Economics Bulletin

Volume 37, Issue 2

Weak Scale Effects in Overlapping Generations Economy

Gilad Sorek
Economics Department Auburn University

Bharat Diwakar
Auburn University

Abstract

We show how the two alternative saving motives - life-cycle consumption smoothing and parental bequests - determine the relation between population growth and R&D-based economic growth, i.e. the sign of the weak scale effect. We take a textbook R&D-based growth model of infinitely lived agents with no life-cycle saving motive and re-analyze it in the Overlapping Generations (OLG) framework, which incorporates both life-cycle and bequest saving motives. We decompose the effect of each saving motive on the sign of the weak scale effect and show that in the presence of both saving motives it is ambiguous in general.

We thank Pietro Peretto for helpful comments.
Citation: Gilad Sorek and Bharat Diwakar, (2017) "Weak Scale Effects in Overlapping Generations Economy", *Economics Bulletin*, Volume 37, Issue 2, pages 962-969
Contact: Gilad Sorek - gms0014@auburn.edu, Bharat Diwakar - bzd0013@tigermail.auburn.edu.
Submitted: January 13, 2017. **Published:** May 04, 2017.

1 Introduction

The second and third generations of R&D-based growth models were criticized for presenting a positive relation between population growth and economic prosperity, i.e. a "weak scale effect", which does not fit the empirical findings of an ambiguous, possibly non-monotonic, relation between these variables[1]. This literature, however, has focused almost exclusively on the analysis of infinitely lived homogenous agents. We study the implications of this demographic structure for the presence of the weak scale effects, through a comparative analysis of the Overlapping Generations (OLG) model of finitely lived agents[2].

The two canonical demographic structures of the macroeconomic workhorse models imply different incentives for saving. The infinitely lived agents are assumed to share their assets (patent ownership in the current context) with their offspring. They fully internalize this into their saving decisions as they maximize the per-capita or aggregate lifetime utility of their dynasty members. Therefore, in this framework savings involve bequests, but they lack a life-cycle saving motive as workers' labor supply does not change with age[3].

By contrast, in the OLG framework saving is aimed to smooth consumption over a finite lifetime, which spans from working years to retirement period, and there are no intergenerational bequests. Hence, in this framework saving is motivated purely by life-cycle considerations. Clearly, the exclusive presentation of each saving motive in its corresponding demographic structure is unrealistically extreme[4].

Our analysis decomposes the implications of the two saving motives for the presence of weak scale effect. First, we show that in the absence of bequest saving-motive, the sign of the *weak scale effect* in the OLG economy depends solely on the degree of intertemporal elasticity of substitution (*IES*). Then, we show how when both saving motives are active, the sign of the *weak effect depends* also on the relative strength of parents' utility from bequest *vs.* utility from their own consumption during retirement.

Our results contribute to a recent line of modified R&D-based growth model with infinitely lived agents, which aimed at aligning the role of population growth in R&D-based growth theory with the empirical evidence[5].

Unlike the present work, these modified models introduce human capital as a productive input in the R&D sector, thereby forming a tension between a positive effect of population growth on saving in the presence of dynastic altruism and its negative (diluting) effect on human capital accumulation.

The present study is also related to the work by Dalgaard and Jensen (2009), hereafter "*DJ*", on the effect of alternative saving motives on the presence of *strong scale effects* - that is the effect

[1] Jones (1999) provides a compact comparative summary of the theoretical literature. Strulik et al. (2013) and Boikos et al.(2013) summarize the empirical literature.

[2] Earlier literature already showed that the different demographic structures have immediate implications for tax policy, convergence patterns, and the feasibility of growth itself. Dalgaard and Jensen (2009, p.1639) summarize this literature. Sorek (2011), and Diwakar and Sorek (2016c) highlight the implications of the OLG demographic structure to patent policy.

[3] The infinitely living agents can be thought equivalently, and more realistically, as finitely living ones with strong altruism toward their offspring.

