (2014) work on developing the bounds-robust version of other unit root tests, such as the ADF test (Dickey and Fuller (1979), Said and Dickey (1984)) and the autocorrelation-robust 'M' unit root test by Perron and Ng (1996), Stock (1999) and Ng and Perron (2001); these, unlike the PP test, are not based on the sum of covariances estimators of the lung run variance, so they should suffer less from problems of finite sample size performance.

One element of novelty in the bounds-robust versions of the ADF and 'M' unit root tests proposed by Cavaliere and Xu (2014) is represented by the new consistent estimators of the nuisance parameters related to the bounds. Moreover, the p-values are now obtained via direct Monte Carlo methods, with a number of possible algorithms allowing also for potential autocorrelation and heteroskedasticity in the error terms. Compared to traditional versions of the same tests as well as with the approach in Cavaliere (2005), these simulation-based tests provide critical values that guarantee a good size performance in case of bounded unit root both asymptotically and in finite samples. It is interesting to observe that another case where traditional unit root tests have been adjusted to avoid overrejecting the unit root is when the data generating process exhibits structural breaks (see Narayan and Pop (2013) for a review). However, processes with structural breaks are very different in nature with respect to processes with bounds and the implied modifications to traditional unit root tests are consequently also different. In the first case, the modification requires the inclusion of structural break dummies; in the second one, it entails an adjustment of the existing test statistics so that the bounds effect is accounted for.

Now, let us provide a few technical details on the simulation-based ADF test, starting with some notation. Given the OLS regression

$$\hat{X}_t = \alpha \hat{X}_{t-1} + \sum_{i=1}^{k} \alpha_i \Delta \hat{X}_{t-1} + \epsilon_{t,k} \tag{3}$$

where $\hat{X}_t$ is the OLS residuals from the regression of X_t on a constant and the traditional ADF statistics, ADF_T is

$$ADF_T = \frac{\hat{\alpha} - 1}{s(\hat{\alpha})} \tag{4}$$

where $\hat{\alpha}$ is the OLS estimate of α and $s(\hat{\alpha})$ is the standard error of $\hat{\alpha}$. The asymptotic null distribution of the ADF_T test statistic in case of $BI(1)$ is based on $B_{\underline{c}}^{\overline{c}}$, a Brownian motion regulated at $\underline{c}, \overline{c}$ (Cavaliere, 2005) that behaves like a standard Brownian motion, but it

[1] This is not surprising since, in general, this is a well-known weakness of the PP test.

reverts in a neighborhood of the bounds. Bearing in mind the dependence of the asymptotic distribution under the null hypothesis upon the nuisance parameters $\underline{c}, \overline{c}$, Cavaliere and Xu (2014) obtain correct p-values for unit root tests after estimating consistently the nuisance parameters $\underline{c}, \overline{c}$ according to the following:

$$\hat{\underline{c}} = \frac{\underline{b} - X_0}{s_{AR}(k)T^{1/2}} \qquad \hat{\overline{c}} = \frac{\overline{b} - X_0}{s_{AR}(k)T^{1/2}} \tag{5}$$

in which the bounds $\underline{b}, \overline{b}$ are assumed to be known and

$$s_{AR}(k) = \frac{\hat{\sigma}^2}{\hat{\alpha}(1)^2} \tag{6}$$

is an autoregressive estimator of the spectral variance where $\hat{\alpha}(1) = 1 - \sum_{i=1}^{k} \hat{\alpha}_i$ and $\hat{\sigma}^2$ is the variance of the OLS regression (3). So, once the $\underline{c}, \overline{c}$ parameters are consistently estimated, the p-values are produced from the approximation of the non pivotal limiting null distribution calculated through a four-step algorithm (see Cavaliere and Xu (2014) for details):

1. *Step 1*: generate an $iid(0,1)$ sequence ϵ_t^*, $t = 1, ..., T$.
2. *Step 2*: obtain recursively X_t^*, $t = 1, ..., n$, $n \geq T$ as

$$X_t^* = \begin{cases} \hat{\overline{c}} & if \quad X_t^* + n^{-1/2}\epsilon_t^* > \hat{\overline{c}} \\ \hat{\underline{c}} & if \quad X_t^* + n^{-1/2}\epsilon_t^* > \hat{\underline{c}} \\ X_t^* + n^{-1/2}\epsilon_t^* & otherwise \end{cases}$$

   with initial condition $X_0 = 0$

3. *Step 3*: compute the Monte Carlo statistic ADF_T^* on the X_t^*
4. *Step 4*: repeat steps *1–3* for B times, then calculate the Monte Carlo p-value.

