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International financial integration: Ramsey vs Solow

Philippe Darreau
University of Limoges

Francois Pigalle
University of Limoges

Abstract

In this didactical exercice we show that the long run welfare gains from international financial integration differ when using the Solow model vis-à-vis the Ramsey model. While the former predicts beneficial effects of financial integration on the wealth and consumption of a poor country borrower in the long run, the latter envisions no change in the borrower's wealth. Moreover, though the Ramsey model presumes an increase in consumption, it is less than what is predicted by the Solow model. We explain this result as follows. With Solow, debt disappears. As income increases with integration, and hence increase national savings, the initial debt transforms itself into national capital and thus there is no more interest burden in steady state. With Ramsey, however, initial debt never disappears. As interest rate and thus national savings decrease with integration, national ownership of capital does not increase and the poor country bears an eternal interest burden.

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Contact: Philippe Darreau - philippe.darreau@unilim.fr, Francois Pigalle - francois.pigalle@unilim.fr.
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1. Introduction

The neoclassical growth model is fundamental in studying the benefits of international financial integration, as in Obstfeld and Rogoff (1996). Recently, many articles have studied this issue using the model of Ramsey (1928), Cass (1965) and Koopmans (1965). For example, Gourinchas and Jeanne (2006), Gourinchas and Rey (2013), Alogoskoufis (2014, 2016), Boucekkine, Fabbri, Pintus (2016) use the so-called Ramsey model to show the weaknesses of capital market integration gains. Darreau and Pigalle (2016) show that this analysis could be illustrated easily with the Solow (1956) model with a constant savings rate. The Solow-Swan (1956) model is a simplification of the neoclassical model, whose core is the neoclassical production function. In a risk-free world, capital market integration promotes efficient allocation of factors of production under the hypothesis of concavity of the production function. As Henry (2007) points out, the Solow model is quite appropriate for studying financial integration in a neoclassical framework.

We show in this didactical framework that the choice of the model to study the effect of integration matters as it has radically different implications on long run welfare gains. In the same context as the rest of the literature, we study the effects of integration on a small open poor country, under the assumption that the rest of the world have already achieved their steady state. We shown that the Solow model predicts more favorable long run welfare gains from international financial integration than the Ramsey model. With Solow, the long-term wealth and consumption of the poor borrower country increase in the same way as they increase in convergence. With Ramsey, the wealth of the poor borrower does not change and though its consumption increases, it increases less than what is predicted in the Solow model. The explanation is as follows. With Solow, debt disappears, since as income increases with integration, and so is national savings, hence initial debt transforms itself into national capital and thus there is no more interest burden in steady state. With Ramsey, however, initial debt never disappears. As interest rate and thus national savings decrease with integration, national ownership of capital does not increase and the poor country bears an eternal interest burden.

The paper is structured as follows. Section 2 presents integration in the Ramsey model. Section 3 presents integration in the Solow model. Section 4 compares the two integration processes. Section 5 concludes.

2. Integration in the Ramsey model

We compare the process of convergence and integration of a "small poor country". At time zero, its capital per capita k_0 is lower than the rest of world k^* which is already at their steady state level, $k_0 < k^*$. Each country has the same parameters. The modeling approach using the Ramsey model is derived from the famous textbook of Barro and Sala-i-Martin (1995 chap 2). For each country, population and labor, N_t grows at the rate n : $N_t = e^{nt}$. The national representative agent maximizes its utility subject to its wealth accumulation constraint:

$$U_t = \int_{t=0}^{+\infty} e^{-\rho t} N_t \ln(c_t) dt \qquad \text{subject to} \qquad Da_t = (r_t - n)a_t + w_t - c_t \tag{1}$$

Where ρ is time preference, a is wealth per capita , r is interest rate, w is wage rate, and c is consumption per capita. For the sake of simplicity, we specify a log utility function. Our demonstration is independant of the parameters values and remains true for a CRRA function. The national representative producer maximizes his profit subject to its production constraint:

$$\Pi_t = Q_t - (r_t + \delta)K_t - w_t N_t \quad \text{with} \quad Q_t = K_t^\alpha N_t^{1-\alpha} \quad \text{or} \quad q_t = k_t^\alpha = f(k_t) \tag{2}$$

For simplicity's sake we retain a Cobb-Douglas function which has the neoclassical properties of concavity. Q is GDP, δ is capital depreciation rate, K is capital, q is GDP per capita and k is capital per capita.

