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Too many traders? On the welfare ranking of prices and quantities

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Abstract

This study conducts a welfare comparison of emission tax and emissions trading in a multi-period model, in which the regulation instruments are chosen through a political process and unregulated participants can trade in the permit market. It is found that under reasonable conditions a tax yields a higher welfare level than emissions trading.

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1 Introduction

"In principle, *anyone* can trade in the carbon market."[1]

This quote describes who can trade in the European Union Emissions Trading System (EU ETS). It is asked here whether the "number" *anyone* is too large. To answer this question, the model of this study is framed as a welfare comparison of prices and quantities with multiple time periods. The feature that differentiates this model from those in the literature including for example Finkelsthain and Kislev (1997), Montero (2002), Newell and Pizer (2003), Baldursson and von der Fehr (2004) and Karp and Zhang (2012) is that the consequences of certain types of perturbations in the permit market are modeled (e.g., trades by unregulated non-polluting participants). In the models that study emissions trading, the market clearing condition is simply one where the number of permits distributed meets the regulated firms' demand. But since *anyone* can trade in the EU ETS, the aggregate demand of permits at some period of time may not equal to the aggregate demand of the regulated firms. Banks, other investors and organizations, for example, may buy and sell permits; therefore, one period may see greater demand for permits and another, an additional supply of permits. This may change the equilibrium price of permits such that the price differs from the tax rate, and in effect, the welfare ordering of the instruments in a multi-period model may change.

As a benchmark, consider the first-best allocation in which the regulator maximizes social welfare defined as the difference between the profits and damages from pollution by choosing the instruments (that is, the tax level or the amount of auctioned permits). Suppose that a perturbation occurs in the permit market.[2] It is intuitive in this case that tax dominates emissions trading: The perturbation induces a change in the permit price from its first-best value, which itself affects the amount of emissions the firms produce. Hence the allocation fails to maximize welfare. But the tax rate is fixed and is unaffected by the perturbation, which consequently leads to a welfare-maximizing emissions level. In the first-best model the number *anyone* is too large.

Although it represents an interesting starting point with a clear intuition, the first-best choice of the instruments leaves out political considerations. These are important since in practice the instrument levels are chosen in some kind of political process. Therefore the purpose of this study is to analyze the same question in a setting where the instruments are chosen through a political process, which implies that the instruments are no longer first-best. To describe the political process, the model of Finkelsthain and Kislev (1997) is used.[3]

It is shown that if the social welfare function is strictly concave with respect to emission price and if the perturbations in the permit market produce equilibrium prices that are *symmetrical around the emission tax*, then the welfare level will be higher with the emission tax than with emissions trading. However, the welfare ranking of the instruments becomes ambiguous in general, and the ranking may even reverse in favor of emissions trading. Indeed, not all perturbations are bad for emissions trading: For example, as the

[1]A quote from European Commission's EU ETS-website, European Commission (2016). Emphasis is added.

[2]Perturbation could be for example a trade by citizens or by an NGO, or "speculation" in general. For example, speculation in the emissions trading market tends to affect the equilibrium permit price (Colla *et al.*, 2012).

[3]Their model is based on Grossman and Helpman (1994), which is a much-used model also in environmental and resource economics. Examples include Finkelsthain and Kislev (1997), Fredriksson (1997), Aidt (1998) and Kawahara (2014).

instruments produce the same emissions level without perturbations, it is possible that emissions trading with perturbations yields a higher welfare level than the tax would; this happens when the perturbation leads to emissions that are slightly closer to the first-best emissions level than the emissions level with the tax. If the perturbations push emissions in the other direction, then the tax dominates.

The next section continues with the description of the assumptions, Section 3 presents the results, and the last section discusses the model.

2 Modeling perturbations

A fixed number of firms are indexed with i, $i = 1, \ldots, N$, and are regulated either by an emission tax or emissions trading. Time is indexed with τ, $\tau = 1, \ldots, T$. The emissions level of firm i at period τ is denoted by e_i^τ, and the aggregate emissions at period τ are $E^\tau := \sum_{i=1}^{N} e_i^\tau$.[4] The profit for firm i as a function of emissions, $\pi_i(e_i^\tau)$, satisfies inequalities $\pi_i' > 0$ and $\pi_i'' < 0$ for every i, and the damage as a function of aggregate emissions, $D\left(E^\tau\right)$, satisfies inequalities $D' > 0$ and $D'' > 0$.[5] The optimal choice of emissions for firm i in any period τ as a function of the permit price p (or emission tax t) is denoted by $e_i^\tau(p)$. This is the unique amount of emissions that maximizes $\pi_i(e_i^\tau) - pe_i^\tau$ and satisfies $\mathrm{d}e_i^\tau/\mathrm{d}p < 0$. Let $E^{0,\tau}$ represent the total number of permits distributed in period τ.

