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Average-opinion group contest

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Abstract

We study the the average-opinion group contest in which each group's effort level is determined as the median of the effort levels of its group members. We find that, in the average-opinion group contest, both free rider and coordination problem exist among the players in each group, and there exist multiple Nash equilibria of the game. This is the mixed characteristics of each Nash equilibria of the perfect-substitutes, the weakest-link, and the best-shot group contest. Also, we specifically figure out the Nash equilibria of the average-opinion group contest in several examples.

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1 Introduction

In a contest between groups the winning probability of each group depends on the collective effort level of the players in that group, and there appear different types of group contests according to the way of aggregating the effort levels of the players in the group.[1] For instance, the group effort level in a contest can be defined as the sum, the minimum, and the maximum of the effort levels chosen by the players in the group, and each case is named perfect-substitutes group contest (Katz et al. 1990, Baik 1993; Baik 2008; Sheremeta 2011), weakest-link group contest (Lee 2012; Chowdhury and Topolyan 2016; Lee and Song 2016), and best-shot group contest (Chowdhury et al. 2013; Barbieri et al. 2014; Lee and Lee 2016), respectively.[2] In this paper, we consider another type of group contest, named "average-opinion group contest", in which each group effort level is defined as the median of players' effort levels in the group. The term "average-opinion" is come from average opinion games of Van Huyck et al. (1991) in which each player's payoff function is increasing in the median of all players' choices and decreasing in the distance between its choice and the median. In our average-opinion group contest, the payoff for each player in a group has similar structure to the one in the average opinion games.

Since the median is not distorted by skewed data and it can be determined even for data measured in a ratio, an interval, and an ordinal scale, the median rule as one of the ways to determine group effort levels in a contest would be applied in various ways. For instance, it would be adopted in the decision-making process of a public authority. Suppose that the government plans to build a public facility in one of several regions and tries to make a decision on its location *on the basis of opinions* collected from the residents in the proposed regions. Then, as one of the ways to gather the opinions of the residents about building the facility in their region, the government can use a questionnaire which asks them to choose one of the following numbers: 5-'more than completely agree', 4-'completely agree', 3-'mostly agree', 2-'agree', 1-'neutral'. Because each respondent is also asked to give some reasonable explanation/evidence for his answer, choosing higher number incurs more effort (cost) to the respondent. In this case, the responses of the residents in each region to the questionnaire reflect their opinions, which are measured *in an ordinal scale*, and they are well summarized by the median of the responses rather than the mean, the maximum, or the minimum. Then the information about the median values for each proposed regions may be useful for the government to make its final decision on where to build the public facility.

We investigate the existence and characteristics of the Nash equilibrium of the average-opinion group contest in a general setting, i.e., n groups compete against and each group i consists of m_i group members for $i = 1, 2, \dots, n$. In our average-opinion group contest, both free rider and coordination problem exist among the players in each group. Namely, at the Nash equilibrium of the game, one subgroup of players in each group do not exert any effort and the other subgroup of players exert the same amount of effort that lies between zero and a certain strictly positive level. The free-riding subgroup doesn't necessarily consists of the low-

[1] The function that translates the effort levels of the players in a group into the group effort level is called as the impact function (Wärneryd 1988).

[2] For more information about contests, see Corchón (2007), Garfinkel and Skaperdas (2007), and Konrad (2009).

valuation players in the group. Thus there exist multiple Nash equilibria of the game. This is a mixed characteristics of the results in the perfect-substitutes, weakest-link, and best-shot group contest. In the perfect-substitutes group contest, only the highest-valuation player in each group expends some efforts and the rest of players do nothing at Nash equilibrium. Similarly, in the best-shot group contest, only one player in each group exerts some efforts but he isn't necessarily the highest-valuation player in the group. Contrary to these contests, there does not exist the free rider problem in the weakest-link group contest, instead, the coordination problem exists among the players in each group. The Nash equilibria of the average-opinion group contest show all these features of the equilibria.

The paper proceeds as follows. Section 2 describes our model and it is analyzed in Section 3. Section 4 provides several examples.

2 The model

Let us consider a contest in which $n \geq 2$ groups compete to win a prize. Each group i consists of $m_i \geq 2$ risk-neutral players who expend effort to win the prize. Players in group i are indexed by ik where $k = 1, 2, \ldots, m_i$. The prize is a public good within a winning group in a sense that all the players in the winning group benefit from the prize. The players' valuations for the prize may differ. Let v_{ik} represent the valuation of player ik. Players' valuations are assumed as follows:

Assumption 1 $v_{i1} \geq v_{i2} \geq \cdots \geq v_{im_i} > 0$ for all $i = 1, \ldots, n$.

Let x_{ik} represent the nonnegative effort level chosen by player ik. Each player's effort is irreversible regardless of whether or not his group wins the prize. Let p_i denote the probability of group i's winning the prize. We define that the winning probability for group i is

$$p_i = p_i(X_1, \ldots, X_n) \text{ where } X_i = \text{median}\left\{x_{i1}, x_{i2}, \ldots, x_{im_i}\right\}.$$

X_i denotes the effort level for group i and it is determined by the median value of the effort levels chosen by the players in group i. I.e., when we rank the effort levels of the players in descending order, X_i is the middle value. If m_i is an even number, we define X_i as the mean of two middle values. The function p_i satisfies the following regularity conditions:

Assumption 2 $0 \leq p_i \leq 1$, $\sum_{j=1}^{n} p_j = 1$, $p_i(0, \ldots, 0) = 1/n$, $\frac{\partial p_i}{\partial X_i} \geq 0$, $\frac{\partial^2 p_i}{\partial X_i^2} \leq 0$, $\frac{\partial p_i}{\partial X_j} \leq 0$, $\frac{\partial^2 p_i}{\partial X_j^2} \geq 0$, $\frac{\partial p_i}{\partial X_i} > 0$ and $\frac{\partial^2 p_i}{\partial X_i^2} < 0$ for some $X_j > 0$, $\frac{\partial p_i}{\partial X_j} < 0$ and $\frac{\partial^2 p_i}{\partial X_j^2} > 0$ for $X_i > 0$, where $i \neq j$.

