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Construction of a dividend index with all the distributed revenues

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Abstract

The existence of concentration in specific months for dividends distribution produces sensible irregularities in its times series. This may happen because some revenues distributed among shareholders are not included (like capital gains and profits). We propose a methodology to construct dividend indices including all the remunerations that shareholders may receive. Using consolidated data from the System of National Accounts and the Central Bank, we obtain well-behaved time series for dividends. We apply the methodology to obtain a dividend index for Brazil in order to test the existence of three types of rational bubbles: explosive bubbles, periodically collapsing bubbles and intrinsic bubbles. The tests indicated that it is not possible to rule out the presence of explosive bubbles and detected evidences of the presence of periodically collapsing bubbles in the Brazilian stock market. However, no evidence of the presence of intrinsic bubbles was found.

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1. Introduction

Stocks with dividends distribution are used as examples where rational bubbles may emerge. Taking the dividends as the fundamentals for the definition of the stock price, the fundamental value is defined as the present value of all the expected future dividends. Any observed deviation of the spot price from the fundamental value is considered a rational bubble in the stock price.

The dynamics of the dividends is important for its expected value calculation. However, a single stock may have sparse dates of dividends distribution and this may bring strong irregularities to the time series. Even if we consider a portfolio of stocks with dividends in order to make a dividend index, all (or almost all) the stocks may concentrate the distribution of those benefits in specifics dates of the year.

In addition, some benefits provided by the stocks may not be in form of dividends; rather they are registered as net profits or capital gains. Thus, all the benefits should be included in the construction of an indicator of the revenues provided by the stocks.

For those reasons, this work proposes a methodology for the construction of a dividend index where dividends and other benefits are distributed among the shareholders. The methodology uses consolidated data from the National Accounts System and from the Central Bank. As we will see, the time series obtained is smoother and more steady, since all the payments are included. In particular, we can test the existence of rational bubbles in the stock markets more confidently.

Rational bubbles in the stock market were widely tested in several works. In Blanchard (1979) and Blanchard and Watson (1982) it is shown that rational behavior and rational expectations can not preclude the existence of deviations from fundamentals. The existence of bubbles can affect the real sector of the economy and their burst could cause failure of financial institutions and downturn in economic activity (Aliber and Kindleberger, 2015).To obtain evidence against existence of explosive bubbles in the stock market, even when some fundamentals are unobservable, Diba and Grossman (1988) proposed an empirical test based in the long term behavior of the spot price and the dividends distributed. In Evans (1991) it is questioned that cointegration methods cannot detect the presence of bubbles with periodically collapsing behavior; then Bohl (2003) provided some tests to detect those types of bubbles. Finally, Froot and Obstfeld (1991) introduced a new type of rational bubbles: the intrinsic bubbles. They theoretically showed the possibility of their existence and provided a simple tests to detect them.

Regarding testing bubbles in the stock market in Brazil, we have some works in the literature. In Nunes and Silva (2009) it is used the monthly Ibovespa and dividends series provided by Standard & Poor's in testing for the presence of rational bubbles in stock prices between July 1994 and December 2006. In turn, Fernandes and Medeiros (2009), Queiroz et al. (2011) and Scolari (2011) built monthly dividend indices for the Brazilian market based on the theoretical portfolio and the methodology of the Ibovespa. All of them highlighted that the dividend indices show great instability between the years 1994 and 1999, with no apparent correspondence with prices or other events. In particular, Queiroz et al. (2011) found that there were high concentrations in the participation of certain shares, because the dividends paid by a single share roughly represented 90% of the index in some months.

Including this introduction, the paper is structured into five sections. Section 2 presents models of rational bubbles, Section 3 describes the construction of the dividend index, Section 4 presents the results of tests to detect bubbles using the provided index, and finally, Section 5 resumes the conclusions.