2.1. Autarky and convergence

For a country under autarky, there is no exchange of capital with the rest of the world and hence national wealth consists only of national productive capital: $a_t = k_t, \forall t$.

The consumer problem solution is :

$$\frac{Dc_t}{c_t} = r_t - \rho \qquad \text{and} \qquad \lim_{t \to +\infty} a_t e^{(n-r_t)t} = 0 \tag{3}$$

and the producer problem solution is :

$$f'(k_t) = r_t + \delta \qquad \text{and} \qquad w_t = f(k_t) - f'(k_t)k_t \tag{4}$$

Equations (3) and (4) imply that consumption increases as long as $f'(k_t) > \rho + \delta$ or $k_t < k^*$. Moreover, equations (1) and (4) imply that capital increases as long as $c_t < f(k_t) - (n + \delta)k_t$.

Analyzing convergence, Barro and Sala-i-Martin (1995) show that the path of growth (near steady state) of consumption is:

$$c_t = c^* + (\rho - n + \beta)(k_0 - k_t)e^{-\beta t} \tag{5}$$

Where β is the speed of convergence[1].
At steady state, consumption and capital no longer grow, $Dc_t = 0$ and $Dk_t = 0$:

$$k^* = \left(\frac{\alpha}{\rho + \delta}\right)^{\frac{1}{1-\alpha}} \qquad \text{and} \qquad c^* = k^{*\alpha} - (n + \delta)k^* \tag{6}$$

In autarky, the small poor country will reach the level of capital and consumption of the rest of the world.

2.2. Integration

Under financial integration, the wealth of a country now consists of national capital and net holdings of foreign securities $a_t = k_t + e_t$. Where e_t is the net international investment position (NIIP) per capita. The variation of e_t is equal to the current account f_t, or trade balance z_t plus net international investment income $e_{t+1} - e_t = f_t = z_t + (r_t + \delta)e_t$. GNP per capita is $y_t = q_t + (r_t + \delta)e_t = w_t + (r_t + \delta)k_t + (r_t + \delta)e_t$. The world is assumed to be in steady state. Since time preference is the same for all countries in the world, world interest rate is $r^w = r^* = \rho$. Since for the small poor country $k_0 < k^*$, before integration, domestic interest rate is $r_0 > r^*$. Integration leads the small poor country to borrow abroad, hence $e_0 < 0$. Within the context of a small open economy, the level of steady state capital of the small open economy is determined instantaneously by the no-arbitrage condition. The interest rate is instantly determined at its steady state level $r_t = r^w = r^*$, which

1. See Barro & Sala-i-Martin (1995), $\beta = \frac{1}{2}\left(\sqrt{4(1-\alpha)(\delta+\rho)\left(-n + \frac{\delta+\rho}{\alpha} - \delta\right) + (\rho - n)^2} + (n - \rho)\right)$

is the same case for capital $k_t = k(r^w) = k^*$, production $q_t = q(r^w) = q^*$ and wage rate $w_t = w(r^w) = w^*$. Capital, production, factor prices are instantly constant at their steady state levels.

With regard to consumption, equation (3) implies that instantaneously $Dc_t = 0$ and thus consumption is constant, $c_t = \bar{c}$. Its level, however, remains to be determined. Since $r^*, w^*, \bar{c}$ are constant, the discounted intertemporal budget constraint of the consumer now becomes under integration : $\int_{t=0}^{\infty} e^{(n-r^*)t} \left(Da_t + (n - r^*)a_t\right) dt = \int_{t=0}^{\infty} e^{(n-r^*)t} \left(w^* - \bar{c}\right) dt = \frac{w^*-\bar{c}}{r^*-n}$. By integrating the left side $a_T e^{(n-r^*)T} - a_0 = \frac{w^*-\bar{c}}{r^*-n}$ and taking into account the transversality condition (3), we find $-a_0 = \frac{w^*-\bar{c}}{r^*-n}$. Solving $\bar{c}$, we obtain :