Let π_{ik} represent the payoff for player ik. Then the payoff function for player ik is given:

$$\pi_{ik} = v_{ik} p_i(X_1, \ldots, X_n) - x_{ik}.$$

We assume that all the players in the contest choose their effort levels independently and simultaneously. All of the above is common knowledge among the players.

3 The equilibria of the game

Let x_{ik}^b denote the best-response of player ik in an imaginary situation where he is a unique player in group i, given the other groups' effort levels. We call x_{ik}^b the "imaginary" best-response of player ik. Specifically, x_{ik}^b is the effort level x_{ik} that maximizes his payoff

$$\pi_{ik}^b = v_{ik} p_i(X_1, \ldots, X_{i-1}, x_{ik}, X_{i+1}, \ldots, X_n) - x_{ik}$$

subject to $x_{ik} \geq 0$. Thus x_{ik}^b satisfies the first-order condition for maximizing π_{ik}^b with respect to x_{ik}:

$$v_{ik} \frac{\partial p_i}{\partial x_{ik}} \leq 1.$$

Assumption 1, 2 and the first-order condition tell us the relative sizes of the imaginary best responses of the players in group i. Lemma 1 shows this.

Lemma 1 *Given the effort levels of the other groups, X_{-i}, the relative sizes of the imaginary best responses of the players in group i are*

$$x_{i1}^b(X_{-i}) \geq x_{i2}^b(X_{-i}) \geq \cdots \geq x_{im_i}^b(X_{-i}) \text{ for all } X_{-i} \geq (0, \ldots, 0),$$

where $X_{-i} = (X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n)$.

Using Lemma 1 and the characteristics of the median function X_i, we obtain group-i-specific equilibrium.[3] A group-i-specific equilibrium is a m_i-tuple vector of effort levels the players in group i will expend, given the other groups' effort levels. So, at group-i-specific equilibrium, every player in group i shouldn't have any incentive to increase or decrease its effort level, given the effort levels of the rest of the players in its group and the other groups' effort levels. Lemma 2 presents the group-i-specific equilibrium. The proof is presented in Appendix.

Lemma 2 *Given effort levels of the other groups, X_{-i}, following group-i-specific equilibria exist.*

(a) When m_i is an odd number:

$$(\underbrace{x_i, x_i, \cdots, x_i, x_i}_{\frac{m_i+1}{2} \text{ players } (A)}, \underbrace{0, 0, \cdots, 0, 0}_{\frac{m_i-1}{2} \text{ players } (B)}) \text{ with } x_i \in \left[0, \min\left\{x_{ik}^b(X_{-i})\right\}_{ik \in A}\right],$$

where A denotes the set of players choosing effort level x_i ($|A| = \frac{m_i+1}{2}$) and B does the set of the other players choosing zero effort level ($|B| = \frac{m_i-1}{2}$). Note that the above m_i-tuple vector of effort levels does not indicate which players compose each subgroup A and B.

[3] We use the term 'group-i-specific equilibrium' that is firstly used in Baik (2008).

(b) *For the convenience of description for an even number m_i, we divide the players into two equally sized subgroups A and B, consisting of contributors and noncontributors respectively. Then group-i-specific equilibria are as follows.*

When m_i is an even number and $\min\{v_{ik}\}_{ik\in A} > \max\{v_{ik}\}_{ik\in B}$ $(v_{im_i/2} > v_{i(m_i+2)/2})$*:*

$$(x_{i1}, x_{i2}, \ldots, x_{im_i}) = (\underbrace{x_i, \cdots, x_i}_{\frac{m_i}{2} \text{ players } (A)}, \underbrace{0, \cdots, 0}_{\frac{m_i}{2} \text{ players } (B)}) \text{ with } x_i \in \left[x^b_{i(m_i+2)/2}(X_{-i}), x^b_{im_i/2}(X_{-i})\right] \cup \{0\}.^4$$

Note that subgroup A consists of player $i1$, $i2$, …, $im_i/2$ and subgroup B consists of player $i(m_i+2)/2$, $i(m_i+4)/2$, …, m_i.

(c) *When m_i is an even number and* $\min\{v_{ik}\}_{ik\in A} = \max\{v_{ik}\}_{ik\in B}$*:*

$$(\underbrace{x_i, x_i, \cdots, x_i, x_i}_{\frac{m_i}{2} \text{ players } (A)}, \underbrace{y_i, 0, \cdots, 0, 0}_{\frac{m_i}{2} \text{ players } (B)}) \text{ with } x_i + y_i = \min\left\{x^b_{ik}(X_{-i})\right\}_{ik\in A} \text{ and } x_i > y_i \geq 0^5$$

and

$$(\underbrace{x_i, x_i, \cdots, x_i, x_i}_{\frac{m_i}{2} \text{ players } (A)}, \underbrace{x_i, 0, \cdots, 0, 0}_{\frac{m_i}{2} \text{ players } (B)}) \text{ with } x_i \in \left[0, \min\left\{x^b_{ik}(X_{-i})\right\}_{ik\in A}\right],^6$$

where nonzero y_i and x_i in subgroup B are chosen by the highest-valuation player (or one of the highest-valuation players) in subgroup B. Note that the above m_i-tuple vectors of effort levels do not indicate which players compose each subgroup A and B.

(d) *When m_i is an even number and* $\min\{v_{ik}\}_{ik\in A} < \max\{v_{ik}\}_{ik\in B}$*:*

$$(\underbrace{x_i, x_i, \cdots, x_i, x_i}_{\frac{m_i}{2} \text{ players } (A)}, \underbrace{x_i, 0, \cdots, 0, 0}_{\frac{m_i}{2} \text{ players } (B)}) \text{ with } x_i \in \left[0, \min\left\{x^b_{ik}(X_{-i})\right\}_{ik\in A}\right],^7$$

where nonzero x_i in subgroup B is chosen by the highest-valuation player (or one of the highest-valuation players) within the subgroup. Note that the above m_i-tuple vectors of effort levels do not indicate which players compose each subgroup A and B.