2. Three types of rational bubbles

A stock is defined by two processes, the price process $(P_t)_{t\geq 0}$ and the dividend process $(D_t)_{t\geq 0}$. Both define the rate of return process $(r_{t,t+1})_{t\geq 0}$ given by $r_{t,t+1} = (P_{t+1} + D_t - P_t)/P_t$. Supposing that the rate of return process is stationary, we have:

$$r = E_t[r_{t,t+1}] = \frac{E_t[P_{t+1}] + D_t - P_t}{P_t} \Rightarrow P_t = aE_t[P_{t+1}] + aD_t, \tag{1}$$

where $a = (1+r)^{-1}$ is the associated discount factor. Solving equation (1) forward we obtain the classical decomposition of prices $P_t = P_t^f + B_t$, the fundamental value P_t^f plus the bubble size B_t, where:

$$P_t^f \equiv \sum_{i=0}^{+\infty} a^{j+1} E_t[D_{t+j}] \text{ and } B_t \equiv \lim_{T\to+\infty} a^T E_t[P_{t+T}]. \tag{2}$$

It is important to notice that the determination of P_t^f depends on the accurate definition of the process $(D_t)_{t\geq 0}$ and the associated discount factor a. On the other hand, the bubble process $(B_t)_{t\geq 0}$ satisfies the explosive equation:

$$E_t[B_{t+1}] = a^{-1} B_t; \tag{3}$$

showing that, once the bubble appears ($B_t \neq 0$) it is expected to grow in time at rate a^{-1}.

Thus, the presence of a bubble satisfying (3) causes divergence between the stock price and the fundamental value. Therefore, cointegration tests can be used to rule out the presence of *explosive bubbles.* Under the hypothesis of the log-dividends process follows a random walk dynamics, the stock prices and dividends must exhibit a long-term equilibrium relationship. In that case, the presence of explosive bubbles is rejected. On the other hand, the absence of cointegration does not necessarily imply the existence bubbles, since that absence may be caused by other factors, as non-stationarity of unobservable variables in market fundamentals (Diba and Grossman, 1988).

A second type of bubbles can be tested: *the periodically collapsing bubbles.* These sort of bubbles were proposed by Evans (1991) and they follow the following dynamics:

$$B_{t+1} = \begin{cases} a^{-1} B_t u_{t+1}; & \text{if } B_t \leq \alpha \\ (\delta + \pi^{-1} a^{-1} \theta_{t+1}(B_t - a\delta)) u_{t+1}; & \text{if } B_t > \alpha \end{cases} \tag{4}$$

where δ and α are parameters satisfying $0 < \delta < a^{-1}\alpha$; $(u_{t+1})_t$ is a positive, independent and identically distributed exogenous random process, with $E_t[u_{t+1}] = 1$; $(\theta_{t+1})_t$ is an exogenous, independent and identically distributed Bernoulli process assuming value 1 with probability π and value 0 with probability $1-\pi$. The intuition behind the process (4) is as follows: while $B_t \leq \alpha$, the bubble grows at a mean rate a^{-1} and the probability of collapse is null. However, if $B_t > \alpha$, the bubble begins to grow at a higher rate and the new mean growth rate will be $a^{-1}\pi^{-1}$. In this case, the bubble may collapse with probability $1-\pi$. Once this occurs B_t does not vanish, rather it takes a (mean) value δ and then the process starts again (see Evans, 1991 for details).

To test periodically collapsing bubbles, Bohl (2003) proposed the threshold autoregressive model (TAR) and the momentum threshold autoregressive model (MTAR). First, it is estimated the following regression and the residuals calculated:

$$P_t = \hat{\beta}_0 + \hat{\beta}_1 D_t + \mu_t$$

The threshold for the change in the dynamics is adjusted by the following regression:

$$\Delta\mu_t = I_t\rho_1\mu_{t-1} + (1 - I_t)\rho_2\mu_{t-1} + \sum_{i=1}^{p-1}\gamma_i\Delta\mu_{t-i} + \epsilon_t$$

in that equation, the indicator function I_t is defined by:

$$I_t = \begin{cases} 1; & \text{if } \mu_{t-1} \geq \tau \\ 0; & \text{if } \mu_{t-1} < \tau \end{cases} \quad \text{or } I_t = \begin{cases} 1; & \text{if } \Delta\mu_{t-1} \geq \tau \\ 0; & \text{if } \Delta\mu_{t-1} < \tau \end{cases}$$

either for model TAR or MATR respectively. The parameter τ assumes the value zero in many economic applications. However Chan (1993) proposed a methodology to estimate it. Using that estimated τ we will have the models TAR consistent (TARC) or MTAR consistent (MTARC). Details on the choice of those values can be seen in Enders and Siklos (2001).

