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"Optimal storage under uncertainty: investigating the implications of frugality and prudence."

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Abstract

I investigate the effects of a convex marginal cost (frugality) and a convex marginal utility (prudence) on energy storage using a simple theoretical model. By characterizing the optimum, I show that prudence and frugality spur precautionary energy storage, and explain the implications of the results for a competitive market equilibrium. The analysis can be of significance for storage decisions in various industries where firms are exposed to production risks. Thus, its scope of application is not limited to the energy markets.

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1 Introduction

Renewable energy's share in power generation is expected rise considerably in the following years. However, as renewable energy (RE), such as wind power and photovoltaic electricity, is inherently variable and uncertain, the overall energy production becomes riskier with the rising renewable share. A number of strategies exist to deal with the challenges posed by intermittent RE.[1] The use of dispatchable generation is one example. It refers to the production of electricity using steam and internal combustion turbines (e.g., natural gas power plants) to align supply and demand. A demand response is another way to enhance the electricity grid resilience. It relates to the presence of end-use consumers who can monitor and change their electricity consumption in response to changes in the price. One other way of enhancing the reliability of the grid is energy storage. Energy storage systems absorb energy during periods of excess generation and release it when RE generation is low and it is costlier to balance the power system with dispatchable generators.

When consumers are responsive, and dispatchable generators are responsible to match electricity supply with demand, I show that two precautionary motives lead to a higher demand for energy storage. The first precautionary motive is prudence with respect to electricity consumption, which is formally equivalent to a positive third derivative of the utility function. The other precautionary motive is frugality. It is formally equivalent to a convex marginal cost of dispatchable generation. I refer to the property of a convex marginal cost function as frugality, since, in the presence of uncertainty, it endows a cost minimizing producer with the same motivations as that of a prudent consumer.

Although it was Kimball (1990) who coined the term prudence, a convex marginal utility has been analyzed earlier by Leland (1968) and Sandmo (1970). Within an expected-utility framework, the authors indicate that a risky future income increases savings if and only if the third-order derivative of the utility function is positive. Frugality, however, has not been fully investigated. Cecchetti et al. (1997) find evidence that firms in various industries can face upward-sloping and convex marginal-cost curves, and note that, from an operational perspective, a firm is capacity constrained when the marginal cost curve is convex. Using bid data, Wolak (2001) obtains a similar result in the Australian electricity market.

To the best of my knowledge, the effect of a convex marginal cost function on precautionary storage has not been investigated before. Additionally, the finding with regard to the impact of the precautionary motives on electricity storage is a novel result in the energy economics literature. As frugality can have implications for inventory decisions in various other industries (petroleum, food and transportation, to name a few) where firms face capacity constraints and are exposed to production risks, its scope of application is not limited to the energy markets.

The remainder of the paper is organized as follows. Section 2 presents the model, states the social planner's problem, and demonstrates the results. Section 3 concludes.

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[1] Intermittency means that RE generation depends on meteorological shocks and is non-controllable.

2 Model and Results

I consider a two-period economy in which energy can be supplied from dispatchable generation, renewable sources, and energy storage systems:

$$q_0 = y_0 + z_0 - \alpha s, \tag{1a}$$

$$q_1 = y_1 + z_1 + s. \tag{1b}$$

where for $j = 0, 1$, q_j is total energy supply, y_j is dispatchable generation, z_j is RE, and s is the level of energy storage. For $\alpha > 1$, $1/\alpha$ is the round-trip efficiency parameter. It is the ratio of energy recovered to the initially stored energy. While z_0 is observed prior to making decisions in the initial period, z_1 is uncertain. In the rest of the analysis, I indicate that a variable is random by placing a tilde over it. Let $\tilde{z}_1$ be independently and identically distributed with a commonly known cumulative distribution function, and a compact support $[0, \bar{z}]$, where $\bar{z}$ is the maximum generating capacity of the RE system. I denote the mean of $\tilde{z}_1$ by μ.