$$\bar{c} = w^* + (r^* - n)a_0 \tag{7}$$

At the time of integration, wealth is $a_0 = k^* + e_0 = k_0$. Consumption is equal to net human and financial wealth at time 0 [2]. Consumption in steady state is higher the greater the wealth is at the time of integration. History leaves traces in the event of financial integration. This is not the case, however, for convergence as described by equation (5). Figure 1 shows consumption time paths, for convergence (5) steady state (6) and integration (7). These time paths are plotted using the following calibration: $\alpha = 0.3, n = 0.01, \delta = 0.06, \rho = 0.04$. These correspond to the study by Gourinchas and Jeanne (2006), however the following propositions (1 to 4) are independent of the parameters values.

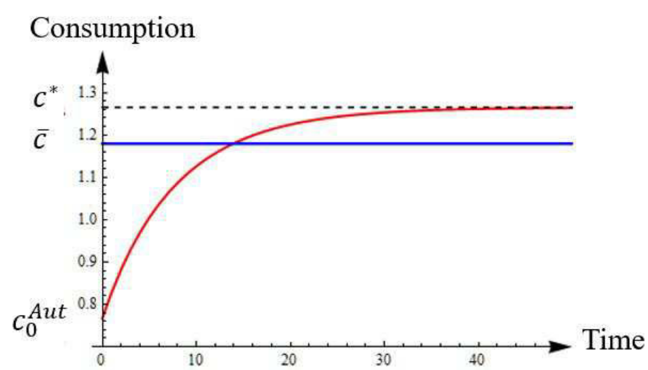

Figure 1 – Consumption time paths using the Ramsey model

2. To our knowledge, this result and the source of the following graph go back to Blanchard and Fischer (1989) pages 62 and 65.

Proposition 1 : *A small poor country with an integrated capital account will have less steady state consumption than it would without integration. For a small poor country ($k_0 < k^*$), steady state consumption under integration is $\bar{c} < c^*$.*

Proof : With integration : $\bar{c} = w^* + (r^* - n)k_0 = w^* + (r^* - n)k^* + (r^* - n)e_0$
Without integration : $c^* = q^* - (n+\delta)k^* = w^* + (r^* + \delta)k^* - (n+\delta)k^* = w^* + (r^* - n)k^*$
Thus $\bar{c} = c^* + (r^* - n)e_0$. Since by (3) $r^* > n$, and since for the small poor country $e_0 < 0$, we have $\bar{c} < c^*$.

The proof allows us to understand the reason for this result. Consumption under integration is less than steady state consumption under autarky because of the flow of interest to the rest of the world. Into $\bar{c} = c^* + (r^* - n)e_0$, the last term represents the net interest expense of the initial debt. It is also the "Lerner's burden", the surplus trade balance implied by the debt of the small poor country. Since current account $f = z + re$ is equal to the change in debt $De = ne$, we have $(r - n)e = -z$. With $e < 0$ we have a perpetual trade surplus $z > 0$. We can still write $\bar{c} = c^* - z$.

Proposition 2 : *A small poor country with an integrated capital account will have less steady state GNP than it would without integration. For a small poor country ($k_0 < k^*$), steady state GNP under integration is $\bar{y} < y^*$:*

Proof : With integration $\bar{y} = q^* + (r^* + \delta)e_0$. Without integration : $y^* = q^*$. Since for the small poor country $e_0 < 0$, we then have $\bar{y} < y^*$.

Integration with respect to convergence allows consumption and welfare gains over a period of time (15 years using our calibration) since GDP and consumption increase at the time of integration. Integration later implies eternal loss of welfare as imposed by the interest burden. This is one of the reasons highlighted by Gourinchas and Jeanne (2006) for the low integration gains, since the discounted sum of gains and losses remains positive but low. Alogoskoufis (2016) remarks a problem of time inconsistency: It's in the small poor country's interest to fail in order not to bear this infinite loss. The lender who anticipates this default can develop an adequate loan contract: Do not lend to the poor country (this explains the paradox of Feldstein and Horioka (1990) and the paradox of Lucas (1990)), or set up mechanisms of engagement and sanctions in case of default. Bussiere and Fratzscher (2008) empirically show that financial openness has led to short-run gain and long-run pain.