Two tests of hypotheses are performed in sequence. First, we verify the null hypothesis of no cointegration $H_0 : \rho_1 = 0$ and $\rho_2 = 0$, using the $t-$statistic, and $H_0 : \rho_1 = \rho_2 = 0$, using the $F-$statistic (that we call $\Phi-$statistic as in Enders and Siklos (2001)). The critical values of t and Φ are provided by Enders and Siklos (2001). If the null hypothesis is rejected, a second test is performed, whose null hypothesis is the presence of symmetry $H_0 : \rho_1 = \rho_2$ and it is tested using the usual $F-$statistic and we call it Φ^{sym}. If the null hypothesis is not rejected, a second test is performed: if the estimated coefficient $\hat{\rho}_1$ is statistically significant, negative and greater in absolute terms than the estimated $\hat{\rho}_2$, the hypothesis of symmetric adjustment is rejected, then there is evidence of the presence of periodically collapsing bubbles.

The third type of bubbles we are going to analyze are the *intrinsic bubbles* presented by Froot and Obstfeld (1991). They show how the fundamentals may feed deviations of stock prices from them. Suppose that the logarithm of the dividends ($d_t = \ln D_t$) follows a random walk with drift, namely, $d_{t+1} = \mu + d_t + \xi_{t+1}$, where $\xi_t \sim N(0, \sigma^2)$. Substituting this in (2) it results:

$$P_t^f = kD_t \text{ and } B_t = B(D_t) = cD_t^\lambda,$$

where $k = (1+r-\exp\{\mu+\sigma^2/2\})^{-1}$ and λ is the positive root of $(\sigma^2/2)\lambda^2+\mu\lambda+\ln(1+r) = 0$. Thus, the spot price of the stock and the price-dividend rate can be written:

$$P_t = kD_t + cD_t^\lambda \text{ and } \frac{P_t}{D_t} = k + cD_t^{\lambda-1}.$$

The significance of c in the estimation of the last equation will test the existence of intrinsic bubbles.

3. Construction of a dividend index

In this section we provide a methodology to construct a dividend index that embodies not only the distributed dividends, but also all the other revenues that stockholders receive by owning shares. The methodology is described using as illustration the Brazilian economy, but it can be used for any other economy since the sources of data (National Accounts System and Central Bank) are widely available.

We consider two time series: the dividends provided by the IBGE[1] in the Integrated Economic Accounts and the series "Income - profits and dividends excluding reinvested

[1] Instituto Brasileiro de Geografia e Estatística, Brazilian Institute of Geography and Statistics, http://www.ibge.gov.br/english/

earnings - total (debit)" provided by the Brazilian Central Bank. We will call them IBGE and BCB time series respectively.

In the IBGE times series we have all the incomes distributed by enterprises as dividends and other payoffs to the shareholders. They are estimated indirectly. For non-financial private enterprises, it is considered 25% of the former year net profit plus the excess withdrawal of partners and owners, the participation of management and the operating surplus of the company. For financial institutions, it is considered 25% of the value of the former year intermediate account savings. Finally, for non-financial public enterprises, the values of the financial report of the former year are considered. The choice of the percentage "25%" is due to the Brazil (1976) Act 6404 revisited in the Brazil (2001) Act 10303, which regulates the minimum percentage to be distributed by the enterprises to the shareholders. The analysis of Martins (2010) shows that most of the enterprises in Brazil approximately distribute that percentage.

Then, the dividend index time series constructed using the IBGE data is:

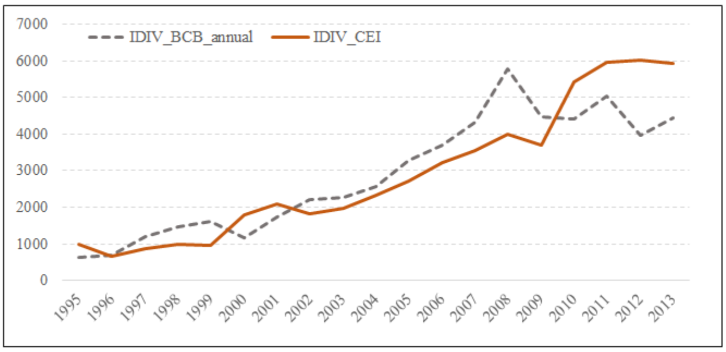
$$IDIV_t^{CEI} = 1000\frac{D_t}{D_0} \tag{5}$$

where D_t is the total dividend reported by the IBGE in year $t = 0, 1, \cdots, 18$ (from 1995 to 2013).[2] Dividends were deflated using the National Consumer Price Index (IPCA).