Lemma 2 says that the free-rider problem occurs within each group at equilibrium. Some players in each group choose zero effort level and those free-riding players are not necessarily the low-valuation players in the group. Note that, except for (b), the players composing of subgroup B are not identified. Lemma 2 also implies that, given X_{-i}, there may exist

[4]Note that X_i is the mean of two middle values, i.e., $x_{im_i/2}$ and $x_{i(m_i+1)/2}$. So, $x^b_{im_i/2}(X_{-i}) = \arg\max_{x_{im_i/2}} \; v_{im_i/2} \cdot p_i(X_1, \ldots, X_{i-1}, \frac{x_{im_i/2}}{2}, X_{i+1}, \ldots, X_n) - x_{im_i/2}$ and $x^b_{i(m_i+2)/2}(X_{-i}) = \arg\max_{x_{i(m_i+2)/2}} \; v_{i(m_i+2)/2} \cdot p_i(X_1, \ldots, X_{i-1}, \frac{x_{i(m_i+2)/2}}{2}, X_{i+1}, \ldots, X_n) - x_{i(m_i+2)/2}$.

[5]$x^b_{ik}(X_{-i}) = \arg\max_{x_{ik}} \; v_{ik} \cdot p_i(X_1, \ldots, X_{i-1}, \frac{x_{ik}}{2}, X_{i+1}, \ldots, X_n) - x_{ik}$.

[6]$x^b_{ik}(X_{-i}) = \arg\max_{x_{ik}} \; v_{ik} \cdot p_i(X_1, \ldots, X_{i-1}, x_{ik}, X_{i+1}, \ldots, X_n) - x_{ik}$.

[7]$x^b_{ik}(X_{-i}) = \arg\max_{x_{ik}} \; v_{ik} \cdot p_i(X_1, \ldots, X_{i-1}, x_{ik}, X_{i+1}, \ldots, X_n) - x_{ik}$.

multiple group-i-specific equilibria. The value of each x_i in the lemma can be any one that belongs to a certain range, which means that the coordination problem exists among the players in subgroup A. And the certain range varies according to who is the lowest-valuation player(s) in subgroup A. As a result, depending on which players compose of subgroup A and B and how the players coordinate with each other in choosing their effort levels, there exist many different group-i-specific equilibria. Hence, the Nash equilibrium of the game, that is derived from group i-specific equilibria for all $i = 1, 2, \ldots, n$, will be characterized by the composition of subgroup A and B and the degree of coordination among the players at each group's specific equilibria, and the multiplicity of each group-i-specific equilibria results in multiple Nash equilibria of the game. Proposition 1 summarizes these.

Proposition 1 *The Nash equilibrium of the average-opinion group contest*

(a) The Nash equilibrium is characterized by the composition of subgroup A and B at each group-i-specific equilibria and the degree of coordination among the players within each group.

(b) There exist multiple Nash equilibria.

(c) At the Nash equilibrium, free rider and coordination problem coexist within each group.

4 Examples

In this section we specifically figure out the Nash equilibria of the average-opinion group contest in which two groups compete against each other, i.e., $n = 2$, and the each group's winning probability follows the Tullock-form contest success function, i.e., $p_i(X_1, X_2) = \frac{X_i}{X_1+X_2}$ for $X_1 + X_2 > 0$ and $\frac{1}{2}$ for $X_1 + X_2 = 0$. We first consider the case in which $m_1 = m_2 = 3$, and then the case in which $m_1 = 3$ and $m_2 = 4$.

4.1 The case in which $m_1 = m_2 = 3$

From (a) in Lemma 2 we obtain group-1-specific equilibria, given X_2. According to the composition of subgroup A and B in group 1, there exist different group-1-specific equilibria. Here we consider the group-1-specific equilibria in which $A = \{11, 12\}$, i.e., subgroup A consists of player 11 and 12. Then player 12 is the lowest-valuation player in subgroup A and those group-i-specific equilibria are as follows:

$$(x_{11}, x_{12}, x_{13}) = (x_1, x_1, 0) \text{ with } 0 \le x_1 \le x_{12}^b(X_2) = \begin{cases} \sqrt{v_{12}X_2} - X_2 & \text{for } X_2 \le v_{12} \\ 0 & \text{for } X_2 > v_{12}. \end{cases}$$

In the same way we have following group-2-specific equilibria in which $A = \{21, 22\}$, given X_1:

$$(x_{21}, x_{22}, x_{23}) = (x_2, x_2, 0) \text{ with } 0 \le x_2 \le x_{22}^b(X_1) = \begin{cases} \sqrt{v_{22}X_1} - X_1 & \text{for } X_1 \le v_{22} \\ 0 & \text{for } X_1 > v_{22}. \end{cases}$$

Figure 1 (a) provides information about the group-i-specific equilibria for $i = 1, 2$. The shadowed area S_i in the figure represents the values of x_i each of which composes each group-i-specific equilibrium, given X_{-i}, i.e., $x_i \in [0, x_{i2}^b(X_{-i})]$. Note that $X_i = x_i$.

[Figure 1 about here.]

