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A Double-Exponential Jump model and its application to risk measure in Wheat spot market

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Abstract

This paper considers the complete feature of commodity spot prices for risk measuring. We use a Double-Exponential Jump model to capture the evolution of wheat spot prices from 2004 to 2014 and utilize the estimated model in the calculation of Value-at-Risk. In the modeling of wheat spot prices, the baseline model outperforms all alternative models. In the case of relative high volatile period, there exists risk underestimation of Value-at-Risk with normal distribution hypothesis. It is suggested to take into account jump risk and other special characteristics of prices in the risk management for agricultural cooperatives.

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1. Introduction

The agricultural industry is highly dependent on the climate, which may lead to unforecastable price variation. Jumps and time-varying volatilities are found to be significant in agricultural commodities price returns. Within this framework, the failure of forecasting risk might produces systemic risk given the role of agricultural cooperatives in the physical commodities markets. This paper discusses a Double-exponential Jump model and its application to the risk measure Value-at-Risk (VaR).
A wide range of literature has developed times series models to simulate the commodity price dynamics and to capture the accurate commodity price behaviour such as skewness right, excess kurtosis and the volatility behaviour. These features may be considered in the stochastic process, including mean reversion, time-varying volatility, discontinuity, etc.
It is stylized empirical evidence that agricultural and other commodity prices returns exhibit mean reversion especially in the competitive markets framework. The Ornstein-Uhlenbeck model first discussed by Ornstein & Uhlenbec (1930) describes the mean-reversion feature. Its application in commodity modelling can be seen, for example, in Gibson & Schwartz (1990), Schwartz (1997) and Schwartz &Smith (2000) Other empirical evidence on returns is revealed, such as excess kurtosis with not normally distributed. On the one hand, ARCH and GARCH models examine the time-varying volatility. On the other hand, jump models could take into account the discontinuity of prices in the price modelling. Models belonging to this category are Beck (2001), Bernard et al. (2008) and Hilliard & Reis (1998), etc. Among these, Kou (2002) and Ramezani & Zeng (1998) propose a Double Exponential Jump model, which indicates that the upward and downward jumps have independently exponential distribution. Some empirical works in the derivatives pricing models support that the double exponential diffusion model outperforms the normal jump diffusion model (Ramezani & Zeng, 2004). Nevertheless, there is not application of this model in the commodity physical market. Using Double-Exponential Jump model deals with the problem that commodity physical market is less liquid and probably have less frequent jumps than other financial markets.
This paper provides the evidence with respect to the features of recent commodity physical market and complements the existing literature by contributing an application in the modelling wheat spot prices using double-exponential jump model. We further apply this model in the risk measure of agricultural cooperatives. We organize the remainder of the article as follows. Section 2 introduces the Double-Exponential Jump model. Section 3 presents the data and estimation results. Section 4 gives an application of model in risk measure VaR. Section 5 concludes.

2. Model specification

In this model, the log price is assumed to follow a Brownian motion with a mean-reversion term, plus a compound Poisson process with jump sizes double exponentially distributed (Kou, 2002; Ramezani & Zeng, 1998). Volatility is supposed to be stochastic (GARCH). With $X_t = ln(S_t)$, log prices and $V_t = \sigma_t^2$, volatility:

$$dX_t = k\left(\mu_x - X_t\right)d_t + \sigma_t dB_t + J_t dN_t \tag{1}$$

$$dV_t = k_v\left(\mu_v - V_t\right)d_t + \sigma_v V_t dB_{vt} \tag{2}$$

In equation (1) and (2), the Ornstein-Uhlenbeck process captures the mean-reversion behaviour of returns. k denotes the rate that the log prices X_t return to a equilibrium or a mean value μ_x; The volatility with GARCH behaviour is modelled as a mean-reversion process. The Brownian motion B_t and B_{vt}, following $Normal \sim (0, dt)$ are assumed to be independent. In the estimation, we can re-write $V_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta V_{t-1}$ with ε_{t-1} the error term at time t-1; $\omega > 0$ et $\alpha \geq 0, \beta \geq 0$; and $\alpha + \beta < 1$ (stationary condition). Transforming to the continuous form of GARCH, we have $k_v = 1 - \alpha - \beta$ and $\sigma_v = \alpha\sqrt{2}$ (Nelson, 1990), where $\frac{\omega}{1-\alpha-\beta}$ measures the long-run average variance per day by using the daily data here. $\alpha + \beta$ is the persistence of volatility.
The discontinuous component of the price process is described by a Poisson counter N_t in the equation (1), with intensity λ and jump size J_t. The intensity λ is the probability that jump occurs, $Prob(\Delta N_t = 1) = \lambda dt$ and probability of no jump, $Prob(\Delta N_t = 0) = 1 - \lambda dt$.
The double exponential distribution of J_t is supposed to be independent of B_t and B_{vt}. The double exponential distribution of J_t is:

$$f_{J_t} = \begin{cases} \eta_1 e^{-\eta_1 J_t} & Probability = p \quad (3) \\ \eta_2 e^{\eta_2 J_t} & Probability = 1 - p \quad (4) \end{cases}$$

The probability p, is the probability that an upward jump occurs, and 1-p is the probability of a downward jump. The means of exponential random variables are $1/\eta_1$ and $-1/\eta_2$ for positive and negative jump sizes respectively. The jump size is not normally distributed but has the leptokurtic feature. And the Double Exponent Jump model (abbreviation DEJM will be in the rest of this paper) gives the feature of overreaction and under-reaction to outside news and also the reaction to good or bad news.

Likelihood function
For the likelihood function, the density of X_t is simplified as a Bernoulli weighted sum of normal and exponential density. The three sources of randomness, B_t,B_{vt} and N_t, are assumed to be independent. Price processes are divided into two regimes: Jumps happen with probability λ; No jumps with probability $1 - \lambda$.
Define a random variable X_t as a sum of independent normal (with mean μ and variance σ^2) and exponential (with parameter δ) random variables. Its density function is:

$$g\left(X'\right) = \frac{\delta}{2} e^{\frac{\delta}{2}\left(2\mu + \delta\sigma^2 - 2X'\right)} erfc\left(\frac{-\mu + \delta\sigma^2 - X'}{\sqrt{2}\sigma}\right), \quad (5)$$

where the complementary error function $erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-s^2} ds$
Then density function can be written also as:
$g\left(X'\right) = \delta e^{\frac{\delta}{2}\left(2\mu + \delta\sigma^2 - 2X'\right)} \Phi\left(\frac{X' + \mu - \delta\sigma^2}{\sigma}\right)$, with $\Phi()$: Cumulative distribution CDF of standard normal variables.
The density function of X_t combined with a Bernoulli distribution in our case can be calculated (see also Ramezani and Zeng (1998) and Kou (2002)):

$$f(X_t) = \lambda \Bigg[p\eta_1 e^{\frac{\sigma^2\eta_1^2}{2}} e^{-(X(t)-(1-k)X(t-1)-k\mu_x)\eta_1} \Phi\left(\frac{-(X(t)-(1-k)X(t-1)+k\mu_x - V_t\eta_1)}{\sqrt{V_t}} \right)$$

$$+(1-p)\eta_2 e^{\frac{\sigma^2\eta_2^2}{2}} e^{-(X(t)-(1-k)X(t-1)-k\mu_x)\eta_2} \Phi\left(\frac{(X(t)-(1-k)X(t-1)+k\mu_x - V_t\eta_2)}{\sqrt{V_t}} \right) \Bigg]$$

$$+(1-\lambda)\frac{1}{\sqrt{2\pi}} e^{\left(-\frac{(X(t)-(1-k)X(t-1)-k\mu_x)^2}{2V_t} \right)} \quad (6)$$

The log likelihood function to be maximized is: $L(P) = \sum_{t=1}^{T} ln[f(X_t)]$ with T the last trading day of the sample.
Parameters $P = (k, \mu_k, p, \eta_1, \eta_2, \lambda, \omega, \alpha, \beta)$ can be estimated by maximization of the likelihood function numerically. The initial parameters are chosen carefully since results are highly dependent on the initial values. Initial values for mean part is obtained with empirical observation of mean return and averaged days of returning to the mean level. Starting value for volatility part is based on the resulting parameters of single GARCH model on return series. Initial value for jump part is based on empirical reasoning on the number and size of jumps[1]. We choose the initial values of parameters that ensure the estimation to have a convergence to a similar and stable estimates.