[4] The empirical literature has not yet reached an agreement regarding their relative importance in driving saving behavior; See De Nardi et al. (2015) for a recent survey.

[5] See for example Dalgaard and Kreiner (2001), Strulik (2005), Bucci (2008, 2015), Bucci and Raurich (2016), and Diwakar and Sorek (2016a,b). Two recent works study this topic within the OLG framework. Prettner (2014) shows that the sign of the weak scale effect depends on the characteristics of the public education sector. Strulik et al.(2013) developed a unified growth model that incorporates endogenous fertility and human capital accumulation, and transition from neoclassical technology to R&D-based growth.

of population *size* on economic growth. Their work adds bequest saving-motive to an otherwise standard OLG model with capital externalities, that is an AK model.

However, our research question differs from DJ's, as we study the effect of alternative saving motives on the presence of *weak scale effects* and our modeling approach differs from DJ's as we incorporate a full-fledged textbook model of R&D-based growth within the OLG framework. Therefore, our results are not fully comparable with those of DJ's. Nevertheless, we reconfirm that the different saving motives implied by the alternative demographic structures are crucial in determining the role of population in R&D-based growth.

2 The Model

We take the variety-expansion model presented in the textbook of Barro and Sala-I-Martin (2004, Chapter 6), hereafter "BS", and accommodate it to the OLG framework: each consumer lives for two periods. In the first period, she supplies one unit of labor and in the second period she retires. Cohort (generation) size is increasing at an exogenous constant rate n, which is also the growth rate of the labor force and overall population.

2.1 Production and Innovation

The final good Y is produced by perfectly competitive firms with labor and differentiated inputs, to which we refer as "machines"

$$Y_t = AL_t^{1-\alpha} \int_0^{M_t} K_{i,t}^{\alpha} \, \mathrm{d}i \quad \alpha \in (0,1) \tag{1}$$

where A is a productivity factor, L_t and $K_{i,t}$ are labor input and the utilization level of machine i in period t, respectively, and M_t measures the number of available machine varieties. The final good price is normalized to one. Machines are capital goods, and thus they are formed one period ahead of utilization, and we assume they fully depreciate after one period. Once invented, the new machine variety is eternally patented. Under symmetric equilibrium, utilization level for all machines is uniform, i.e. $K_{i,t} = K_t \;\forall\; i$, and thus total output is

$$Y_t = AM_t {K_t}^{\alpha} L_t^{1-\alpha} \tag{1a}$$

The labor market is perfectly competitive, and therefore the equilibrium wage and aggregate labor income are $w_t = A(1-\alpha)M_t {K_t}^{\alpha} L_t^{-\alpha}$ and $w_t L_t = A(1-\alpha)M_t {K_t}^{\alpha} L_t^{1-\alpha}$, respectively. The profit for the final good producer is $\pi_{i,t} = AL_t^{1-\alpha} \int_0^{M_t} K_{i,t}^{\alpha} \, \mathrm{d}i - \int_{i=1}^{M_t} p_{i,t} K_{i,t} \, \mathrm{d}i - w_t L_t$, where $p_{i,t}$ is the price of input i. Profit maximization yields the demand for each machine: $K_{i,t}^{d} = A^{\frac{1}{1-\alpha}} L_t \left(\frac{\alpha}{p_{i,t}}\right)^{\frac{1}{1-\alpha}}$, for which the periodic producer-surplus from machine i, denoted PS_i, is[6] $PS_{i,t} = [p_{i,t} - (1+r_t)]\, K_{i,t}^{d}$. This surplus is maximized by the standard monopolistic price $p_{i,t} = \frac{1+r_t}{\alpha} \forall i, t$[7].

[6] Total surplus is the given by per-unit surplus times demand, and per-unit surplus is the selling price minus the marginal cost of capital, that is $\delta + r$ (full depreciation is assumed here).