Once the RE system is installed, the unit cost of generating RE becomes so low that I consider it as zero (Ambec and Crampes, 2012). Thus, the renewable system always operates at its capacity. However, as the weather is uncertain, so is the RE generation. Because RE has a zero operating cost, it must be dispatched earlier. Since RE is uncertain, its earlier dispatch causes the residual demand to be uncertain. Therefore, after accounting for RE, a dispatchable generator has to supply an uncertain residual demand. The cost function of the dispatchable generator is denoted by $C(y_j), j = 0, 1$. Assume that $C(\cdot)$ is thrice continuously differentiable with $C' > 0$ and $C'' > 0$.

I consider quasi-linear preferences and no income effects. In economic theory, using such preferences is a standard assumption when discussing issues related to a single market in a general equilibrium framework. The gross surplus function over kilowatt-hour consumption of energy, e_j, is, $U(e_j)$, $j = 0, 1$. It is assumed that $U(\cdot)$ is thrice continuously differentiable with $U' > 0$ and $U'' < 0$.

To simplify the analysis, I focus on cases where dispatchable generation is efficient. Thus, $U'(z_0) > C'(0)$ and $U'(z_1 + s) > C'(0)$. This assumption, incidentally, overlaps with the observation that the existing energy systems worldwide are generally characterized by small shares of RE and, hence, dispatchable generation always supplies the residual demand (Joskow, 2011; Lund et al., 2012; Tsitsiklis and Xu, 2015).

In studying the implications of prudence and frugality, I solve a benevolent social planner's problem. Let $\mathbb{E}[\cdot]$ denote the expected value operator. The problem is

$$\max_{\{q_0, q_1, y_0, y_1, s\}} \quad U(q_0) - C(y_0) + \mathbb{E}\left[U(\tilde{q}_1) - C(\tilde{y}_1)\right]$$

subject to Eqs. (1a) and (1b), and $q_0 \geq 0$, $\tilde{q}_1 \geq 0$, $y_0 \geq 0$, $\tilde{y}_1 \geq 0$, $s \geq 0$ and $s \leq \bar{s}$, where $\bar{s}$ is the storage capacity.

In the following, I shall investigate whether the optimal level of energy storage under uncertain RE generation is greater than the corresponding level without uncertainty. Let s^+ denote the optimal level of energy storage when RE generation in the final period, $\tilde{z}_1$, is certain, and equals μ. Following this definition, I present the major result of the study:

Theorem 1. *$s \geq s^+$ if and only if*

$$\frac{\left(C''(y_1)\right)^3}{\left(C''(y_1) - U''(q_1)\right)^3} U'''(q_1) + \frac{\left(-U''(q_1)\right)^3}{\left(C''(y_1) - U''(q_1)\right)^3} C'''(y_1) \geq 0, \tag{2}$$

where U''' and C''' denote the third-order derivative of the surplus function and the cost function, respectively. $U''' \geq 0$ and $C''' \geq 0$ are sufficient for $s \geq s^+$.

Proof. The proof is provided in Appendix A. □

Theorem 1 indicates that a higher level of energy storage will be welfare improving if and only if Eq. (2) holds. There can be precautionary storage even if $U''' < 0$ and $C''' > 0$ or $C''' < 0$ and $U''' > 0$. Furthermore, prudence and frugality are sufficient to generate precautionary storage, that is, $s \geq s^+$. When $U''' = 0$, frugality alone will lead to precautionary energy storage. The same is true when $C''' = 0$ and $U''' \geq 0$.[2] Absent precautionary motives, there will be no precautionary storage: $s = s^+$.

To provide further intuition, I consider an interior solution for s. The first-order condition with respect to s is

$$\alpha U'(q_0) = \mathbb{E}\left[U'(\tilde{q}_1)\right].$$

Note that $\tilde{q}_1 = \tilde{y}_1 + \tilde{z}_1 + s$, where, slightly abusing the notation, $\tilde{y}_1 = y_1(\tilde{z}_1)$.[3] Taking the expectation of a second-order Taylor approximation of $U'(\tilde{y}_1 + \tilde{z}_1 + s)$ around μ yields

$$\alpha U'(q_0) = \left[U'(\tilde{q}_1) + \frac{1}{2}\sigma^2 \kappa(\tilde{q}_1, \tilde{y}_1)\right]\Bigg|_{\tilde{z}_1 = \mu}$$

where σ^2 is the variance of $\tilde{z}_1$ and $\kappa(q_1, y_1)$ is given by left-hand side of (2). This shows that the weighted sum of prudence and frugality give a mark-up on the marginal surplus at the expected energy production. The higher the mark-up, the higher will be the level of storage. Moreover, a higher level of volatility in RE will induce further storage.