Cavaliere and Xu (2014) show (Theorem 2, page 263) that under the null hypothesis ADF_T^* has the same asymptotic distribution as ADF_T. Hence, even if the unit root statistics are not pivotal in the presence of bounds, ADF_T^* has correct asymptotic size.

Cavaliere and Xu (2014) also consider the case of correlated errors. Under this circumstance, the ADF_T^* in the third step can be replaced with ADF_T^{**}, the corresponding statistic from the following OLS regression

$$\hat{X}_t^* = \alpha \hat{X}_{t-1}^* + \sum_{i=1}^{k} \alpha_i \Delta \hat{X^*}_{t-1} + e_t^* \tag{7}$$

where $\hat{X}_t^*$ is the de-meaned counterpart of X_t^*. An alternative way to deal with correlated ϵ_t suggested by the same authors is to include in the above algorithm a sieve (or recolouring) intermediate step. This does not alter the underlying theory and, operatively, it can be done by fitting the regression:

$$\hat{X}_t^* = \alpha \hat{X}_{t-1}^* + \sum_{i=1}^{k_{rc}} \alpha_i \Delta \hat{X}^*{}_{t-1} + \epsilon_{t,k_{rc}}^* \tag{8}$$

where k_{rc} is the lag truncation for recolouring. To accommodate this, *Step 2* of the algorithm becomes:

$$X_t^* = \begin{cases} \hat{\bar{c}} & if \quad X_t^* + n^{-1/2} u_{t,k_{rc}}^* > \hat{\bar{c}} \\ \hat{\underline{c}} & if \quad X_t^* + n^{-1/2} u_{t,k_{rc}}^* > \hat{\underline{c}} \\ X_t^* + n^{-1/2} u_{t,k_{rc}}^* & otherwise \end{cases}$$

where $u_{t,k_{rc}}^*$ is the recoloured innovation

$$\frac{\hat{\alpha}_{k_{rc}}(L)}{\hat{\alpha}_{k_{rc}}(1)} u_{t,k_{rc}}^* = \epsilon_t^*$$

$\hat{\alpha}_{k_{rc}}(L)$ being $1 - \sum_{i=1}^{k_{rc}} \hat{\alpha}_i L^i$. Clearly, since the simulated errors are correlated, the ADF_T^{**} must be employed in the fourth step.

3. Monte Carlo experiment

To study the power of the simulation based ADF unit root test by Cavaliere and Xu (2014), we consider bounded near unit root data generating processes (DGPs) and bounded fractional DGPs for the sample sizes $T = 100, 200, 500$. The number of Monte Carlo replications is 2000. While the idea of considering fractional alternatives in the power study comes from a work by Hassler and Wolters (1994), here, given the theme we are working on, we focus on bounded fractionally integrated processes introduced by Trokic (2013).[2]

As for the first group of DGPs, we generate bounded AR(1) time series X_t following Cavaliere (2005) algorithm that begins with the generation of an AR(1) process S_T:

$$\Delta S_t = \frac{a}{T} S_{t-1} + \epsilon_t, \quad a \geq 0, \quad S_0 = 0 \tag{9}$$

and then maps S_T onto an interval $[\underline{b}, \bar{b}]$ so that $X_t \in [\underline{b}, \bar{b}]$ (hence $\Delta X_t \in [\underline{b} - X_{t-1}, \bar{b} - X_{t-1}]$). This can be done by either truncating (e.g. according to a reflection principle) or

[2] Actually, Trokic (2013) call these processes *regulated* fractionally integrated processes, but we use the name bounded fractionally integrated as a synonym.

censoring ΔS_t between $[\underline{b} - X_{t-1}, \overline{b} - X_{t-1}]$.[3] In what follows, we will focus on values of the autoregressive coefficient that gradually approach 1, namely $\frac{a}{T} = 0.7, 0.9, 0.95$. This, in turn, given the adopted sample sizes, leads to $a = 30, 10, 5$ (for $T = 100$), $a = 60, 20, 10$ (for $T = 200$), $a = 150, 50, 25$ (for $T = 500$).