Let $P \subseteq \mathbb{R}_{++}$ be the set of possible emissions prices, and let Θ be a set that includes the possible perturbations. Define *demand function of permits* M^τ as $M^\tau \colon P \times \Theta \to \mathbb{R}_{++}$ for any τ. The following assumption characterizes the market-clearing condition in the permit market and the market-clearing permit price.

Assumption 1.

(*i*) (*Market clearing*) There exists for any τ and for any $\theta \in \Theta$ a price $p \in P$ such that

$$M^\tau(p, \theta) = E^{0,\tau}. \tag{1}$$

(*ii*) (*Clearing price*) Equation (1) defines for any τ the price as a one-to-one function of θ, $p^\tau \colon \Theta \to P$. Its value at θ is denoted by p_θ^τ.

These assumptions can be interpreted as follows. The elements of Θ describe the perturbations that can happen in the permit market. The content of parts (*i*) and (*ii*) is only that some perturbation affects market clearing or price formation in the permit market. In particular, the permit market clears in such a way that different perturbation values yield different clearing prices. Define $\theta^{*,\tau}$ as the value of θ that produces the price without perturbations at period τ.

Example. The above situation leaves unspecified the elements of set Θ. For example, let $\Theta := \mathbb{N}_0 = \{0, 1, \ldots\}$, and let θ represent the number of unregulated participants who trade in the permit market. Then $\theta^{*,\tau} := 0$ and $M^\tau(p, 0) := \sum_{i=1}^{N} e_i^\tau(p)$ for all τ. In this case, the market-clearing condition can be expressed as

$$M^\tau(p, \theta) := \sum_{i=1}^{N} e_i^\tau(p) + \sum_{j=1}^{\theta} d_j^\tau(p) = E^{0,\tau}, \tag{2}$$

[4] Superscript τ is used to denote time period.

[5] The analysis is simplified by assuming a flow pollutant. Also, the profit and damage functions are assumed to be time invariant.

where d_j^τ is the demand (or supply) function of an unregulated participant j at period τ.

3 Results

Next, the political process is briefly described in the current context.[6] The model in this section, which originates from Grossman and Helpman (1994) and Finkelsthain and Kislev (1997), consist of three stages. First, the lobby group with $L \leq N$ firms decides its political contribution C, which is contingent on the instrument level of the regulator.[7] In the second stage, the regulator maximizes its utility, defined as a weighted combination of the aggregate social welfare (the difference between profits and damages) and the contribution, by choosing the instrument.[8] In the third stage, the firms choose their optimal emissions individually and perturbations occur. However, the perturbations are not expected by the regulator.

The analysis begins from the third stage. Here, the following equation describes the firm's choice:

$$\pi_i'(e_i) = k, \tag{3}$$

in which k is the price of emissions. This price is either the tax t or the equilibrium permit price p. Following Finkelsthain and Kislev (1997), in the second stage, the regulator maximizes its utility under the constraint that the lobby group's profit is greater than or equal to the reservation profit Y. The weight is given by the parameter $\alpha > 0$. Let c_i be firm i's contribution or "political bribes", and $C = \sum_{i=1}^{L} c_i$. As in Finkelsthain and Kislev (1997), the maximization problem for the regulator is, in the case of an emission tax,

$$\max_{\{t\}} \left\{ \sum_{i=1}^{N} \pi_i(e_i) - D(E) + \alpha C \right\}, \quad \text{subject to} \quad \sum_{i=1}^{L} (\pi_i(e_i) - te_i - c_i) \geq Y.$$

As in Finkelsthain and Kislev (1997), the problem in which the constraint binds is analyzed. Thus a necessary condition for a maximum is

$$\sum_{i=1}^{N} \pi_i'(e_i)\frac{\mathrm{d}e_i}{\mathrm{d}t} - D'(E)\frac{\mathrm{d}E}{\mathrm{d}t} + \alpha \left(\sum_{i=1}^{L} \left(\pi_i'(e_i)\frac{\mathrm{d}e_i}{\mathrm{d}t} - e_i - t\frac{\mathrm{d}e_i}{\mathrm{d}t} \right) \right) = 0. \tag{4}$$

Let E^L denote the aggregate emissions of the firms that participate in the lobby group. Given the optimal behavior of the firms (equation (3)), the emission tax satisfies equation

$$t - D'(E) = \frac{\alpha t \frac{E^L}{E}}{\mathrm{El}_t E} < 0, \tag{5}$$

where $\mathrm{El}_t E$ is the elasticity of aggregate emissions with respect to an emission tax. The tax rate is set below the marginal damages and differs from the first-best emission price level. These are well-known results from Finkelsthain and Kislev (1997).[9] Finkelsthain and Kislev (1997) compare an emission tax with non-tradable quantities,

[6] Time index τ is dropped for now, because it only clutters the notation. However, it is still used at some places to avoid possible confusion.