[7] BS abstract from the timing of investment, setting the cost of each machine (in terms of output units) to one and therefore having the optimal monopolistic price $p = \frac{1}{\alpha}$ (equations 6.9-6.10 on pp. 291-292 there). In their continuous time framework this abstraction has no effect on any of the results.

Plugging this price in $K_{i,t}^{d}$, and then back in (1a), we obtain per-worker output, y_t :

$$y_t \equiv \frac{Y_t}{L_t} = A^{\frac{1}{1-\alpha}} \left(\frac{\alpha^2}{1+r_t}\right)^{\frac{\alpha}{1-\alpha}} M_t \tag{1b}$$

The innovation technology follows the specification of *BS* in the analysis of scale effects (see p.302 in sub-chapter 6.1.7 there)[8]:

$$\eta_t = \eta A^{\frac{1}{1-\alpha}} \left(\frac{\alpha^2}{1+r_t}\right)^{\frac{\alpha}{1-\alpha}} L_t \tag{2}$$

where η_t the cost of innovating a new variety ($\eta > 0$). This innovation technology implies that variety expansion (i.e. productivity growth) in this model depends positively on the share of output devoted to R&D[9]. As we assume machine-varieties are patented forever, patents are being traded inter-generationally - young buy patents from old. New and old varieties play equivalent roles in the production, as reflected in their symmetric presentation in (1). Therefore the market value of old varieties equals the cost of inventing a new one - η_t. Hence the return on patent ownership - over old and new technologies is $1 + r_{t+1} = \frac{PS_{i,t+1}+\eta_{t+1}}{\eta_t}$. Plugging the explicit expressions for the surplus and the innovation cost, we obtain the stationary interest rate[10]:

$$1 + r = (1+n)\left[\frac{\alpha(1-\alpha)}{\eta} + 1\right], \forall t \tag{3}$$

Hence, population growth works to increase the rate of return on capital, due increased demand for patented machines. Following (1b), per-capita output growth (which coincides with per-worker output growth), denoted g_y, is determined by the expansion rate of machine-varieties range, g_M:

$$1 + g_{y,t+1} \equiv \frac{y_{t+1}}{y_t} = 1 + g_{M,t+1} \tag{4}$$

2.2 Preferences

Lifetime utility, for an agent born in period t, is derived from consumption over two periods, and bequest:

$$u(c_{t,1}, c_{t,2}, b_t) = \frac{(c_{t,1})^{1-\theta}}{1-\theta} + \beta\left[\frac{(c_{t,2})^{1-\theta}}{1-\theta} + \kappa\frac{\left(\frac{b_t}{1+n}\right)^{1-\theta}}{1-\theta}\right] \tag{5}$$

[8] Equation (2) implies that variety expansion rate, which defines productivity growth in this model, depends positively on the share of output devoted to R&D. This specification aligns with the empirical regularities summarized in that chapter, which were originally presented by Jones (1995). See chapter6.1.7 in Barro and Sala-I-Martin (2004) for a detailed discussion.

[9] This specification aligns with the empirical regularities summarized in that chapter, which were originally presented by Jones (1995). See chapter6.1.7 in Barro and Sala-I-Martin (2004) for a detailed discussion.

[10] Any non-staiotnary interest rate path should satisfy $(1+r_{t+1})^{\frac{1}{1-\alpha}} = (1+n)\,\frac{\alpha(1-\alpha)+\eta}{\eta}\,(1+r_t)^{\frac{\alpha}{1-\alpha}}$. Our results would hold if we assume that patents ownership is transferred from parents to offspring, like in the model with infinitely living agents. Then, however, the interest rate would be $1+r = \frac{(1+n)\alpha(1-\alpha)}{\eta}$, which corresponds to the one presented in *BS* (adjusted for continuous time).

where $\beta \in (0,1)$ is the subjective discount factor, $\frac{1}{\theta}$ is the IES[11], c_1, c_2 denote consumption when young and old, respectively, and b_t is the total bequest left by a representative parent in period t (hence $\frac{b_t}{1+n}$ denotes per-child bequest). The parameter $\kappa \geq 0$ measures the weight placed on utility from the bequest. This specification of the bequest motive for saving, which resembles a 'joy-of-giving' is similar to *DJ* and is common to the literature written in the OLG framework (see for example Strulik et al. 2013). It implies that parents care about the per-child bequest level, which is in line with the Millian type of parental preferences employed by *BS.* In the extended working-paper version of this study[12] we explore also the Benthamite and "Beckerian" types of parental preferences.