The other time series is constructed from Central Bank data of Brazil. Its advantage with respect to the former index is that its frequency is monthly. Thus, with a short horizon of time, the $IDIV^{CEI}$ may be not suitable for robust analysis. Anyway, we perform the analysis with both indices. Thus, we elaborated another index using the BCB records of the payments to foreign investors due to investments in Brazil (call it $IDIV^{BCB}$). They are reported as "Income - profits and dividends excluding reinvested earnings - total (debit)". Since those payments only represent a portion of all dividends and gains distributed, we perform some tests to corroborate if this is an appropriate proxy for the total dividends distribution in the country. The values are in millions of dollars, so we used the floating Exchange rate - United States dollar series provided by Central Bank to transform the values to the Brazilian currency (Real) and then the values were deflated by the IPCA.

The Figure 1 shows the evolution of both annual indices, $IDIV^{CEI}$ and $IDIV^{BCB}$. The Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) Unit Root tests point out

Figure 1: Comparing both annual indexes: $IDIV^{CEI}$ and $IDIV^{BCB}$



the presence of unit root in the time series in level and reject that hypothesis for their

[2]The term CEI is the acronym in Portuguese of Integrated Economic Accounts.

first differences. The Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) test rejects the stationarity of those time series in level but not in their first differences.

Despite the small size of the data, the time series cover a period of almost two decades, therefore, we perform the Johansen cointegration test to assess the existence of a long term relationship between them. The hypothesis of absence of cointegration is rejected using both the statistical tests λ-trace and λ-max. The values for the two statistics were 20.907 and 20.906 respectively, and the critical value at 5%, 12.32 and 11.22, respectively. Therefore, we can use of the BCB as a proxy for dividend distribution in Brazil.

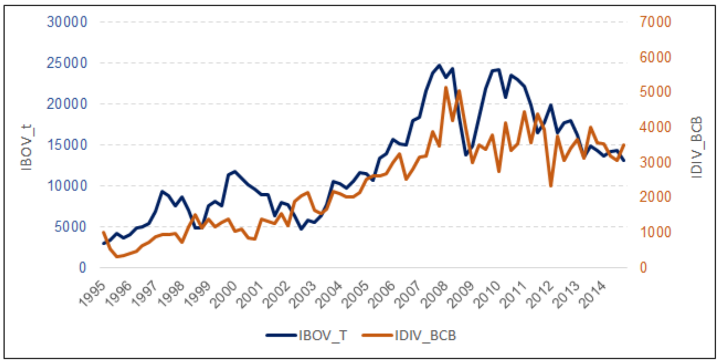
The BCB time series index is constructed from quarterly data for the period 1995 to 2014. After applying a seasonal adjustment using the X12-ARIMA methodology, the index is defined by:

$$IDIV_t^{BCB} = 1000 \frac{D_t}{D_0} \tag{6}$$

where D_t is the seasonally adjusted value of the Income - profits and dividends excluding reinvested earnings - total (debit) reported by the Central Bank.

As a proxy for stock prices, we used the stock market index of Brazil, the Ibovespa. The quarterly Ibovespa time series was deflated by the IPCA. We denote that time series by $IBOV_t$. Figure 2 shows both quarterly indices, the $IBOV_t$ and the $IDIV_t^{BCB}$ in different scales to perceive the tendencies.

Figure 2: Stock price index and dividend index in Brazil (quarterly)



4. Testing rational bubbles

In this section we will perform the tests of existence of the three types of bubbles described in Section 2. By the reasons presented in Section 3 we will only consider as dividend index that constructed from the Central Bank data. However, for countries with longer data histories, it will be preferred the use of the data from the National Account System to obtain more robust results.