3. Model estimation and diagnostics check

3.1 Data

We consider the prices of wheat delivered in Rouen (France) from 2004 to 2014 in figure 1. The estimation are conducted for different harvest periods [2]. Prior to 2004, the wheat price is found at the lowest level, which corresponds to the recession in the USA. The prices attain a peak of 292 Euro/ton in mid-2007. From January 2006 to April 2008, the prices of wheat increase about 159%, and then decrease abruptly to the long-term level in the second semester of 2008. Meanwhile, the prices in the period of 2007/2008 become volatile compared to the previous harvest. Another peak is in 2010/2011 where prices are also volatile and a higher price level is maintained in the next harvest of 2011/2012, when Russia banned the wheat exportation due to an extremely dry climate. Descriptive statistics for the log return (defined as the difference of log prices of two opening days) is given in Table I. The return series are not normally distributed, as they have non-zero skewness and positive excess kurtosis. The normality hypothesis on the returns series is rejected for all harvests by Kolmogorov-Smirnov test, which confirms results on the observation of Kurtosis and Skewness.

[1] We define firstly different thresholds on the return dynamics, for example: mean ± 1 standard deviation; ± 1.2 standard deviation. The price returns exceeding the thresholds are considered as a jump occurring. We have compared the results of different initial values and retained those that give reasonable and stable estimates of all other parameters. The final initial parameters that maximize the likelihood function are preserved.

[2] One harvest is from July to June the next year. As we consider that wheat in different harvests are distinct products.

3.2 Model estimation

The Table II gives the results from the maximum likelihood function. All the estimated parameters are significant, with the level at least 10%. The standard errors reported are calculated using the Hessian matrix.

Prices in the harvest during 2007/2008 and 2008/2009 revert to equilibrium more slowly than in other years, and also for another period, 2004/2005, which may be with respect to the lower storage pressure in the previous harvest. Regarding the equilibrium level of prices, peaks are found in 2007/2008, 2010/2011 and 2012/2013. The 2007/2008 harvest coincides with the crisis from the financial industry with the fact of financialization of the commodity market. And also the higher volatility is found in 2004/2005, 2007/2008 and 2010/2011.

Jumps have happened more frequently since 2006. Prices increase from 2006 and oscillate along a high level during 2007/2008. Tow kinds of cases can be distinguished according to the results: firstly, the harvest of 2004/2005 is categorized by jump with large size and lower frequency. Secondly, in the years of 2006/2008 and 2010/2011, the jumps happen more frequently but have lower value. The results of the jumps are confirmed with those of mean reversion and volatility: In the higher volatile period, the jumps are more frequent and prices deviate to the market equilibrium much longer than in other periods.

3.3 Diagnostic checks

In this section, to examine the impact of various features in the calibration of Double-exponential Jump model, we also fitted the following three more restrictive models to prices returns: ARCH model with stochastic volatility (without mean reversion); Mean-reversion model with stochastic volatility (without jump term); and Jump diffusion model with constant volatility (without stochastic volatility).[3]

Likelihood ratio is defined as $LR = -2[L(\widehat{p}, X) - L(P, X)]$, with $\widehat{p}$ the parameters estimated by restricted models, P the parameters estimated by the complete model, and X the log prices. LR is Chi-square distributed with k degree of freedom (k is the number of restrictions).

According to the likelihood ratio test in table III, the null hypothesis that the constrained model fits sample data better than the unconstrained model is rejected. These models do not outperform the baseline DEJM.

On the basis of the results using DEJM, it reveals that it is effective to investigate the wheat prices by the complete model according to the likelihood ratio test.