If ϵ_t is WN, X_t is a bounded near $I(1)$ with uncorrelated errors (we denote this as DGP1) and under this circumstance $\underline{b} = \underline{c}\sqrt{T}$ and $\overline{b} = \overline{c}\sqrt{T}$ (because s_{AR} is equal to 1). On the other hand, generating ϵ_t as an AR(1), the resulting X_t will be a bounded near unit root process with autocorrelated errors (DGP2). In this second case, where $\epsilon_t = \phi\epsilon_{t-1} + \eta_t$, $\eta_t \sim WN$, we set $\phi = 0.5$ and we have that $\underline{b} = \frac{\underline{c}\sqrt{T}}{(1-\phi)^2}$ and $\overline{b} = \frac{\overline{c}\sqrt{T}}{(1-\phi)^2}$ (Cavaliere (2005)).[4]

As for bounded fractional DGPs (DGP3), we concentrate on nonstationary processes. To do this, we adopt the algorithm by Trokic (2013), who modifies Cavaliere (2005)'s in equation (9) by generating the error terms $\epsilon_t \sim I(d)$, i.e. as a factional noise. More in details, in order to generate nonstationary bounded fractional DGPs, i.e. processes whose overall long memory parameter $d + 1$ is larger than 1/2, we consider $\epsilon_t \sim I(d)$ where $d = -0.1, -0.2, -0.3$; this leads to nonstationary S_t processes with long memory parameter, respectively equal to 0.9, 0.8, 0.7. We then map S_T onto the interval $[\underline{b}, \overline{b}]$, in the same fashion as for bounded AR(1) to obtain a bounded fractionally integrated process X_t. In general, for bounded fractionally integrated processes Trokic (2013) shows that $\underline{b} = \underline{c}\sqrt{\frac{T^{2(d+1/2)}}{\Gamma^2(d+1)}}$ and $\overline{b} = \overline{c}\sqrt{\frac{T^{2(d+1/2)}}{\Gamma^2(d+1)}}$.[5]

For all models, innovations are distributed as $N(0, 1)$. The parameters $\underline{c}$ and $\overline{c}$ are set equal to, respectively, $\pm 0.4, 0.6, 0.8$ as in Cavaliere and Xu (2014), corresponding to increasingly wider symmetric bounds. Also the case of one single (positive) bound has been considered, *i.e.* $\overline{c}$=0.4, 0.6, 0.8. The bounded near unit root DGPs have been generated using a censoring algorithm, whereas the fractionally bounded DGPs have been generated according to a reflection algorithm.

As for the implementation of the simulation-based ADF test, for all cases, the number of replications is $B = 499$, the significance level is 0.05, the Monte Carlo errors $\epsilon^* \sim N(0, 1)$. When the recolouring algorithm is used, $k_{rc} = 4$.[6]

Power results for DGP1, DPG2 and DGP3 are shown, respectively, in tables I, II, III. Each table reports the percentages of rejection of the null hypothesis of unit root for increasingly wider bounds, both in case of two symmetric bounds and one single bound. Three algorithms of the simulation-based ADF test have been employed. In case of DGP1, ADF_T^* has been adopted. In case of DGP2 and DGP3, due to the correlation in the error terms, the ADF_T^{**} version of the test has been adopted, also augmented by

[3] Details on the censoring and truncating algorithm are in Cavaliere and Xu (2014).
[4] In this case the long run variance s_{AR} is equal to $\frac{1}{(1-\phi)^2}$.
[5] The long run variance s_{AR} is, yet again, equal to one.
[6] Most of these settings are as in Cavaliere and Xu (2014) experiment.

the recouloring algorithm (denoted as $ADF_T^{**} - rec$ in the tables). As a benchmark, the traditional ADF has also been computed both for the bounded DGPs and for their corresponding unbounded versions (table IV).