3 Life-Cycle Saving

In the absence of bequest saving motive, which is the case we analyze first, we have $\kappa = 0$ and lifetime utility boils down to the standard form:

$$U(c_{t,1}, c_{t,2}) = \frac{c_{t,1}^{1-\theta}}{1-\theta} + \beta \frac{c_{t,2}^{1-\theta}}{1-\theta} \tag{5a}$$

Under (5a) young agents allocate their labor income between consumption and saving, denoted s. The solution for the standard optimal saving problem is $s_t = \frac{w_t}{1+\beta^{-\frac{1}{\theta}}(1+r)^{\frac{\theta-1}{\theta}}}$. Hence, aggregate saving is $S_t = \frac{w_t L_t}{1+\beta^{-\frac{1}{\theta}}(1+r)^{\frac{\theta-1}{\theta}}}$, which after substituting the explicit expressions for w_t becomes

$$S_t = \frac{M_t(1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha^2}{1+r}\right)^{\frac{\alpha}{1-\alpha}} L_t}{1+\beta^{-\frac{1}{\theta}}(1+r)^{1-\frac{1}{\theta}}} \tag{6}$$

The saving from labor income in (6) is allocated to three types of investment: buying patents over old varieties, inventing new varieties, and forming specialized machines. Hence aggregate investment in each period, I_t , satisfies

$$I_t = M_{t+1}\left[\eta_t + A^{\frac{1}{1-\alpha}} L_{t+1}\left(\frac{\alpha^2}{1+r}\right)^{\frac{1}{1-\alpha}}\right] \tag{7}$$

Notice that a higher population growth rate between period t and $t+1$, has a direct positive effect on the demand for each machine variety - due to the increase in L. However, following (3), a higher population growth rate also increases the interest rate, which thereby increases machine prices and therefore decreases the demand for each machine variety. By equalizing (6) and (7), we impose the equilibrium condition $I_t = S_t$, to obtain the dynamic equation that governs the variety expansion rate:

$$1 + g_y = \frac{(1-\alpha)A^{\frac{1}{1-\alpha}}\left(\frac{\alpha^2}{1+r}\right)^{\frac{\alpha}{1-\alpha}} L_t}{\left[\eta_t + A^{\frac{1}{1-\alpha}} L_{t+1}\left(\frac{\alpha^2}{1+r}\right)^{\frac{1}{1-\alpha}}\right]\left[1+\beta^{-\frac{1}{\theta}}(1+r)^{1-\frac{1}{\theta}}\right]} \tag{8}$$

Plugging (2) and (3) in (8) yields

[11] The empirical literature suggests that the IES is lower than one; See Hall (1988), Beaudry and Wincoop (1996), Ogaki and Reinhart (1998), Engelhardt and Kumar (2009).

[12] Available at: http://cla.auburn.edu/econwp/Archives/2016/2016-12.pdf

$$1+g_y = \frac{\left(\frac{\alpha(1-\alpha)}{\eta}+1\right)(1-\alpha)}{(\alpha+\eta)\left[1+\beta^{-\frac{1}{\theta}}\left[(1+n)\left(\frac{\alpha(1-\alpha)}{\eta}+1\right)\right]^{1-\frac{1}{\theta}}\right]} \tag{8a}$$

Proposition 1 *With no bequest motive, the sign of the weak scale effect depends on the $IES \equiv \frac{1}{\theta}$: for $\frac{1}{\theta}<1$ ($\frac{1}{\theta}>1$) there is negative (positive) weak scale effect, i.e. $\frac{\partial g_y}{\partial n}<0$ ($\frac{\partial g_y}{\partial n}>0$).*