4. Application in risk measure of agricultural cooperatives

In this section, we turn to apply the estimated model in the risk measure of agricultural cooperatives. Cooperatives play an important intermediary role in the agricultural industry or agribusiness. As Declerck and Mauget (2008) indicates: "The job of agricultural cooperatives is to collect agricultural commodities from farmers and commercialize these commodities in the market." In France, cooperatives usually pay an "average price" to the members. This payment contract can be explained as: Cooperatives pay the members for an advance price when the contract is established between

[3]For brevity, the estimation results for these models are not presented

them. At the end of a harvest, cooperatives may pay for a complement price (difference between average price and advance price) when the average price is higher than the advance price; cooperatives do not need to pay for a complement price if average price is lower than the advance price.
We discuss the Value-at-Risk (VaR), a measure of the potential loss given a probability within a period (one to ten days); In this paper, we consider the 1% VaR with a horizon of 10 trading days. VaR is actually a quantile of portfolio returns, which is a risk measurement widely used by financial institutions and the agricultural industry. This section focus on the calculation of VaR of the last three periods from 2011 to 2014 for simplicity. And also these three periods involve different jump and volatility behaviors from previous estimation. In the 2011/2012 harvest, one of the volatile periodes, jumps happens frequently with large size. The frequently jumps continue in the next harvest of 2012/2013. While the 2013/2014 is one of the quite period with less frequent jumps happens. An appropriate VaR applied in agricultural cooperatives should deal well with the trade-off between sufficient risk prevision and not overusing resources. We are going to compare two VaRs under different prices process: DEJM and Black-Scholes [4] in different portfolios. Portfolios try to mime cooperatives' activities, including contract between farmers and cooperatives, and contract of hedging.

4.1 Portfolio specification

In the first portfolio, cooperatives buy wheat from farmers and sign a contract to guarantee for farmers that selling prices are not smaller than the average prices of the year. This contract can be considered as short Asian Call. Moreover, cooperatives are assumed not to hedge risk; thus, they face the risk when price decreases. Meanwhile, cooperatives sell the same proportion of wheat every day. Following this, in the second portfolio besides the guaranteeing contract and the selling activity, cooperatives long a put for hedging risk. Profit and loss of two portfolios are given in Table IV.
The specification of Asian call Y (guaranteeing contract) is: Maturity T for one year/one harvest; K strike price is supposed to be 190 Euros, which is the advanced prices to pay for farmers.

4.2 Backtesting

Backtesting techniques are needed for testing the performance of Normal-VaR and Jump-VaR. One test is based on the number of exceptions. Kupiec's test is to determine whether the observed frequency of exceptions is consistent with the frequency of expected exceptions according to the VaR model and chosen confidence. The LR test statistic is:

$$LR = -2ln\left(\frac{(1-\widehat{p})^{n-x}\,\widehat{p}^{x}}{(1-p)^{n-x}\,p^{x}}\right) \backsim X^{2}(1), \tag{7}$$

With $\widehat{p}$ the VaR stated probability level; p the exception frequency; $\frac{x}{n}$, n the total trading days and x the number of exceptions. The null hypothesis under this test is that the realised number of exceptions is equal to the theoretical number of exceptions defined in the VaR model and model is efficient.

[4]Commodities price returns S_t have a lognormal distribution with constant drift and volatility: $S_t = S_{t-1}exp\left(\left[\mu - 0.5\sigma^2\right]\frac{1}{T} + \sigma\frac{1}{T}Z_t\right)$

A larger difference between VaR and real Profit/Loss in the case of exception will be a disaster for cooperatives. For this purpose, we calculate a Root-Mean-Square Deviation (RMSD), which is the difference between Expected shortfall and the P/L in the case of exceptions. RMSD is inspired from Angelidis and Degiannakis (2006), where they point out that, given an identical number of exceptions, one model underestimates the risks in the case of exception, which is costly as well.

$$RMSE = \begin{cases} \sqrt{\frac{\sum_{i=1}^{T} (Expected\ short\ fall_i - P/L_i)^2}{T}} & \text{if exception accurs} \quad (8) \\ 0 & \text{if no exception} \quad (9) \end{cases}$$

with T the total trading days.