In general, over all considered DGPs we observe that, as expected, the power grows with the sample size but it worsens the closer the DGP is to the unit root case, *i.e.* when the autoregressive parameter approaches 1 or the long memory parameter also approaches 1. Moreover, the power performance is poorer the tighter are the bounds. More precisely, the smaller is the absolute value of $(\underline{c}, \overline{c})$, the tighter are the bounds and, consequently, the stronger is the mean reversion induced by the bounds themselves. This is rather as expected: as remarked by Cavaliere (2005), under this circumstance the discrimination between H_0 and H_1 is even more difficult than in the usual case of unbounded DGPs. So, it is of no surprise that the power performance of the simulation-based ADF test in presence of bounds (table I–III) can actually be even poorer than that of the traditional ADF in the analogous case with no bounds (table IV). On the other hand, compared to the power of the traditional ADF in case of the same bounded DGPs, the simulation-based ADF test exhibits no loss, being sometimes even able to outperform it. This is an important result, especially if it is read together with the results in Cavaliere and Xu (2014) about the better size performance of the simulation-based ADF test.

In comparison to the case of two symmetric bounds, the effect of one single bound varies with the DGPs, but appears actually rather small. In case of near unit root DGPs, it seems that the power is slightly higher for the case of one bound compared to the case of two bounds. This is a rather logical behaviour considering that the mean reversion induced by the bounds is much stronger when the regulation of the dynamics of the time series is governed by two bounds instead of just one. On the contrary, in case of bounded fractionally DGPs, the two bounds case seems to lead to a better power performance but, actually, the power level is always extremely low.

Finally, the recolouring algorithm is a good option, able to slightly improve the performance of the simulation-based test in case of near unit root DGPs. The same holds in case of bounded fractional DGPs.

Moving now to the specific results, in case of near unit root DGPs with uncorrelated errors (table I), the ADF_T^{*} test has a performance that is in line with that of the traditional ADF_T. The only exception is represented by the case (indeed, a tough one) that combines the smallest sample size with the tightest bounds ($T = 100$, $c = 0.4$, in particular a= 5, 10). Also for near unit root DGPs with correlated errors (table II), the performance of the ADF_T^{**} test is very similar to that of ADF_T. Again, the case where $T = 100$, $c = 0.4$, a= 5, 10 is the most difficult to treat: ADF_T^{**} is outperformed by ADF_T, but it also should be observed that the power level is extremely low for ADF_T anyway. Even the recolouring version of the test is unable to improve the performance in this very difficult case.

As for bounded fractional DGPs, the case $c = 0.4$ over all sample sizes is the most difficult one, not only for the simulation-based ADF (both ADF_T^{*} and ADF_T^{**}), but also for the traditional ADF, which, although managing to have slightly better percentages of rejection, is still features rather unsatisfactory power levels. From $c = 0.6, 0.8$ things improve for all tests and no difference can be noticed between simulation based and traditional ADF.

Table I: Percentage of rejections of the unit root null hypothesis for DGP1 (bounded nearly integrated with uncorrelated errors) in which: c is the parameter expressing the tightness of the bounds (in case of two bounds, the value should be read as $\pm$); T is the sample size; the parameter a, for a given T, leads to values of the autoregressive parameter $\alpha = a/T$ equal to 0.7,0.9,0.95. The nominal level is 0.05.