Then the Nash equilibrium of the game are consist of x_1 and x_2 which belong to the overlapped area from S_1 and S_2, i.e. $S_1 \cap S_2$. Figure 1 (b) provides information about the Nash equilibria of the game. Each pair of x_1 and x_2, belonging to the deviant-crease-line area, compose of the Nash equilibrium, $((x_1, x_1, 0), (x_2, x_2, 0))$. So, there exist infinitely many Nash equilibria. Formally, the set of the pure-strategy Nash equilibria is

$$\left\{((x_1, x_2, 0), (x_2, x_2, 0)) \mid x_1 \in [0, x_{12}^b(x_2)] \text{ and } x_2 \in [0, x_{22}^b(x_1)]\right\}.$$

Among the infinitely many Nash equilibria, there is a coalition-proof Nash equilibrium and it is denoted by N. It is determined at the intersection of the imaginary best responses of player 12 and 22, i.e., $x_{12}^b(X_2)$ and $x_{22}^b(X_1)$. Specifically, the unique coalition-proof Nash equilibrium is

$$\left((\frac{v_{12}^2 v_{22}}{(v_{12}+v_{22})^2}, \frac{v_{12}^2 v_{22}}{(v_{12}+v_{22})^2}, 0), (\frac{v_{12} v_{22}^2}{(v_{12}+v_{22})^2}, \frac{v_{12} v_{22}^2}{(v_{12}+v_{22})^2}, 0)\right).$$

4.2 The case in which $m_1 = 3$, $m_2 = 4$, and $v_{22} > v_{23}$

As in the previous section, we have following group-1-specific equilibria in which $A = \{11, 12\}$, given X_2:

$$(x_{11}, x_{12}, x_{13}) = (x_1, x_1, 0) \text{ with } 0 \le x_1 \le x_{12}^b(X_2) = \begin{cases} \sqrt{v_{12}X_2} - X_2 & \text{for } X_2 \le v_{12} \\ 0 & \text{for } X_2 > v_{12}. \end{cases}$$

From (b) in Lemma 2 we have following group-2-specific equilibria, given X_1:

$$(x_{21}, x_{22}, x_{23}, x_{24}) = (x_2, x_2, 0, 0) \text{ with } x_{23}^b(X_1) \le x_2 \le x_{22}^b(X_1),$$

$$x_{23}^b(X_1) = \begin{cases} \sqrt{2v_{23}X_1} - 2X_1 & \text{for } X_1 \le \frac{v_{23}}{2} \\ 0 & \text{for } X_1 > \frac{v_{23}}{2} \end{cases}$$

and

$$x_{22}^b(X_1) = \begin{cases} \sqrt{2v_{22}X_1} - 2X_1 & \text{for } X_1 \le \frac{v_{22}}{2} \\ 0 & \text{for } X_1 > \frac{v_{22}}{2}. \end{cases}$$

Figure 2 (a) provides information about group-i-specific equilibria. The shadowed area S_1 in the figure represents the values of x_1 each of which composes each group-1-specific equilibrium, given X_2, i.e., $x_1 \in [0, x_{12}^b(X_2)]$. The area S_2 represents the values of $\frac{x_2}{2}$ each of which composes each group-2-specific equilibrium, given X_1, i.e., $\frac{x_2}{2} \in [\frac{x_{23}^b(X_1)}{2}, \frac{x_{22}^b(X_1)}{2}]$. Note that $X_1 = x_1$ and $X_2 = \frac{x_2}{2}$.

[Figure 2 about here.]

The Nash equilibria of the game are consist of x_1 and $\frac{x_2}{2}$ which belong to the overlapped area from S_1 and S_2, i.e. $S_1 \cap S_2$. Figure 2 (b) provides information about the Nash equilibria of the game. Each pair of x_1 and $\frac{x_2}{2}$, belonging to the deviant-crease-line area, compose of the Nash equilibrium, $((x_1, x_1, 0), (x_2, x_2, 0, 0))$. So, there exist infinitely many Nash equilibria. Formally, the set of the pure-strategy Nash equilibria is

$$\left\{ ((x_1, x_2, 0), (x_2, x_2, 0, 0)) \mid x_1 \in \left[0, x_{12}^b(\frac{x_2}{2})\right] \text{ and } \frac{x_2}{2} \in \left[\frac{x_{23}^b(x_1)}{2}, \frac{x_{22}^b(x_1)}{2}\right]\right\}.$$

Among the infinitely many Nash equilibria, there is a coalition-proof Nash equilibrium and it is denoted by M. It is determined at the intersection of the imaginary best responses of player 12 and 22, i.e., $x_{12}^b(X_2)$ and $x_{22}^b(X_1)/2$. Specifically, the unique coalition-proof Nash equilibrium is

$$\left((\frac{2v_{12}^2 v_{22}}{(2v_{12}+v_{22})^2}, \frac{2v_{12}^2 v_{22}}{(2v_{12}+v_{22})^2}, 0), (\frac{2v_{12} v_{22}^2}{(2v_{12}+v_{22})^2}, \frac{2v_{12} v_{22}^2}{(2v_{12}+v_{22})^2}, 0, 0) \right).$$

4.3 The case in which $m_1 = 3$, $m_2 = 4$, and $v_{22} = v_{23}$

We have following group-1-specific equilibria in which $A = \{11, 12\}$, given X_2:

$$(x_{11}, x_{12}, x_{13}) = (x_1, x_1, 0) \text{ with } 0 \le x_1 \le x_{12}^b(X_2) = \begin{cases} \sqrt{v_{12}X_2} - X_2 & \text{for } X_2 \le v_{12} \\ 0 & \text{for } X_2 > v_{12}. \end{cases}$$

From (c) in Lemma 2, we have following two types of group-2-specific equilibria in which $A = \{21, 22\}$ and $B = \{23, 24\}$, given X_1:

$$(x_{21}, x_{22}, x_{23}, x_{24}) = (x_2, x_2, y_2, 0) \text{ with } x_2 > y_2 \text{ and } x_2 + y_2 = x_{22}^b(X_1) = \begin{cases} \sqrt{2v_{22}X_1} - 2X_1 & \text{for } X_1 \le \frac{v_{22}}{2} \\ 0 & \text{for } X_1 > \frac{v_{22}}{2} \end{cases}$$

and

$$(x_{21}, x_{22}, x_{23}, x_{24}) = (x_2, x_2, x_2, 0) \text{ with } 0 \le x_2 \le x_{22}^b(X_1) = \begin{cases} \sqrt{v_{22}X_1} - X_1 & \text{for } X_1 \le v_{22} \\ 0 & \text{for } X_1 > v_{22}. \end{cases}$$

First, we consider the Nash equilibrium which consists of the group-2-specific equilibrium $(x_2, x_2, y_2, 0)$. Figure 3 (a) provides information about the group-i-specific equilibria. The shadowed area S_1 in the figure represents the values of x_1 each of which composes each group-1-specific equilibrium, given X_2, i.e., $x_1 \in [0, x_{12}^b(X_2)]$. The curve S_2 represents the values of $\frac{x_2+y_2}{2}$ that compose each group-2-specific equilibrium, given X_1, i.e., $\frac{x_2+y_2}{2} = \frac{x_{22}^b(X_1)}{2}$. Note that $X_1 = x_1$ and $X_2 = \frac{x_2+y_2}{2}$.