Proposition 3 : *In the Ramsey model, for a small poor country financial integration has no effect on the steady state GDP. It only accelerates the convergence (which becomes instantaneous). But the initial small wealth of the country determines a level of steady state of consumption and GNP which is lower than they would have been without financial integration. History has therefore left traces.*

3. Integration in the Solow model

Gains from integration in the Solow model were shown by Gale (1974). We propose a graphic presentation of this model identical to Darreau and Pigalle (2016) reprinted from Schröder (1972). The same production function (2) is used as in the previous section. The demand side is replaced by a Keynesian savings hypothesis on income and equation (1) is replaced by the wealth accumulation constraint:

$$Da_t = s(w_t + (r_t + \delta)a_t) - (n + \delta)a_t \tag{8}$$

Where s is the constant savings rate of the consumer, which is the same for all countries in the world, and $y_t = w_t + (r_t + \delta)a_t$ is the income of the national representative agent.

3.1. Autarky and convergence

Consider Figure 2, a small open economy with inital wealth k_0 and the rest of the world at steady state k^*. The small economy can either remain in autarky and converge "naturally" to its steady state, or financially integrate with the rest of the world and transit to its steady state as a small open economy. If it does not integrate $(a_t = k_t, \forall t)$, the country will converge towards the steady state k^*. Its capital will increase gradually from k_0 to k^*. Its consumption will increase from BD to FG while its interest rate r_0, will gradually decrease until it reaches r^*.

3.2. Immediate effects of integration

If it is integrated $(a_t = k_t + e_t)$, factor prices are immediately r^* and w^*. According to the hypothesis of a small open economy, the level of steady state capital of the small open economy is determined instantaneously by the no-arbitrage condition. The straight line $y = w^* + (r^* + \delta)a$, tangent to $q = f(k)$ in F has the slope $r^* + \delta$

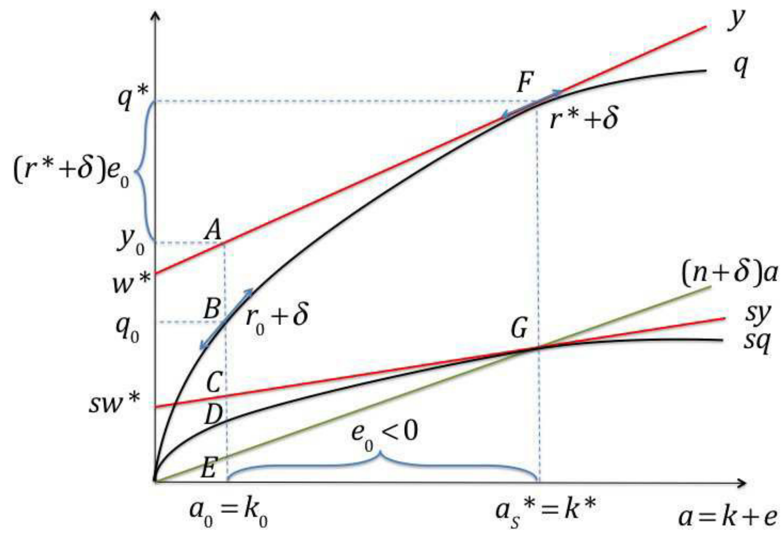

Figure 2 – Integration and convergence in the Solow model

and ordinate w^*. It represents national income. The straight line sy, tangent to sq in G has the slope $s(r^* + \delta)$ and ordinate sw^*. It represents national savings. As $r_0 > r^*$, the small economy borrows $e_0 < 0$, and its capital immediately becomes k^*. Its GDP is immediately q^*, and its GNP is equal to y_0. The country would pay $-(r^* + \delta)e_0$ in interest. The country benefits from higher wages but suffers from a lower interest rate. The immediate effect on income, however, is positive since $y_0 > q_0$. This result (Gale (1974)) is due to the concavity of the neoclassical production function. Fig.2 graphically shows this result. Consumption increases immediately from BD to AC.