Proof. Proof is by inspection of equation (8a). ■

The counterpart model with infinitely lived agents (presented in *BS*) yields no relation between population growth and economic growth regardless of the *IES* value[13]. In both models, population growth increases future demand for patented machines, thereby increasing the equilibrium interest rate. However, for the infinitely lived agents, population growth works also as a demographic discounting factor which discourages saving, and thus the two effects cancel out. In the OLG economy, population growth does not generate direct negative effects on saving and, due to the life-cycle structure of this framework, the effect of the increased interest rate on saving depends on the *IES*.

4 Bequests

In the presence of bequest saving motive, each young agent maximizes her lifetime utility (5), subject to the budget constraint: $w_t + \frac{b_{t-1}}{1+n} = c_{t,1} + \frac{c_{t,2}+b_t}{1+r}$. Applying this budget constraint to (5) we write the indirect utility function

$$u(s_t, w_t, b_{t-1}, b_t, r) = \frac{\left(w_t + \frac{b_{t-1}}{1+n} - s_t\right)^{1-\theta}}{1-\theta} + \beta\left[\frac{[s_t(1+r)-b_t]^{1-\theta}}{1-\theta} + \kappa\frac{\left(\frac{b_t}{1+n}\right)^{1-\theta}}{1-\theta}\right] \tag{9}$$

Differentiating (9) with respect to s and b we obtain the following first order conditions

$$s_t = \frac{w_t + \frac{b_{t-1}}{1+n}}{\frac{\beta^{-\frac{1}{\theta}}(1+r)^{\frac{\theta-1}{\theta}}}{1+(1+n)^{\frac{\theta-1}{\theta}}\kappa^{\frac{1}{\theta}}}+1}, \quad b_t = s_t\frac{1+r}{\frac{(1+n)^{\frac{1-\theta}{\theta}}}{\kappa^{\frac{1}{\theta}}}+1} \tag{10}$$

Optimal saving is still a fraction of the resources available to the young (worker), which now combine labor income and her inherited bequest. Hence, the operative bequest motive relaxes the former dependency of saving (and thereby investment and innovation rate) on labor income. Saving depends now not only on the interest rate and the *IES*, but also on the bequest motive parameter κ, through the expression $(1+n)^{\frac{\theta-1}{\theta}}\kappa^{\frac{1}{\theta}}$.

The effect of population growth rate on this expression (and thereby on saving) depends on the *IES*. Here, the population growth rate works as a depreciation rate that erodes the per-child bequest level. Hence, its effect is inverse to the effect of the interest rate. This effect has life-cycle saving properties due to the timing of parents utility from bequest-giving during the second period of life. The second factor has a positive effect on saving, due to the increased marginal utility from per-child bequest.

[13] The growth equation for this model, defined by the regular Euler condition, are presented in the following section.

The optimal per-child bequest level is a certain fraction of capital income, $s_t(1+r)$. This fraction is a function of the population growth rate and the bequest motive. As explained above, the population growth rate erodes the per-child bequest level, and thus works like a decrease in the interest rate: as the utility from bequest takes place during retirement, the effect of lower return on the bequest per-child depends on the *IES*. The effect of the strength of bequest motive, κ, on per-child optimal bequest is positive.

The first condition in (10) implies that aggregate savings is given by

$$S_t = \frac{(1-\alpha)A^{\frac{1}{1-\alpha}} M_t \left(\frac{\alpha^2}{1+r}\right)^{\frac{\alpha}{1-\alpha}} L_t + B_{t-1}}{\frac{\beta^{-\frac{1}{\theta}}(1+r)^{\frac{\theta-1}{\theta}}}{1+(1+n)^{\frac{\theta-1}{\theta}}\kappa^{\frac{1}{\theta}}} + 1} \tag{11}$$