[Figure 3 about here.]

Then the Nash equilibria of the game are consist of x_1 and $\frac{x_2+y_2}{2}$ which belong to the overlapped curve from S_1 and S_2, i.e. $S_1 \cap S_2$. Figure 3 (b) provides information about the

Nash equilibria of the game. Each pair of x_1 and $\frac{x_2+y_2}{2}$, belonging to the green bold line, compose of the Nash equilibrium, $((x_1, x_1, 0), (x_2, x_2, y_2, 0))$. So, there exist infinitely many pure-strategy Nash equilibria. Formally, the set of the pure-strategy Nash equilibria is

$$\left\{((x_1, x_2, 0), (x_2, x_2, y_2, 0)) \mid x_1 \in \left[0, x_{12}^b(\frac{x_2+y_2}{2})\right], x_2 > y_2, \text{ and } \frac{x_2+y_2}{2} = \frac{x_{22}^b(x_1)}{2}\right\}.$$

Among the infinitely many Nash equilibria, the coalition-proof Nash equilibrium is denoted by L which is determined at the intersection of the imaginary best responses of player 12 and 22, i.e., $x_{12}^b(X_2)$ and $x_{22}^b(X_1)/2$. Specifically, the coalition-proof Nash equilibria are

$$\left((\frac{2v_{12}^2 v_{22}}{(2v_{12}+v_{22})^2}, \frac{2v_{12}^2 v_{22}}{(2v_{12}+v_{22})^2}, 0), (x_2, x_2, y_2, 0) \right) \text{ with } \frac{x_2+y_2}{2} = \frac{v_{12}v_{22}^2}{(2v_{12}+v_{22})^2} \text{ and } x_2 > y_2.$$

Lastly, we consider the Nash equilibrium which consists of the group-2-specific equilibrium $(x_2, x_2, x_2, 0)$. Figure 4 (a) provides information about the group-i-specific equilibria. The shadowed area S_i in the figure represents the values of x_i each of which composes a group-i-specific equilibrium, given X_{-i}, i.e., $x_i \in [0, x_{i2}^b(X_{-i})]$. Note that $X_1 = x_1$ and $X_2 = x_2$.

[Figure 4 about here.]

Then the Nash equilibria of the game are consist of x_1 and x_2 which belong to the overlapped area from S_1 and S_2, i.e. $S_1 \cap S_2$. Figure 4 (b) provides information about the Nash equilibria of the game. Each pair of x_1 and x_2, belonging to the deviant-crease-line area, compose of the pure-strategy Nash equilibrium, $((x_1, x_1, 0), (x_2, x_2, x_2, 0))$. So, there exist infinitely many Nash equilibria. Formally, the set of the pure-strategy Nash equilibria is

$$\left\{((x_1, x_2, 0), (x_2, x_2, x_2, 0)) \mid x_1 \in [0, x_{12}^b(x_2)] \text{ and } x_2 \in [0, x_{22}^b(x_1)]\right\}.$$

Among the infinitely many Nash equilibria, there is a coalition-proof Nash equilibrium. The coalition-proof Nash equilibrium is denoted by K, which is determined at the intersection of the imaginary best responses of player 12 and 22, i.e., $x_{12}^b(X_2)$ and $x_{22}^b(X_1)$. Specifically, the coalition-proof Nash equilibrium is

$$\left((\frac{v_{12}^2 v_{22}}{(v_{12}+v_{22})^2}, \frac{v_{12}^2 v_{22}}{(v_{12}+v_{22})^2}, 0), (\frac{v_{12}v_{22}^2}{(v_{12}+v_{22})^2}, \frac{v_{12}v_{22}^2}{(v_{12}+v_{22})^2}, \frac{v_{12}v_{22}^2}{(v_{12}+v_{22})^2}, 0) \right).$$ [8]

[8] Armed with the coalition-proof Nash equilibrium concept as an equilibrium selection principle, it would be an interesting question to study the group-size effect in the contest, i.e., whether the larger group is advantaged or disadvantaged, by comparing the case in which $m_1 = m_2 = 3$ with the case in which $m_1 = 3, m_2 = 4$ in our examples. We have found that the larger group is indeed disadvantaged in the symmetric setting where players in each group have the same valuation, and that the same is true under certain conditions in the asymmetric setting. We leave a rigorous analysis on the group-size effect in our average-opinion group contest for future research. We are grateful to the anonymous reviewer for giving this interesting question.

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Appendix: The proof of Lemma 2