To test for explosive bubbles, first, we performed the ADF, PP and KPSS tests and estimated the Johansen and Engle-Granger models. The results, in Table 1, show that all the series are non-stationary in levels; however, they are stationary in the first difference. In the sequence, we estimate using ordinary least squares, the possible long term relationship between dividends and prices:

$$IBOV_t = \beta_0 + \beta_1 IDIV_t^{BCB} + u_t.$$

The test of unit root applied to the residuals u_t does not reject the null hypothesis (the t statistic is -3.18 and the 5% critical value reported by Mackinnon, 2010 is -3.14). Thus,

Table 1: ADF, PP and KPSS tests: $IBOV$ and $IDIV^{BCB}$

	Models	ADF*		PP*		KPSS**	
		Level	1st Dif	Level	1st Dif	Level	1st Dif
IBOV_t	Constant and trend	-2.07	-7.54	-1.64	-7.46	0.34	0.06
	Constant	-1.94	-7.50	-1.86	-7.45	0.92	0.13
	None	-0.24	-7.53	-0.30	-7.47	-	-
IDIV_BCB	Constant and trend	-1.59	-6.28	-5.13	-16.81	0.16	0.04
	Constant	-1.38	-6.23	-1.49	-16.68	1.13	0.04
	None	0.69	-6.00	0.04	-16.30	-	-

* Critical values ADF and PP tests for each model respectively, series in level: -3.47,-2.89 and -1.95.
** Critical values KPSS test for each model respectively: 0.15 and 0.46

the series are not cointegrated. To corroborate the no-cointegration we also apply the Johansen test and the hypothesis of no cointegration is not rejected by any criteria, as Table 2 reports.

Table 2 - Johansen's test: $IBOV$ and $IDIV^{BCB}$

Number of cointegration vectors	λ trace	Critical Values (5%)	λ max	Critical values (5%)
None	11.39	15.49	9.16	14.26
At most 1	2.23	3.84	2.23	3.84

Based on the results of the cointegration tests, there is not a long-run equilibrium relationship between stock prices and dividends. Therefore, the presence of explosive bubbles in the Brazilian stock market can not be ruled out.

To test for the existence of periodically collapsing bubbles we use the TAR, MTAR, TARC and MTARC models described in Section 2. The critical values for the statistics of those models were tabulated by Ender and Siklos (2001).

Then, using the $IBOV$ and $IDIV^{BCB}$ series we estimate the four models. In Table 3 we present the estimated parameters and important statistics. The $t-$statistics are not reported, since all the parameters, but ρ_2 in MTAR and MTARC, were significant. In all models, the null hypothesis of no cointegration is rejected at a significance level of 5%, since Φ is greater than its critical value. The MTAR and MTARC models rejected the symmetric adjustment hypothesis, since Φ^{sym} is greater than its critical value. The analysis show that the coefficient $\hat{\rho}_1$ is negative and greater than $\hat{\rho}_2$ in absolute terms. Thus there is evidence of periodically collapsing bubbles.

Table 3: Testing for periodically collapsing bubbles: $IBOV$ and $IDIV^{BCB}$ quarterly series

	TAR	MTAR	TARC	MTARC
$\hat{\rho}_1$	-0.55	-0.59	-0.56	-0.69
$\hat{\rho}_2$	-0.35	**-0.27**	-0.34	**-0.17**
Φ *	12.42	12.87	12.55	16.16
Φ – crit (5%)	5.98	6.51	6.95	6.78
Φ^{sym} **	1.05	5.04	1.26	7.54
Φ^{sym} – crit	4.0	4.0	4.0	4.0

* Φ statistics used to test the null hypothesis of cointegration absence.
** Φ^{sym} statistics used to test the null hypothesis of symmetric adjustment.
In **bold** script are the non-significant parameters.

Finally, we will test for the existence of intrinsic bubbles in the Brazilian stock market. The first step is to verify that the logarithm of the dividend index $IDIV^{BCB}$ follows a random walk. Applying the ADF, PP and KPSS tests to that time series we obtain that the series has a unit root in level but it is stationary in its first difference. After that routine tests, we proceed to estimate the model $d_{t+1} = \mu + d_t + \xi_{t+1}$ to obtain $\hat{\mu} = 0.0$ and $\hat{\sigma}^2 = 0.05$. We use as the rate of return r the average rate of return of Ibovespa to calculate $\hat{k} = (1 + r - \exp\{\hat{\mu} + \hat{\sigma}^2/2\})^{-1} = 8.10$ and the positive root of $(\hat{\sigma}^2/2)\lambda^2 + \hat{\mu}\lambda + \ln(1 + r) = 0$ is $\lambda = 2.31$. Hence, we estimate the constant c in the following regression:

$$\frac{P_t}{D_t} = 8.10 + cD_t^{1.31} + \nu_t.$$

As result, we found a non significant coefficient c, so the intrinsic bubbles are precluded.