where $B_{t-1} = \frac{L_t b_{t-1}}{1+n}$, is aggregate bequests given to workers who were born in period t. Notice that for $\kappa = 0$ the aggregate saving level defined in (11) falls back to the one presented in (6). The second condition in (10) implies that $B_{t-1} = \frac{1+r}{(1+n)^{\frac{1-\theta}{\theta}}\kappa^{-\frac{1}{\theta}}+1} S_{t-1}$, and the equilibrium condition $S_{t-1} = I_{t-1}$ requires

$$B_{t-1} = \frac{1+r}{(1+n)^{\frac{1-\theta}{\theta}}\kappa^{-\frac{1}{\theta}}+1} M_t \left(\eta_{t-1} + A^{\frac{1}{1-\alpha}} L_t \left(\frac{\alpha^2}{1+r}\right)^{\frac{1}{1-\alpha}}\right) \tag{12}$$

Substituting (12), along with (3), back into (11) and equalizing to (7), i.e. setting $S_t = I_t$, we obtain

$$1+g_y = \frac{\left[\frac{\alpha(1-\alpha)}{\eta}+1\right]\left[\frac{(1-\alpha)}{\alpha+\eta}(1+n)^{\frac{1-\theta}{\theta}} + \frac{(1+\eta)}{\alpha+\eta}\kappa^{\frac{1}{\theta}}\right]}{\beta^{-\frac{1}{\theta}}\left(\frac{\alpha(1-\alpha)}{\eta}+1\right)^{\frac{\theta-1}{\theta}} + \left[(1+n)^{\frac{1-\theta}{\theta}} + \kappa^{\frac{1}{\theta}}\right]} \tag{13}$$

Proposition 2 *In the presence of bequest saving-motive, the sign of the weak scale effect,* $\frac{\partial g_y}{\partial n}$ *is positive (negative) for* $\theta > 1$ $(\theta < 1)$ *and sufficiently strong (weak) bequest motive.*

Proof. Differentiating (13) for n reveals that, for $\theta > 1$ $(\theta < 1)$, $\frac{\partial g_y}{\partial n} > 0$ *iff* $\beta^{-1}\left(\frac{1-\alpha}{\alpha+\eta}\right)^{\theta}\left[\frac{\alpha(1-\alpha)}{\eta}+1\right]^{\theta-1} < \kappa$ $(\beta^{-1}\left(\frac{1-\alpha}{\alpha+\eta}\right)^{\theta}\left[\frac{\alpha(1-\alpha)}{\eta}+1\right]^{\theta-1} > \kappa)$. for $\theta = 1$, we have $\frac{\partial g_y}{\partial n} = 0$ independently of κ. ■

In the counterpart model of infinitely lived agents, presented in *BS*, households maximize per-capita utility of their dynasty members (following Millian preferences). Hence, aggregate consumption growth follows the standard Euler equation[14]: $\frac{\dot{C}}{C} = \frac{1}{\theta}(r-\beta)$, and per-capita consumption follows $\frac{\dot{c}}{c} = \frac{1}{\theta}(r-\beta-n)$ where the interest rate is given by[15] $r = n + \frac{\alpha(1-\alpha)}{\eta}$. Combining the two latter conditions yields the stationary growth rates for per-capita income: $g_{c,y} = \frac{1}{\theta}\left[\frac{\alpha(1-\alpha)}{\eta} - \beta\right]$. Hence, in the counterpart economy of infinitely lived homogeneous agents the *IES* plays no role in the presence (or sign) of the weak scale effect. Notice that, by Proposition 2, for $\theta = 1$ the weak scale effect is also muted in our model, for any κ.

In reference to the results obtained by *DJ*, it is worthwhile noting that under the technological parameters used in our model, they find a strong scale-effect will prevail for any for $\theta \leq 1$. However,

[14] Equation (6.22) on p.295 there, in which the parameter ρ the time preference parameter (denoted here as β).

[15] Equation (6.35) on p. 302 there.

if $\theta < 1$ is sufficiently larger (smaller) than one, strong scale effect in their model will prevail only if κ is sufficiently large (small)[16].