The last backtesting VaR, Performance Criterion (VPC), used also in Fuss et al. (2006), and Bao and Saltoglu (2006), considers "the trade-off between efficient capital allocation and sufficient reserves". This measure includes the distance between returns and VaR, correlation between return volatility and VaR, and penalties when there is exception. VPC is calculated as follow:

$$VPC = \alpha_1 \frac{1}{n} \sum_{x=1}^{X} (Real\ return_x - VaR_x)^2 + \alpha_2 \frac{1}{n} \sqrt{\sum_{k=1}^{K} |VaR_k - Real\ return_k| I\,(Real\ return_k < 0)}$$
$$+ \alpha_3 cor\left(Real\ return^2, VaR\right) + \alpha_4 |\widehat{p} - p| \quad (10)$$

n is the number of observations; X the number of exceptions; K the numbers of times when there is no exception and real return is negative. The second term measures the cost of reserves. It is also undesirable if VaR overestimates market risks. $\widehat{p}$, p are defined as in the Topiec's test. α_1, α_2, α_3, α_4 are the weight of each component and sum to unity. They can be adjusted according to the risk managers' preference. In our case, we use an equal weight, with $\alpha_i = 0.25$.

In general, a good VaR estimation should explain accurate risk level under both volatile market conditions and a quiet market without unnecessary reserve costs. In the following section, we will estimate VaR in different scenarios and compare the results with three backtesting techniques.

4.3 Implication results

Implication results are given in table V and table VI. In the portfolio 1, Jump-VaR has less exception times than Normal-VaR except for the year 2012/2013, where Jump-VaR has one more violation. Normal-VaR and Jump-VaR are efficient for the Kupiec's test; both results have accepted the hypothesis that the VaR models are accurate. However, for the first harvest and in the case of exception, Normal-VaR has underestimated much more risks than Jump-VaR, as the difference between real loss and expected shortfall is much higher. And concerning the VPC test, the results of both VaRs are quite close in terms of efficiency.

Without hedging risk, estimation results and backtesting results are quite close in both VaRs. In the first two harvests, Jump-VaR outperforms Normal-VaR with less violations and more efficient resource allocation. Jump-VaR will be more desirable for cooperatives in the first two harvests. In the 2013/2014 harvest, Jump-VaR overestimates risk, which is costly from the point of view of cooperative managers.

Regarding the portfolio 2, Jump-VaR has lower frequency of violation in all of the three periods. On average, Jump-VaR is larger than Normal-VaR and frequency of real loss exceeding expected shortfall is higher in the Normal model. Normal-VaR has exceeded the real P/L in the 2013/2014 harvest 5 times and has passed the theoretical level of exception. Normal-VaR and Jump-VaR are accepted to be correct according to Kupiec's test. Performance of VaR is similar for the first two harvests. In the 2013/2014 harvest, Normal-VaR has underestimated risk with expected shortfall lower than real loss in the case of exceptions. Jump-VaR is also more efficient during this harvest on the basis of VPC.

In the second portfolio, cooperatives start to hedge risk with an Asian Put. Risk hedging is not perfect but portfolio return volatility decreases. Normal-VaR and Jump-VaR work well in the first two harvests; however, in the last year, Normal-VaR underestimates risk.

5. Conclusion

In this paper, a stochastic jump process is used to capture the French wheat spot prices from 2004 to 2014. The stochastic process incorporates the recent characteristics of price variation, including mean reversion, jump behaviour and time-varying volatility. The modelling of jump as an exponential distribution deals with the problem of less liquidity in physical market and allows to distinguish between positive and negative jump. The parameters are estimated by a numerical maximization of the likelihood function of different harvests independently. According to the diagnostic tests, Double-Exponential Jump is efficient to capture the evolution of wheat spot prices in the application of risk measure for agricultural cooperatives. The difference is small in the two portfolios. However, it should be noted that there is one occasion when Normal-VaR has passed the theoretical level of exception. Normal-VaR might bring about underestimation risk problems in the case of a highly volatile market or other extreme market condition. Overall, given the changing market environment and uncertain price distributions, taking into account the other characteristics of prices returns can be complementary for examining the market and risk management of commodity companies.

Figure 1: Daily prices series

The graph gives the daily spot prices (Euro/ton) of French soft wheat and it is also the data that are used in the paper. The sample period is from 2004 to 2014, 10 harvests in total.

Table I: Descriptive statistics

Descriptive statistic are calculated from log-return series for every harvests. The P-value of Kolmogorov-Smirnov test is given inside the brackets, the test rejects the null hypothesis at the 5% significance level.