3.3. Effects of integration on long-run steady state

The level of steady state capital of the world k^* (and of the small open economy) is determined by the dynamic equation $Dk_t = sk_t^\alpha - (n + \delta)k_t$ whose solution is :

$$k^* = \left(\frac{s}{n+\delta}\right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad c^* = k^{*\alpha} - (n+\delta)k^* \tag{9}$$

The Solow steady state wealth for the poor small country under integration a_S^* is determined by its dynamic equation $Da_t = s(w^* + (r^* + \delta)a_t) - (n + \delta)a_t = 0$

whose solution is : $a_S^* = sw^*/((n+\delta) - s(r^*+\delta))$. Replacing $w^* = (1-\alpha)k^{*\alpha}$ and $r^* + \delta = \alpha k^{*\alpha-1}$ we find :

$$a_S^* = k^* \tag{10}$$

The small poor country thus converges towards the steady state, which it would have achieved nevertheless even without integration. In particular, its wealth is equal to what it would have had without integration $a_S^* = k^*$, its debt is zero $e^* = 0$, its consumption converges to c^* (FG on figure 2), and its GNP converges to $y^* = q^*$.

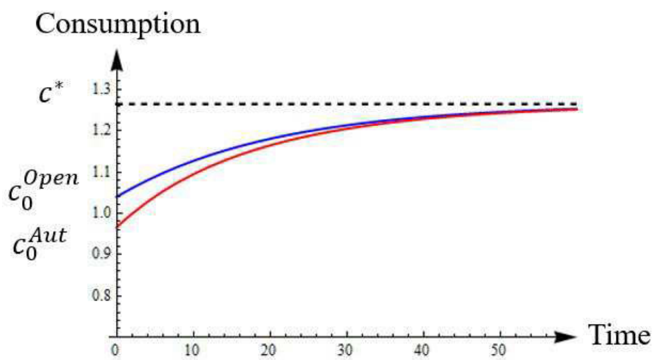

Figure 3 – Consumption time paths using the Solow model

To summarize, consumption experiences whith integration an initial jump and then converges towards its steady state value. Figure 3 shows the time path of consumption using the Solow model with and without financial integration [3].

Proposition 4: *Integration in the Solow model has no effect on the steady state value of GDP. The initial wealth has no effect on the steady state of consumption and GNP. History hence does not leave traces.*

4. Comparison of the two processes

The major difference between the predictions of the two models lies in the determination of the long-term level of consumption, GNP and wealth under the

3. These time paths are plotted for the same calibration as before. To obtain the same levels for c* we took: $s = \alpha((n+\delta)/(\rho+\delta)) = 0.3((0.01+0.06)/(0.04+0.06)) = 0.21$. If one compares this time paths with those of Fig.1 : At time zero, autarky consumption level in the Ramsey model is obviously lower than that of the Solow model and convergence is quicker in the Ramsey model.

integration regime. They are weaker in the Ramsey model than in the Solow model. This difference in predictions is evidently due to differences in assumptions. The savings rate is constant in the Solow model while the savings rate depends on the interest rate in the Ramsey model. Integration reduces this interest rate instantaneously for the small poor country and thus the incentive to save for the national representative agent of the small poor country.

In the Solow model, at the time of integration, the savings of the national representative agent increase by $sy(k_0) - sq(k_0)$ (shown in Figure 2 from ED to EC) as the income increases. This increase in savings makes it possible for nationals to increase their capital holdings and hence to reduce their debt. As is true throughout the convergence process (savings increase of the CDG triangle) the external debt disappears gradually. Debt is null at steady state as well as interest burden. This explains why consumption reaches its steady state c^*.