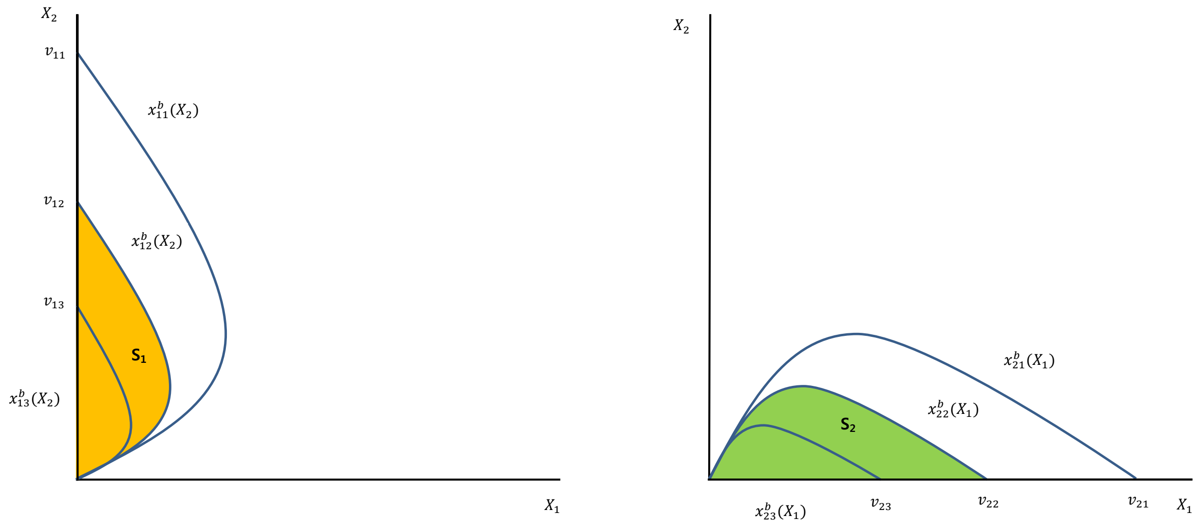
Proof. (a) We show that each player in group i does not have any incentive to change its effort level stipulated in the m_i-tuple vector of effort levels above, given the other players' effort levels within its group and X_{-i}. First, any player $ik \in B$ does not have an incentive to increase its effort level, because increasing its effort level does not change its group effort level X_i, which is equal to x_i. Now let us consider the deviation incentive of each player $ik \in A$. Since $X_i = x_i \leq \min \left\{ x_{ik}^b(X_{-i}) \right\}_{ik \in A}$, the player(s) whose valuation is the lowest in subgroup A has no incentive to decrease its effort level, and does not have any incentive to increase due to the invariant X_i. The other players in subgroup A have the imaginary best responses (to X_{-i}) that are greater than or at least equal to x_i, and thus they do not have any incentive to decrease their effort level. They do not have any incentive to increase due to the invariant X_i, either. Therefore, every player in group i has no incentive to change its effort level stipulated in the vector of effort levels in (a). Lastly, we show that $|A| = \frac{m_i+1}{2}$ should be held in equilibrium. Suppose that $|A| > \frac{m_i+1}{2}$ and $x_i > 0$. Then, in subgroup A, there exists at least a player who has an incentive to reduce his effort level to 0. Now suppose that $|A| < \frac{m_i+1}{2}$. In this case, the median value X_i is 0 and thus every player in subgroup A has the incentive to decrease his effort level to 0. So, in equilibrium, $|A|$ should be equal to $\frac{m_i+1}{2}$.

(b) The vector of effort levels says that half of the players in group i, i.e., player $i(m_i + 2)/2$, $i(m_i + 4)/2$, $\ldots$, im_i, choose zero effort level, and the other half choose an equal effort level x_i. The highest-valuation player in subgroup B is player $i(m_i + 2)/2$, and he has no incentive to increase its effort level, because x_i is greater than or at least equal to his imaginary best response or equal to zero. The other players in subgroup B do not have any incentive to increase, for their imaginary best responses are less than or at most equal to player $i(m_i + 2)/2$'s. Now let us consider the players in subgroup A. Player $im_i/2$ is the lowest-valuation player in the subgroup, and he has no incentive to decrease its effort level, because x_i is less than or equal to his imaginary best response to X_{-i}. Due to the invariant X_i, there is no incentive for him to increase his effort level. Since the other players in subgroup A have imaginary best responses that are higher than or at least equal to player $im_i/2$'s, they do not have any interest to change their effort levels. All players in group i do not have any incentive to deviate from the suggested vector of effort levels in (b).

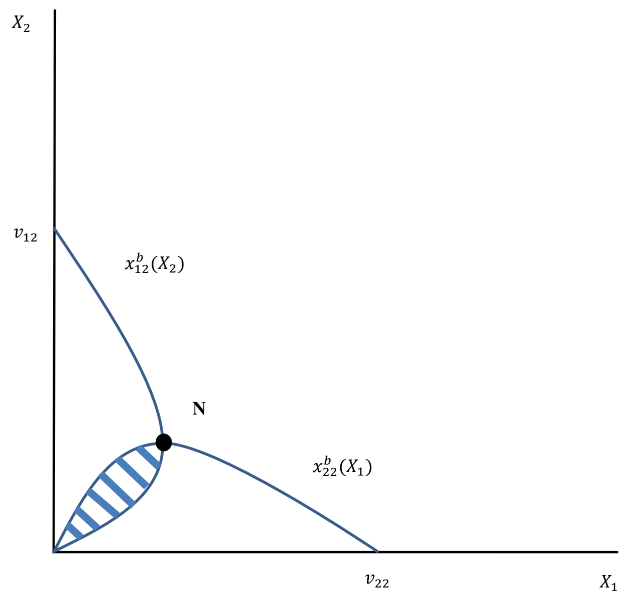
(c) We show that the first vector of effort levels, $(x_i, x_i, \cdots, x_i, x_i, y_i, 0, \cdots, 0, 0)$, constitutes the group-$i$-specific equilibrium. The lowest-valuation player(s) in subgroup A does not have any incentive to increase or decrease his effort level, because $x_i + y_i$ is equal to his imaginary best response to X_{-i}. The other players in subgroup A do not have any incentive to decrease their effort levels, because their imaginary best responses are greater than or at least equal to $x_i + y_i$. They also do not have any interest to increase their effort levels because of the invariant X_i. The highest-valuation player who chooses y_i in subgroup B has no incentive to change its effort level because $x_i + y_i$ is equal to its imaginary best response to X_{-i}. And the other players in subgroup B have no incentive to increase their effort levels since their imaginary best responses are less than or at most equal to $x_i + y_i$.