Log return	**Mean**	**Std Deviation**	**Variance**	**Excess Kurtosis**	**Skewness**	**Kolmogorov-Smirnov test *(P-value)***
2004/2005	-0.0011	0.0196	0.0004	9.7423	-0.4120	0.4783 *(<0.001)*
2005/2006	0.0004	0.0092	8.43E-05	26.6499	3.5099	0.4925 *(<0.001)*
2006/2007	0.0026	0.0174	0.0003	3.9229	0.9290	0.4813 *(<0.001)*
2007/2008	0.0004	0.0237	0.0006	0.6912	-0.3691	0.4751 *(<0.001)*
2008/2009	-0.0022	0.0226	0.0005	10.1772	1.5842	0.4769 *(<0.001)*
2009/2010	-0.0002	0.0144	0.0002	0.7993	-0.1566	0.4826 *(<0.001)*
2010/2011	0.0013	0.0249	0.0006	0.7192	-0.0771	0.4735 *(<0.001)*
2011/2012	0.0009	0.0162	0.0003	0.5689	0.3988	0.4852 *(<0.001)*
2012/2013	-0.0007	0.0168	0.0003	6.1398	-0.0939	0.4770 *(<0.001)*
2013/2014	-0.0011	0.0115	0.0001	0.2206	0.0211	0.4878 *(<0.001)*

Table II: Estimated parameters

This table gives all estimated parameters using maximum likelihood function. Standard error given in parentheses is calculated by Hessian matrix. All the parameters are significant – at least 10% level.

Campagne		k	μ_x	p	η_1	η_2	λ	ω	α	β	**-Log Likelihood**
2004/2005	**Value**	**0.0218**	**4.5796**	**0.4011**	**28.1493**	**24.5404**	**0.0489**	**8.10E-05**	**0.2867**	**0.0059**	**-452.7**
	std error	(3.66E-03)	(1.05E-02)	(0.0021)	(0.0901)	(0.0126)	(0.0023)	(1.25E-05)	(5.13E-03)	2.14E-03	
2005/2006	**Value**	**0.1003**	**4.6478**	**0.5398**	**63.8271**	**87.1363**	**0.0507**	**8.49E-06**	**0.1985**	**0.5775**	**-453.95**
	std error	(8.91E-03)	4.24E-03	0.0266	0.9895	1.00	0.0303	1.62E-02	3.38E-02	5.45E-02	
2006/2007	**Value**	**0.0403**	**4.9832**	**0.5997**	**28.1841**	**36.2241**	**0.0310**	**2.84E-05**	**0.1379**	**0.6682**	**-607.8**
	std error	(6.15E-03)	(3.23E-02)	(0.0515)	(1.03)	(1.03)	(1.2E-02)	(7.24E-06)	(3.03E-02)	(0.0463)	
2007/2008	**Value**	**0.0239**	**5.4754**	**0.451**	**23.603**	**21.1662**	**0.035**	**0.0002**	**0.2119**	**0.3798**	**-527.1**
	std error	(0.0052)	(0.0771)	(0.8920)	(1.2288)	(1.69)	(0.0210)	(2.53E-05)	0.0564	0.0508	
2008/2009	**Value**	**0.0226**	**4.8342**	**0.4339**	**17.9475**	**23.6587**	**0.0634**	**1.04E-04**	**0.0212**	**0.6463**	**-498.2**
	std error	(9.92E-03)	(0.0579)	(0.0412)	(1.09)	(1.42)	(0.0406)	(1.16E-05)	(0.0183)	(0.0351)	
2009/2010	**Value**	**0.0600**	**4.7706**	**0.5013**	**33.3163**	**30.7647**	**0.0600**	**8.79E-05**	**0.0282**	**0.4277**	**-570.05**
	std error	(0.0015)	(1.21E-02)	(0.3746)	(1.0048)	(1.01)	(2.21E-02)	(4.20E-06)	(0.0129)	(5.88E-02)	
2010/2011	**Value**	**0.0606**	**5.4719**	**0.6165**	**26.8196**	**19.2339**	**0.0472**	**3.51E-05**	**0.2012**	**0.7367**	**-514.4**
	std error	(0.0106)	(2.09E-02)	(0.1105)	(0.9451)	(1.01)	(0.0364)	1.15E-05	(0.0433)	(0.0307)	
2011/2012	**Value**	**0.0903**	**5.2874**	**0.5798**	**27.9485**	**30.9311**	**0.0601**	**3.19E-06**	**0.0040**	**0.9743**	**-419.7**
	std error	(2.26E-02)	(1.29E-02)	(0.3540)	(4.7725)	(0.0331)	(0.0573)	(7.01E-07)	(2.90E-03)	(3.08E-03)	
2012/2013	**Value**	**0.0200**	**5.4491**	**0.400**	**15.00**	**14.996**	**0.0600**	**1.50E-04**	**0.1464**	**0.0415**	**-492.2**
	std error	(6.66E-03)	4.15E-02	(0.0145)	(0.4089)	(0.0365)	(0.0226)	(2.14E-05)	(1.78E-02)	(1.26E-02)	
2013/2014	**Value**	**0.0725**	**5.2110**	**0.5872**	**39.997**	**29.9998**	**0.0300**	**9.47E-05**	**0.1000**	**0.1119**	**-489.4**
	std error	(1.34E-02)	(5.28E-02)	(0.185)	(10.4418)	(9.74)	(0.0464)	(1.17E-05)	(8.95E-02)	(8.9E-02)	