[7] Lobby formation is exogenous as it is also in Grossman and Helpman (1994).

[8] It is assumed that the regulator does not discount.

[9] Regarding the first stage lobby contributions, the lobbying firms capture all the surplus, since there is only one lobby group (Grossman and Helpman, 1994; Finkelsthain and Kislev, 1997).

but this study is interested in auctioned permits. When auctioning the permits, the number of permits auctioned is set such that

$$p - D'(E) = \alpha \sum_{i=1}^{L} \frac{\mathrm{d}p}{\mathrm{d}E^0} e_i = \frac{\alpha p \frac{E^L}{E}}{\mathrm{El}_p E}. \tag{6}$$

Note that the emission prices are equal with either instrument, when no perturbation occurs in the permit market at the third stage. This implies that the welfare levels are the same with the instruments. However, according to Assumption 1, if a perturbation occurs in the permit market, the equilibrium permit price diverges from the emission tax. This may change the welfare ordering of the instruments. The following assumption is used to obtain a clear-cut welfare ordering. Define first function $Z_\tau \colon P \to \mathbb{R}$ with

$$Z_\tau(p) = \sum_{i=1}^{N} \pi_i(e_i^\tau(p)) - D\left(\sum_{i=1}^{N} e_i^\tau(p)\right). \tag{7}$$

This function measures the aggregate social welfare at period τ.[10]

Assumption 2.

(i) Function Z_τ is strictly concave.

(ii) The perturbed price can obtain either value $\underline{p} < t$ or value $\overline{p} > t$ with $|t - \underline{p}| = |t - \overline{p}|$.

(iii) The number of perturbations in either direction are the same.

It is easily verified that Z_τ is strictly concave for example when the marginal profit functions are linear functions. Parts *(ii)* and *(iii)* include an important special case, in which an unregulated firm buys permits in some period and later sells them in some other period. The main result is the following:

Proposition 1. *Suppose that Assumptions 1 and 2 hold. If a perturbation occurs in the permit market at least in one period of time, then the aggregate welfare under an emission tax is strictly greater than under auctioned permits.*

Proof. Define $A = \{\tau \in \{1, \dots, T\} \mid \theta^\tau \neq \theta^{*,\tau}\}$, that is, A consists of all the periods with perturbed prices. Set A is non-empty by assumption. Let the number of elements in A be m. For all periods $\tau \notin A$, the welfare difference between the emission tax and emissions trading is zero, because by equations (5) and (6) $p = t$. Define function $K \colon P^m \to \mathbb{R}$ with

$$K(p^1, \dots, p^m) = \sum_{\tau \in A} Z_\tau(p^\tau), \tag{8}$$

where p^τ means the price at period τ. This function measures the aggregate welfare over the periods with perturbed prices. Let $p_\theta = (p^1_{\theta^1}, \dots, p^m_{\theta^m})$ be the perturbed price vector and g be an m-vector $(t, \dots, t)$. Note that $K(p_\theta) - K(g)$ measures then the welfare difference between the instruments. Since K is a strictly concave function,

$$K(p_\theta) - K(g) < \nabla K(g)(p_\theta - g). \tag{9}$$

Denote by K'_τ the partial derivative with respect to variable number τ, $\tau = 1, \dots, m$.

[10] Note that the function Z_τ is not necessarily concave.