Now consider the vector of effort levels, $(x_i, x_i, \cdots, x_i, x_i, x_i, 0, \cdots, 0, 0)$. The lowest-valuation player(s) in subgroup A does not have an incentive to decrease his effort level because x_i is less than or equal to his imaginary best response to X_{-i}. He does not have any incentive to increase his effort level, either, because of the invariant X_i. The other players in subgroup A do not have any incentive to decrease their effort levels, because their imaginary best responses are greater than or at least equal to x_i. They also do not have any interest to increase their effort levels due to the invariant X_i. Similarly, the highest-valuation player who chooses x_i in subgroup B has no incentive to decrease its effort level, because x_i is less than or equal to its imaginary best response to X_{-i}. He has no incentive to increase due to the invariant X_i. The other players in subgroup B have no incentive to increase their effort levels because of the invariant X_i.

(d) The lowest-valuation player(s) in subgroup A does not have an incentive to decrease his effort level because x_i is less than or equal to his imaginary best response to X_{-i}. He does not have any incentive to increase his effort level, due to the invariant X_i. The other players in subgroup A do not have any incentive to decrease their effort levels, because their imaginary best responses are greater than or at least equal to x_i. They also do not have any interest to increase their effort levels due to the invariant X_i. The highest-valuation player who chooses x_i in subgroup B has no incentive to decrease its effort level, because x_i is less than its imaginary best response to X_{-i}. Also, due to the invariant X_i, he has no interest to increase. And the other players in subgroup B have no interest to increase their effort levels due to the invariant X_i, either. ■