Table III: Likelihood ratio tests(LR)

Likelihood ratios are calculated for every harvest by comparing with the models without jump, models without mean reversion and models with constant volatility

	Without jump	**Without mean reversion**	**Constant volatility**	**MC p-value**
2004/2005	49.579	9.620	6.896	0.01
2005/2006	59.963	1.907	3.109	0.01
2006/2007	29.470	5.923	5.890	0.01
2007/2008	13.224	4.042	23.254	0.01
2008/2009	39.970	1.227	1.844	0.01
2009/2010	8.486	2.511	32.174	0.01
2010/2011	4.991	8.152	23.492	0.01
2011/2012	2.865	1.033	1.596	0.01
2012/2013	18.571	1.492	2.058	0.01
2013/2014	4.343	3.770	1.148	0.01

Table IV: Portfolio Specification

	Setting-up	**Profit & Loss**
Portfolio 1	Long wheat S + Short Asian call Y	$t : (S_{t+10} - S_t) + (0 - Y_{t+10})$ $t+1 : (S_{t+11} - S_{t+1})(1 - 1/T) - (Y_{t+11} - Y_{t+1})$
Portfolio 2	Long wheat S + Short Asian call Y + Long Asian Put	$t : (S_{t+10} - S_t) + (0 - Y_{t+10})$ $t+1 : (S_{t+11} - S_{t+1})(1 - 1/T) - (Y_{t+11} - Y_{t+1})$ $+(X_{t+11} - X_t)$

Table V: Number of exceptions

		Portfolio 1 **Number of exceptions**	**Portfolio 2** **Number of exceptions**
2011/2012	VaR(Black-Scholes)	3	3
	VaR(Jump)	2	1
2012/2013	VaR(Black-Scholes)	3	3
	VaR(Jump)	4	2
2013/2014	VaR(Black-Scholes)	4	5
	VaR(Jump)	3	4

Table VI: Backtesting

			Kupiec's test	**RMSE**	**VPC(VaR performance criterion)**
Portfolio 1	**2011/2012**	VaR(Black-Scholes)	20.775	0.541	0.188
		VaR(Jump)	24.931	0.020	0.142
	2012/2013	VaR(Black-Scholes)	20.772	0.063	0.321
		VaR(Jump)	.306	0.089	0.332
	2013/2014	VaR(Black-Scholes)	17.245	0.414	0.023
		VaR(Jump)	20.726	1.783	0.023
Portfolio 2	**2011/2012**	VaR(Black-Scholes)	24.993	0.123	0.131
		VaR(Jump)	30.143	0.153	0.157
	2012/2013	VaR(Black-Scholes)	20.772	0.058	0.345
		VaR(Jump)	24.929	0.049	0.301
	2013/2014	VaR(Black-Scholes)	14.288	0.211	0.025
		VaR(Jump)	17.251	0.168	0.010

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