5 Conclusion

This study highlights the implications of alternative demographic structures, and the saving motives they imply, to the presence of weak scale effect on R&D-based growth models. To this end, we have placed a basic variety-expansion textbook model (without human capital accumulation) in the overlapping-generations demographic framework, and showed how the interaction between the two alternative saving motives - life-cycle consumption smoothing and parental bequests - determine the sign of the weak scale effect. In particular, for the empirically valid degree of the *IES*, positive (negative) weak scale effect presents in the *OLG* economy only if parental-bequest saving motive is sufficiently strong (weak).

References

[1] Barro R.J., and X. Sala-i-Martin, (2004). "Economic growth", Second Edition. MIT Press. Cambridge. MA

[2] Beaudry P., and E. Wincoop, (1996). "The intertemporal elasticity of substitution: an exploration using a US panel of state data". *Economica* 63, 495–512

[3] Boikos S., Bucci A., Stengos T., 2013. Non-monotonicity of fertility in human capital accumulation and economic growth. *Journal of Macroeconomics* 38, 44–59

[4] Bucci A., 2008. Population growth in a model of economic growth with human capital accumulation and horizontal R&D. *Journal of Macroeconomics* 30, 1124-1147

[5] Bucci A., 2015. Product proliferation, population, and economic growth. *Journal of Human Capital* 9. 170-197

[6] Bucci A., Raurich X., 2017. "Population and economic growth under different growth engines". *German Economic Review* 18, 182-211

[7] Dalgaard C.J., and M.K. Jensen (2009). "Life-cycle savings, bequest, and a diminishing impact of scale on growth". *Journal of Economic Dynamics&Control* 33,1639–1647

[8] Dalgaard C.J, and C.T Kreiner (2001). "Is declining productivity inevitable?". *Journal of Economic Growth* 6, 187–203

[9] De Nardi M, E. French, and J.B. Jones (2015). "Savings After Retirement: A Survey". NBER Working Paper #21268

[10] Diamond P. (1965). "National debt in a neoclassical growth model". *American Economic Review* 55, 1126–1150

[11] Diwakar B. and G. Sorek (2016a). "Human-capital spillover, population, and economic growth". *The B.E Journal of Macroeconomics*, volume 17 issue 1

[16] See Theorem 1 and Colloraly 2 (on pp. 1642 and 1643 respectively) there, for $\sigma = 1$ (by their notation), which is the elasticity of substitution between labor and capital in our model.

[12] Diwakar B. and G. Sorek (2016b). "Dynastic altruism, population growth, and economic prosperity". *Economics Bulletin* 36, p. A196.

[13] Diwakar B. and G Sorek (2016c). "Finite lifetimes, patents' length and breadth, and growth". Auburn University Economics Working Papers # 2016-08

[14] Engelhardt G.V. and A. Kumar (2009). "The elasticity of inter temporal substitution: new evidence from 401(k) participation". *Economics Letters* 103, 15–17

[15] Hall R., 1988. "Intertemporal substitution in consumption". *Journal of Political Economy* 96, 339–357

[16] Jones C.I. (1999). "Growth: with or without scale effects". *American Economic Review* 89, 139-144

[17] Ogaki M., and C.M. Reinhart (1998). "Measuring intertemporal substitution: the role of durable goods". *Journal of Political Economy* 106, 1078–1098

[18] Prettner K. (2014). "The non-monotonous impact of population growth on economic prosperity". *Economics Letters* 124, 93–95

[19] Strulik H. (2005). "The role of human capital and population growth in R&D-based models of economic growth". *Review of International Economics* 13, 129–145

[20] Strulik H., K. Prettner, and A. Prskawetz (2013). "The past and future of knowledge-based growth". *Journal of Economics Growth* 18, 411–437

[21] Sorek G. (2011). "Patents and quality growth in OLG economy". *Journal of Macroeconomics* 33, 690-699