In the Ramsey model, at the time of integration, the savings of the national representative agent is: $y_0 - c = (w^* + (r^* + \delta)a_0) - (w^* + (r^* - n)a_0) = (n + \delta)a_0$. This savings allows just the required investment so that the wealth per capita remains constant at its steady state value. As this is true from period to period, the wealth held by the nationals remains equal to $a_0 = k^* + e_0 = k_0$. Since there is in the country, from the moment of integration, capital equal to k^*, the difference ($e_0 = k_0 - k^*$), which is debt held by the rest of the world, remains from period to period. The Ramsey steady state wealth for the poor small country in integration is:

$$a_R^* = k^* + e_0 \tag{11}$$

The debt of the poor country never disappears, neither does its interest burden, which, as we have seen, explains that consumption is below its steady state level. The classic representation of the Solow model in Figure 2 above can be compared with the classic representation of the Ramsey model in Figure 4.

If it does not integrate, starting from c_0, k_0, the country will converge towards the steady state c^*, k^* at point J, where $Dk = 0$ and $Dc = 0$. If it is integrated, the country benefits immediately from r^* and w^*. The straight line $c = w^* + (r^* - n)a$ with the slope $(r^* + \delta) - (n + \delta)$ is tangent to $Dk = 0$ in J. It has been shown that this line represents the bunch of points where $Dc = 0$ under integration and where $Da = w^* + (r^* - n)a - c = 0$. The steady state $\bar{c}, a_R^*$ is reached immediately at point I. The figure finally shows that e_0 is the steady state debt.

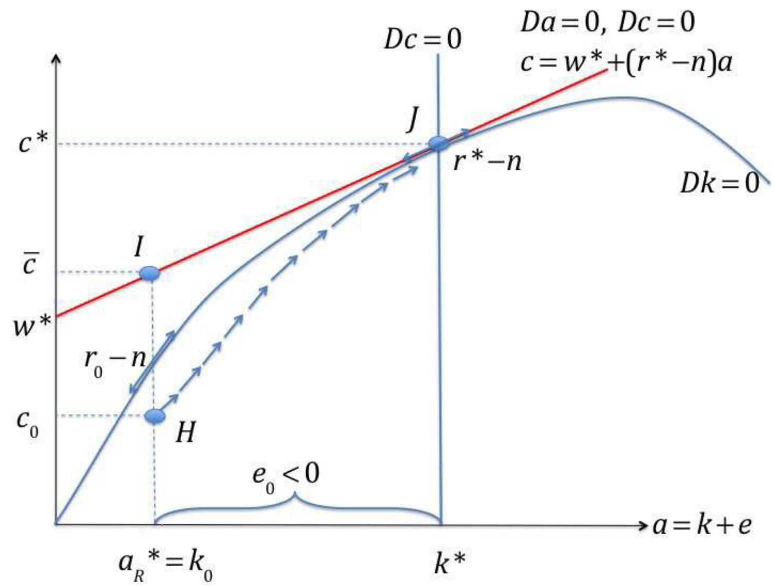

Figure 4 – Integration and convergence in the Ramsey model

5. Conclusion

In this paper, the impact of financial integration was compared between the Ramsey model which assumes that savings are sensitive to interest rates and the Solow model which supposes a constant savings rate. In the Ramsey model integration implies an instantaneous adjustment of all the variables. In the Solow model, r, w, k, q have an instantaneous fit but wealth and consumption increase monotonically in a transitional dynamics. We have shown that integration is a best thing for long run, if it does not reduce the incentive to save. This is the case in the Solow model, but not in the Ramsey model. It is strange that the Ramsey model (purely neoclassical) is, in long run, less favorable to financial integration (to free trade of capital) than the Solow model (impregnated with Keynesianism in savings behavior).

This paper makes it possible to distinguish the two reasons for the elusive gains from international financial integration. Integration does the job that convergence would have done anyway. Integration reduces the incentive to save, and the borrower cannot get rich. The first reason is explained by the Solow model, and the second by the Ramsey model.

From a theoretical point of view the Ramsey model implies by construction maximum well-being on the whole of the consumption time path [4]. The absence of discounting in the Solow model implies a saving effort at the beginning of the time path which is rewarded by higher consumption at the end of the time path. From an empirical point of view, the question regarding which of the two models should be chosen depends on what was the immediate effect of financial integration on saving. The savings glut speaks up for Solow model.

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