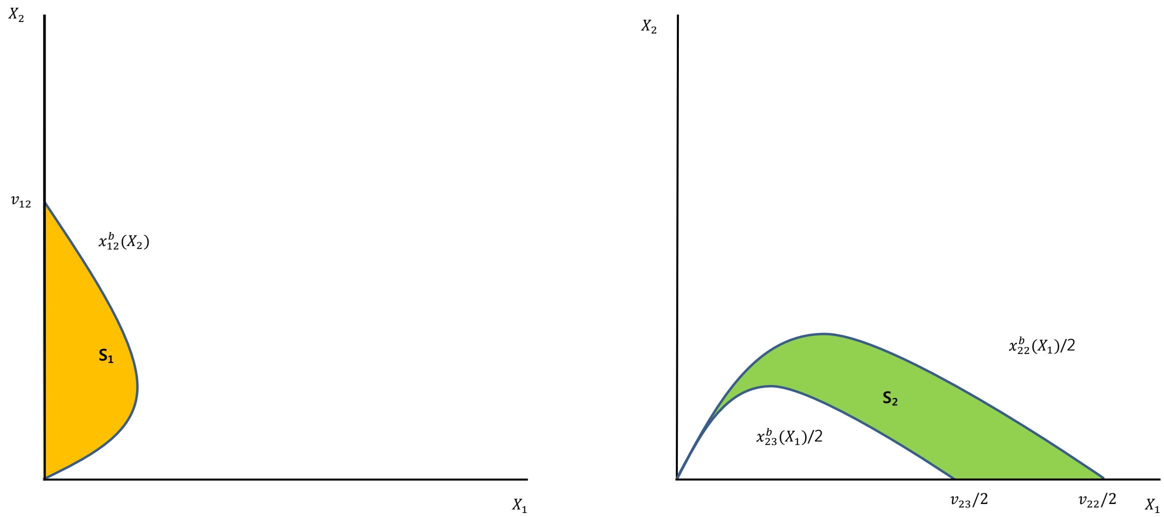


(a) The group-i-specific equilibria, given X_{-i}

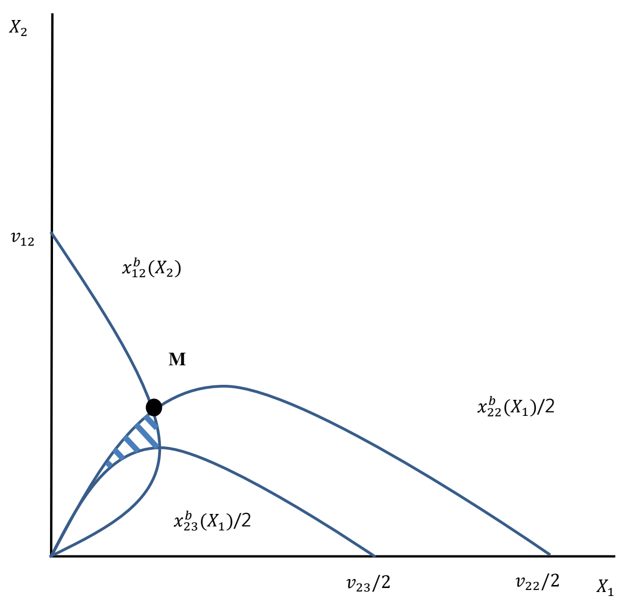


(b) The Nash equilibria of the game

Figure 1: $m_1 = m_2 = 3$

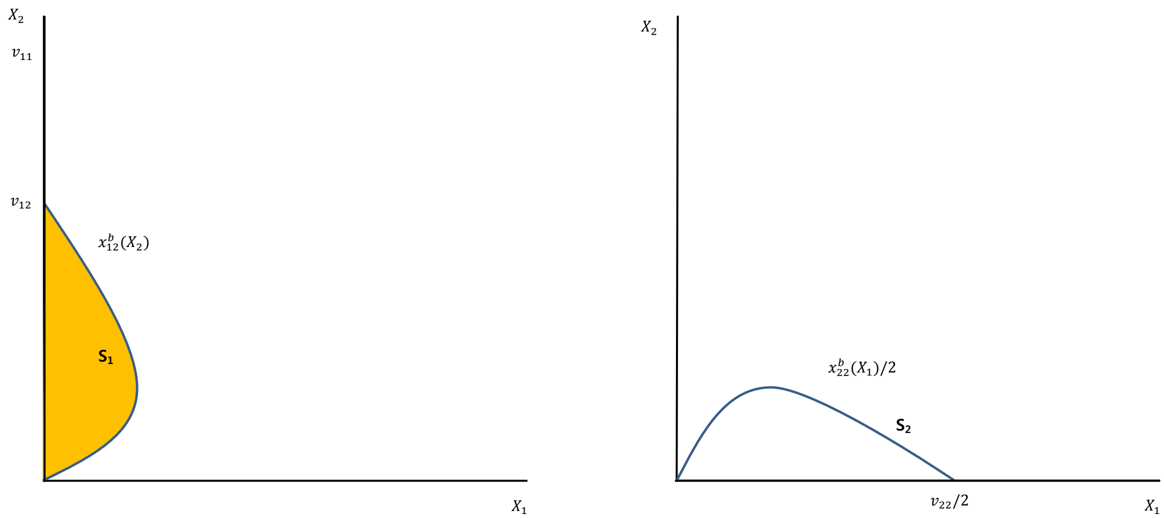


(a) The group-i-specific equilibria, given X_{-i}

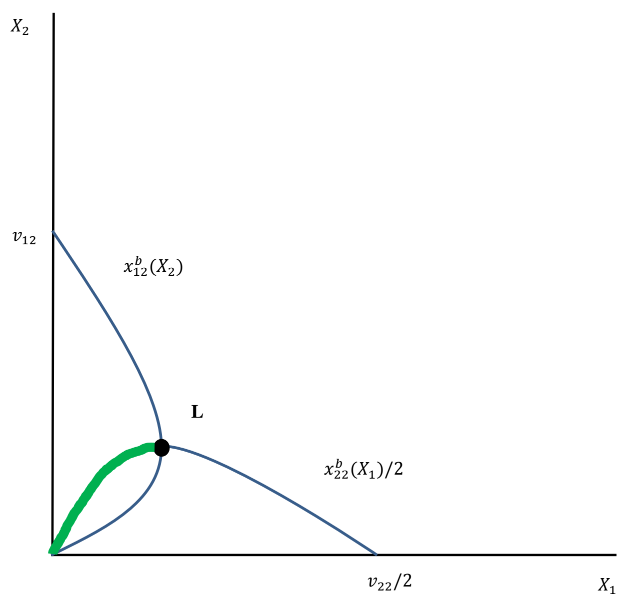


(b) The Nash equilibria of the game

Figure 2: $m_1 = 3, m_2 = 4$ and $v_{22} > v_{23}$

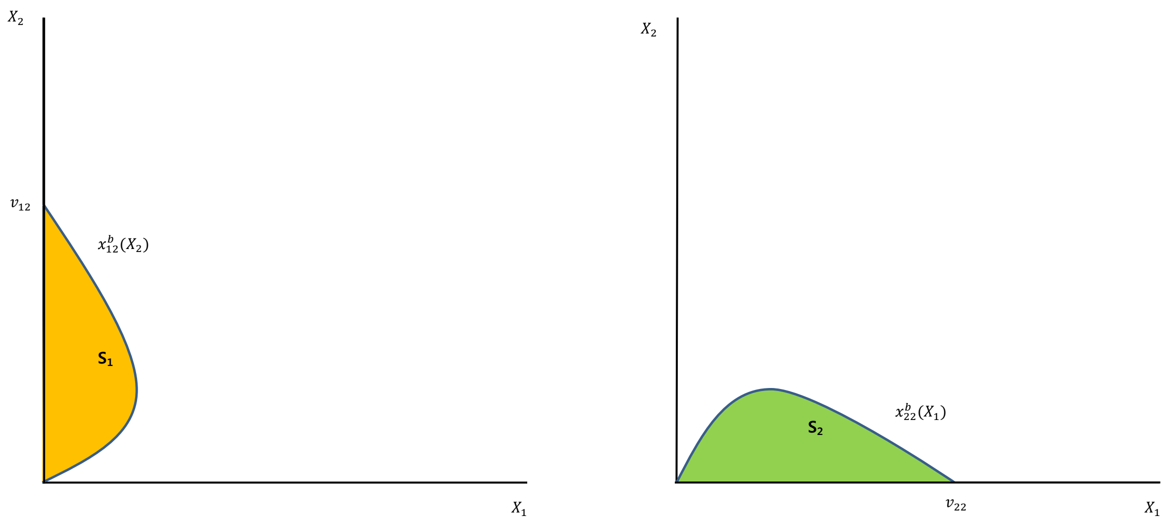


(a) The group-i-specific equilibria, given X_{-i}

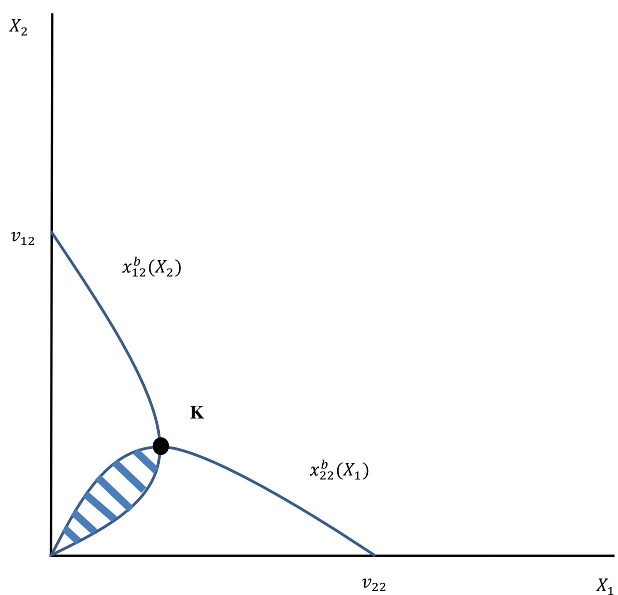


(b) The Nash equilibria of the game

Figure 3: $m_1 = 3, m_2 = 4$, $v_{22} = v_{23}$, and $(x_2, x_2, y_2, 0)$



(a) The group-i-specific equilibria, given X_{-i}



(b) The Nash equilibria of the game

Figure 4: $m_1 = 3, m_2 = 4$, $v_{22} = v_{23}$, and $(x_2, x_2, x_2, 0)$