			Two bounds		*Single bound*	
c	T	a	ADF_T^*	ADF_T	ADF_T^*	ADF_T
0.4	100	5	0.140	0.358	0.189	0.408
		10	0.499	0.799	0.616	0.816
		30	1.000	1.000	1.000	1.000
	200	10	0.628	0.801	0.693	0.792
		20	0.947	0.999	0.985	0.997
		60	1.000	1.000	1.000	1.000
	500	25	0.993	1	0.999	1.000
		50	1.000	1.000	1.000	1.000
		150	1.000	1.000	1.000	1.000
0.6	100	5	0.300	0.374	0.310	0.364
		10	0.763	0.758	0.762	0.808
		30	1.000	1.000	1.000	1.000
	200	10	0.709	0.761	0.755	0.790
		20	0.998	0.999	0.994	0.998
		60	1.000	1.000	1.000	1.000
	500	25	1.00	1.000	1.000	1.000
		50	1.000	1.000	1.000	1.000
		150	1.000	1.000	1.000	1.000
0.8	100	5	0.365	0.352	0.367	0.354
		10	0.765	0.781	0.782	0.798
		30	1.000	1.000	1.000	1.000
	200	10	0.781	0.790	0.800	0.807
		20	0.997	0.999	0.995	1.000
		60	1.000	1.000	1.000	1.000
	500	25	1.000	1.000	0.999	1.000
		50	1.000	1.000	1.000	1.000
		150	1.000	1.000	1.000	1.000

Table II: Percentage of rejections of the unit root null hypothesis for DGP2 (bounded nearly integrated with AR(1) errors, $\epsilon_t = 0.5\epsilon_{t-1} + \eta_t$) in which: c is the parameter expressing the tightness of the bounds (in case of two bounds, the value should be read as $\pm$); T is the sample size; the parameter a, for a given T, leads to values of the autoregressive parameter $\alpha = a/T$ equal to 0.7,0.9,0.95. The number of lags in the recolouring algorithm is $k_{rc} = 4$. The nominal level is 0.05.

			Two bounds			*Single bound*		
c	T	a	ADF_T^{**}	$ADF_T^{**} - rec$	ADF_T	ADF_T^{**}	$ADF_T^{**} - rec$	ADF_T
0.4	100	5	0.093	0.130	0.249	0.156	0.198	0.273
		10	0.217	0.332	0.546	0.328	0.338	0.557
		30	0.761	0.746	0.947	0.854	0.843	0.942
	200	10	0.549	0.561	0.627	0.421	0.557	0.637
		20	0.685	0.731	0.930	0.822	0.941	0.914
		60	0.995	0.992	1	0.999	0.996	1.000
	500	25	0.905	0.927	0.995	0.968	0.979	0.999
		50	1.000	1.000	1.000	1.000	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000
0.6	100	5	0.225	0.243	0.274	0.249	0.258	0.265
		10	0.456	0.499	0.56	0.535	0.551	0.553
		30	0.916	0.902	0.946	0.937	0.961	0.942
	200	10	0.573	0.571	0.619	0.602	0.604	0.610
		20	0.900	0.913	0.938	0.921	0.930	0.932
		60	1.000	0.999	1.000	1.000	1.000	1.000
	500	25	0.992	0.991	0.996	0.992	0.996	0.997
		50	1.000	1.000	1.000	1.000	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000
0.8	100	5	0.286	0.303	0.271	0.268	0.286	0.289
		10	0.568	0.565	0.555	0.579	0.627	0.579
		30	0.950	0.941	0.936	0.942	0.961	0.953
	200	10	0.641	0.645	0.618	0.659	0.689	0.623
		20	0.939	0.931	0.933	0.931	0.966	0.948
		60	1.000	1.000	1.000	1.000	1.000	1.000
	500	25	0.993	0.995	0.994	0.992	0.998	0.999
		50	1.000	1.000	1.000	1.000	1.000	1.000
		150	1.000	1.000	1.000	1.000	1.000	1.000

Table III: Percentage of rejections of the unit root null hypothesis for DGP3 (bounded fractionally integrated) in which: c is the parameter expressing the tightness of the bounds (in case of two bounds, the value should be read as $\pm$); T is the sample size; d is the parameter expressing the long memory level. The number of lags in the recolouring algorithm is $k_{rc} = 4$. The nominal level is 0.05.