Alternatively, we may jointly estimate the non-linear regression:

$$\frac{P_t}{D_t} = k + cD_t^{\lambda-1} + \nu_t.$$

this may be more appropriate if there is not confidence in the estimation of the parameters μ or σ^2 or even if the value of r does not seem appropriate. The estimated residuals of the regression above were autoregressive, then we performed new estimations using the Maximum Likelihood method with autoregressive errors, as shown by Davinson and Mackinnon (2003). The results of the estimation are in Table 4. Positive, but not

Table 4: Estimation of $\frac{P_t}{D_t} = k + cD_t^{\lambda-1} + \nu_t$

	Estimated Parameter	Standard error
$\hat{k}$	5.80	2.50
$\hat{c}$	0.16	0.21
$\hat{\lambda} - 1$	0.001	0.01

significant values of c were found, thus the hypothesis of the existence of intrinsic bubbles in the Brazilian stock market is rejected.

Therefore, we can summarize the results as follows: using the new methodology of constructing a dividend index, which includes all the revenues distributed to shareholders, the Brazilian stock market does not preclude the existence of explosive bubbles and there is evidence of the presence of periodically collapsing bubbles in the stock market price. Finally, there is not evidence of existence of intrinsic bubbles in those prices. All those conclusions are true for the period 1995 to 2014.

5. Conclusions

Conventionally, bubbles in prices are associated to strong price increases followed by a sharp drop. Properly speaking, they occur when asset prices deviates from their fundamentals. The presence of rational bubbles in the stock market can be detected by analyzing the relationship between stock prices and its dividends, hence the accurate selection of time series of dividends is important to analyze whether the bubbles exist or not.

The objective of this study was to propose the construction of a dividend index which is more suitable as a *proxy* for that fundamental. The motivation was the observation that most of the dividend indices do not include all the payments received by the owners of stocks, so some fundamentals could not be included in the *proxy*. In addition, the time series for pure dividends may exhibit strong irregularities, since dividends distributions are usually concentrated in some months of the year. For those reasons we adopt data for dividends and withdrawals reported in the National Accounts System of the countries as well as in their Central Banks.

We illustrate the construction of those indices for the case of the Brazilian stock market and used it to verify the presence of rational bubbles in it. Specifically, two new dividend indices were constructed from data provided by the IBGE in its Integrated Economic Accounts ($IDIV^{CEI}$) and by the Brazilian Central ($IDIV^{BCB}$) and we tested for existence of price bubbles in the Brazilian stock market.

We run the tests for both indices, however for the sake of saving space in the paper, we only report the results for $IDIV^{BCB}$. Notwithstanding, in this conclusions we report both results in the following last table:

Table 5: Tests of bubbles using the dividend indexes $IDIV^{CEI}$ and $IDIV^{BCB}$

Bubble type	**Period**	**Dividend index**	**Existence of Bubbles**
Explosive bubbles			Not rejected
Intrinsic bubbles	1995-2013	IDIV_CEI	Rejected
Periodically collapsing bubbles			Rejected
Explosive bubbles			Not rejected
Intrinsic bubbles	1995-2014	IDIV_BCB	Rejected
Periodically collapsing bubbles			Accepted

As we can see, when using any of the indices, the tests do not reject the existence of explosive bubbles in stock prices; however, the deviations from fundamentals may due to other variables rather than bubbles. Intrinsic bubbles are rejected when we use any of the indices. Since the methodology for both tests rely on the long run behavior of the model, we are confident with both coinciding results. However, for the case of periodically collapsing bubbles, it is necessary to have more short run information of index variation. Because the index $IDIV^{CEI}$ only have annual observations and we only have 18 observations for it, we are more confident to the test results using the index $IDIV^{BCB}$, which has quarterly information for a (little) longer period.

In comparing our results to others in the Brazilian literature about existence of bubbles (Fernandes and Medeiros, 2009, Nunes and Silva, 2009, Queiroz et al., 2011 and Scolari, 2011), we can find some coincident and some opposite results. However, we can say that a more precise concept of dividends are stated in this study, which goes in the direction of more accurate specification of fundamentals in the stock markets.

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