Conducting a comparative-statics analysis yields that $q_0 \leq q_0^+$ (see Appendix B). This result indicates that precautionary storage causes a lower energy consumption and, in turn, a lower welfare, in the initial period. Nevertheless, by transferring the social surplus to the next period using energy storage systems (i.e., $\partial \tilde{q}_1 / \partial s > 0$), the benevolent social planner expects that a higher welfare in the future will more than compensate for this loss.

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[2] These results hold under risk aversion, that is, $U'' < 0$, which I assume throughout the paper.

[3] To be more precise, $\tilde{y}_1 \equiv y_1(\tilde{z}_1, s)$.

Competitive market equilibrium

A few comments on the relevance of the results for a competitive market equilibrium are in order. As there are no externalities or other distortions in the model, the competitive equilibrium quantities correspond to the social planner's allocation. I assume that consumers have identical preferences and model their behavior by a representative consumer. Given the retail prices P_0 and $\tilde{P}_1$, the marginal surplus functions must satisfy

$$P_0 = U'(q_0) \quad \text{and} \quad \tilde{P}_1 = U'(\tilde{q}_1).$$

Thus, $q_0 \equiv q(P_0)$ and $\tilde{q}_1 \equiv q(\tilde{P}_1)$ are the aggregate demand functions. When the aggregate demand and aggregate supply functions are convex, the expected value of storing one more unit of energy increases with the degree of precaution in the market, that is, with Eq. (2). Subsequently, precautionary storage increases P_0 and decreases $\tilde{P}_1$ until all arbitrage opportunities are exhausted, that is, until

$$\alpha P_0 = \mathbb{E}\big[\tilde{P}_1\big]$$

is satisfied.

3 Conclusion and Discussion

In this study, I show that frugality and prudence spur precautionary energy storage. The former is due to the structure of the energy markets where, after accounting for the RE, the capacity constrained dispatchable generators supply the uncertain residual demand. Thus, energy storage depends not only on the curvature of the demand function, but also on the curvature of the supply function.

Because devices such as smart meters and batteries can still be viewed as costly, the ability of consumers to respond to price changes and to store energy can be questioned. Nevertheless, smart meters are becoming relatively widely used in the world (e.g., more than half of the households in the United States now have smart meters installed, IEI, 2016). Furthermore, with new developments in the home batteries (e.g., Tesla Powerwall), the home energy storage option is no longer far-fetched. From the point of view of the supply side, pumped storage is considered the most mature method for electricity storage. Nonetheless, because of their small spatial requirements, mechanical simplicity, and flexibility in sitting (Wang et al., 2014), the most obvious alternative for energy storage will be (high-energy) batteries. Although they are still costly to install, maintain and operate, it is expected that the cost of these storage systems will fall rapidly over time.[4] An important consequence of these developments is that a policy maker, when formulating his electricity policies, can greatly benefit from the knowledge of the effects of prudence and frugality on storage and electricity generation decisions.

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[4] The cost of energy storage is expected to decrease by 40% by 2020 (Ortiz, 2016).

As the empirical evidence suggests, a highly convex marginal cost function can be an indicator of a high level of market power (Bunn et al., 2000). Although this note demonstrates that additional storage is welfare improving when the supply function is convex, a generator with market power can prefer to use storage strategically to increase its power further in the market. This generator, on the other hand, can also prefer to store energy on precautionary grounds and be assured of meeting its commitment to provide electricity to the real-time market. Accordingly, it is of great interest to investigate these rather contrasting decisions and their possible consequences in markets that are characterized by imperfect competition. Whether precautionary motives can help the guiding hand of regulation is another interesting issue for future research.

Lastly, precautionary storage can also have implications for allocation decisions, commodity prices and price variations in industries other than electricity generation. Therefore, further investigation of precautionary storage as well as its economic effects for different sectors of the economy is worthwhile.

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