			Two bounds			*Single bound*		
c	T	d	ADF_T^{**}	$ADF_T^{**} - rec$	ADF_T	ADF_T^{**}	$ADF_T^{**} - rec$	ADF_T
0.4	100	-0.3	0.227	0.182	0.357	0.137	0.184	0.192
		-0.2	0.253	0.225	0.356	0.135	0.181	0.195
		-0.1	0.145	0.147	0.292	0.130	0.171	0.188
	200	-0.3	0.304	0.237	0.467	0.214	0.234	0.312
		-0.2	0.342	0.298	0.481	0.197	0.227	0.284
		-0.1	0.229	0.228	0.423	0.155	0.188	0.23
	500	-0.3	0.418	0.372	0.632	0.346	0.421	0.48
		-0.2	0.483	0.467	0.606	0.304	0.407	0.393
		-0.1	0.323	0.297	0.571	0.216	0.356	0.309
0.6	100	-0.3	0.475	0.45	0.508	0.302	0.326	0.301
		-0.2	0.395	0.335	0.42	0.253	0.283	0.265
		-0.1	0.15	0.146	0.215	0.121	0.196	0.123
	200	-0.3	0.595	0.552	0.66	0.395	0.415	0.413
		-0.2	0.584	0.531	0.611	0.342	0.387	0.337
		-0.1	0.224	0.254	0.301	0.19	0.197	0.189
	500	-0.3	0.742	0.738	0.738	0.52	0.583	0.574
		-0.2	0.724	0.719	0.735	0.47	0.485	0.464
		-0.1	0.363	0.342	0.463	0.237	0.336	0.262
0.8	100	-0.3	0.593	0.587	0.581	0.369	0.382	0.339
		-0.2	0.356	0.341	0.334	0.2	0.292	0.204
		-0.1	0.113	0.121	0.11	0.086	0.177	0.094
	200	-0.3	0.748	0.735	0.753	0.488	0.502	0.487
		-0.2	0.531	0.505	0.536	0.315	0.369	0.330
		-0.1	0.157	0.155	0.146	0.112	0.188	0.123
	500	-0.3	0.848	0.846	0.879	0.602	0.632	0.611
		-0.2	0.759	0.746	0.761	0.481	0.502	0.467
		-0.1	0.207	0.213	0.203	0.156	0.193	0.169

Table IV: Traditional ADF test power for all considered DGPs when the series are generated with no bounds. Parameters T, a and d as defined in Tables I–III. The nominal level is 0.05.

T	a	DGP1	DGP2	d	DGP3
100	5	0.357	0.258	-0.3	0.127
	10	0.788	0.571	-0.2	0.092
	30	1.000	0.959	-0.1	0.071
200	10	0.792	0.610	-0.3	0.233
	20	0.998	0.944	-0.2	0.139
	60	1.000	0.999	-0.1	0.072
500	25	1.000	0.993	-0.3	0.357
	50	1.000	1.000	-0.2	0.191
	150	1.000	1.000	-0.1	0.109

4. Conclusions

In this work we proposed a Monte Carlo study to investigate the power of the simulation-based version of the ADF test for unit root developed by Cavaliere and Xu (2014) that is valid in case the time series is characterized by the presence of bounds. In our Monte Carlo experiment we consider bounded near unit root and bounded fractionally integrated processes and apply the simulation based ADF test in a variety of algorithms as well as the traditional ADF test, for comparative purpose.

Results show a good performance of the simulation-based ADF test, particularly for near unit root alternatives. Indeed, no loss is registered in terms of power levels when we compare the simulation-based ADF tests with the traditional ADF test, sometimes the rormer being even able to outperform the latter. This is a very important result given that the presence of the bounds, acting as regulators, induces mean-reversion and makes distinguishing unit root from near unit root a possibly even more difficult task. The same type of results is also found in the case of fractionally integrated alternatives, but here the power level of all unit root tests is extremely low: none of them has a satisfactory performance.

As a final comment for practitioners, it must be emphasized that the results indicate that a reasonably large sample size, of at least $T = 200$, is recommended to make sure that the test is capable of recognizing the nonstationary behaviour far from the bounds.

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