It is shown that the right-side of (9) is zero. To this end, define A^- as the subset of A that includes all periods with $\underline{p} < t$, and A^+ as the subset of A that includes all periods with $\overline{p} > t$. Since by Assumption 2 the number of perturbations in either of the directions is the same, $\#A^- = \#A^+$ and $A = A^- \cup A^+$. This allows to rewrite the right-side of (9) as

$$\nabla K(g)(p_\theta - g) = \sum_{\tau \in A} K'_\tau(t)(p^\tau_{\theta^\tau} - t) \tag{10}$$

$$= \sum_{\tau \in A^+} K'_\tau(t)(\overline{p} - t) + \sum_{\tau \in A^-} K'_\tau(t)(\underline{p} - t). \tag{11}$$

This equals zero, since $\overline{p} - t = -(\underline{p} - t)$ by Assumption 2. Hence $\sum_{\tau \in A} Z_\tau(t) > \sum_{\tau \in A} Z_\tau(p^\tau_{\theta^\tau})$. $\square$

The result is intuitive. Although some perturbations push the equilibrium in the permit market towards the price and aggregate emissions that maximize the social welfare, the same number of perturbations push the equilibrium away from aggregate emissions under an emission tax. The latter perturbations have more bearing on the value of welfare than do the former ones due to the strict concavity of the social welfare function. Proposition 1 focused on perturbations that can result in only two different prices. In Appendix A.1, a similar situation is analyzed in which perturbed prices can assume multiple values and the same result is obtained.

4 Discussion

This paper studied perturbations stemming, for example, from the trading of unregulated participants in the permit market and found that with these perturbations, emissions trading is welfare inferior to an emission tax, when instrument is chosen through a political process that allows firms to lobby the regulator. However, to obtain this result, the perturbations were constrained in a way that they yield symmetric prices around the emissions tax. It should be noted that without Assumption 2, welfare can just as well be greater with emissions trading than with an emissions tax. For example, if perturbations result in permit prices that only lower the aggregate emissions such that they fall between the first-best values and emissions under an emission tax, emissions trading yields greater welfare. This could be the case, when for example an NGO buys permits with the aim to keep the permits in order to lower the level of total emissions and hence the level of damages.

The model can be compared to the "prices versus quantities"-literature. There, uncertainty stemming for example from uncertain final product demand affects both instruments and the demand of emissions, but yields different impacts on the aggregate social welfare. In the current model, the perturbations affect not the demand of emissions (by the regulated firms) but the market clearing condition, which is a part of the definition for the equilibrium in the permit market. Further work should include a specific analysis of the incentives for the trades by unregulated participants in the market.

Appendix

A.1 More than two types of perturbations

Define $B := \{\tau_1, \ldots, \tau_w\}$ as the set of time periods in which perturbations occur. Suppose that $2 \leq w \leq T$. (If $w = 1$, either an emission tax or emissions trading will yield a higher welfare level depending on the direction of the perturbation.) Define $\mathcal{P} := \{p_{\theta^{\tau_1}}^{\tau_1}, \ldots, p_{\theta^{\tau_w}}^{\tau_w}\}$ as the set of perturbed prices. The following assumption characterizes set $\mathcal{P}$:

Assumption 3. Let $p_{\theta^u}^u \in \mathcal{P}$ for any $u \in B$. There exists an odd number of $p \in \mathcal{P} \setminus \{p_{\theta^u}^u\}$ such that $|p - t| = |p_{\theta^u}^u - t|$.

Denote $[p_{\theta^u}^u] := \{p \in \mathcal{P} \mid |p - t| = |p_{\theta^u}^u - t| \text{ for some } u \in B\}$. This is an equivalence class relative to relation $\sim$ defined on $\mathcal{P}$ as $p \sim q$ iff $|p - t| = |q - t|$ for all $p, q \in \mathcal{P}$. Thus, the union of $[p_{\theta^u}^u]$ is $\mathcal{P}$. Note that for every $[p_{\theta^u}^u]$ there exists a corresponding set of periods $[u] = \{\tau \in B \mid |p_{\theta^\tau}^\tau - t| = |p - t|, p \in [p_{\theta^u}^u]\}$, which form a partition of B.

Assumption 4.

(i) $\sum_{p \in [p_{\theta^u}^u]} (p - t) = 0$.

(ii) Function Z_τ is strictly concave.

Let the representatives of classes $[u]$ be indexed with index set U.

Proposition 2. *If Assumptions 1, 3 and 4 hold, the aggregate welfare is greater with an emission tax than with auctioned permits.*

Proof. As in the proof of Proposition 1,

$$K(p_\theta) - K(g) < \nabla K(g)(p_\theta - g) = \sum_{u \in U} \sum_{\tau \in [u]} K_\tau'(t) \left(p_{\theta^\tau}^\tau - t\right). \tag{A.12}$$

The right side is zero, since $\sum_{\tau \in [u]} K_\tau'(t) \left(p_{\theta^\tau}^\tau - t\right) = 0$ for any class $[u]$ (the proof of this is the same as the proof of Proposition 1 after replacing A, A^+ and A^- with $[u]$, $[u]^+$ and $[u]^-$). $\square$

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