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Trade Induced Firm Productivity and Division of Labor in a General Equilibrium with Oligopoly

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Abstract

We construct a general equilibrium model with oligopoly that embeds Smith's (1776) famous theory of the division of labor under vertical specialization. Firms have to perform their sequential tasks in their product chain to produce the final good. To perform their tasks, firms optimally organize the teams and assign them tasks but they incur costs in performing this organizational work. If the number of teams increases, then firm productivity also improves. We show that, if the number of firms is exogenous, trade liberalization does not affect firm productivity. Meanwhile, if the number of firms is endogenous, trade liberalization improves firm productivity. This occurs because pro-competitive trade liberalization decreases the number of domestic firms. The surviving firms can then employ more labor and increase their productivity because, using new employment, they organize the new teams to promote a deeper division of labor in relation to their product lines.

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1 Introduction

Does trade liberalization increase firm productivity? Recent empirical studies show that pro-competitive trade liberalization increases plant-level productivity as well as the aggregate productivity arising from resource reshuffling.[1)] In our study, we focus on the relationship between firm productivity and resource reshuffling.

To investigate the relationship between resource reshuffling and firm productivity, we construct a model based on the general oligopolistic equilibrium (GOLE) model of Neary (2016) with a division of labor under the vertical specialization of Chaney and Ossa (2013). The main results are summarized as follows. This model reveals that trade liberalization reduces the number of firms. Surviving firms then acquire additional workers previously employed in exiting firms, and the increase in employment in each surviving firm promotes a deeper division of labor on that firm's product line and increases firm productivity. Consequently, the total welfare of each country improves.

The division of labor is an important firm characteristic. Using the example of a pin factory, Smith (1776) emphasized that a deeper division of labor increases firm productivity. He stated that "one man draws out the wire; another straights it; a third cuts it; a fourth points it; a fifth grinds it at the top for receiving the head (...)." Ethier (1982) rationalizes the story of the pin factory in a monopolistically competitive model on the basis of Krugman's framework (1980). His rationalization depends on the average costs falling with horizontal specialization as the variety of intermediate inputs increases. In contrast, Chaney and Ossa (2013) introduce vertical specialization of tasks into the monopolistic competitive model in Krugman's (1979) framework. They demonstrate that an increase in market size promotes a deeper division of labor, thus increasing firm productivity and number of firms. In their framework, the extent of the division of labor is measured by the (optimal) number of teams in a firm's production line.

Kamei (2014) develops the general oligopolistic equilibrium (GOLE) model constructed by Neary (2016) with a division of labor under the vertical specialization of Chaney and Ossa (2013) to show a relationship between pro-competitive policy and firm productivity.[2)] Kamei (2014) finds that a pro-competitive policy disrupts a deeper division of labor and thus decreases firm productivity because such a pro-competitive policy increases the number of domestic firms and decreases the level of employment within each firm.

However, Kamei (2014) only investigates the case of an autarky economy. This paper modifies the autarky economy of Kamei (2014) into an open economy and investigates how trade

1) Wagner (2012) surveys recent seminal papers on the relationship between international trade and firm productivity.

2) Colacicco (2014) surveys Neary (2016) and subsequent research on the GOLE model.

liberalization affects firm productivity. We reveal that, if the number of firms is determined exogenously, trade liberalization does not affect firm productivity, while if the number of firms is endogenous, trade liberalization increases firm productivity. This is because, under the free entry condition, pro-competitive trade liberalization that implies an increase in the number of foreign firms decreases the number of domestic firms and, hence, the surviving domestic firms employ more labor. The surviving firms then increase the number of teams involved in the division of labor on their product line, increasing firm productivity.

In recent years, a considerable number of theoretical studies have examined trade-induced firm productivity. Trade liberalization promotes a focus on core competency products of multi-product firms (Eckel and Neary, 2010; Bernard, Redding, and Schott, 2011); increases innovation incentives among domestic firms to deter the entry of foreign rivals (Aghion et al, 2005); and increases R&D investment depending on the cost of education in a general oligopolistic equilibrium model (Morita, 2010). Our approach differs from the above approaches.

The remainder of this paper is structured as follows. Section 2 constructs a basic model for a situation of autarky. Section 3 shows that if the number of firms is exogenous, then trade liberalization does not affect firm productivity and welfare in each country Section 4 considers the free entry and exit case and reveals that trade liberalization promotes a deeper division of labor, and thus an increase in firm productivity and welfare. Section 5 concludes.

2 The Model

In this section, we merge the general oligopolistic equilibrium (GOLE) model with the vertical specialization of tasks in Chaney and Ossa (2013). In the economy there is a continuum of sectors, $z \in [0, 1]$ whose products are assumed to be tradable. In this section we construct the basic model under autarky. Next, we investigate the open economy case in Section 3.

2.1 Preferences

There are $L > 0$ consumers in each country and the total consumption in sector z is $x(z)$ with a price of $p(z)$. The representative consumer then maximizes the following utility function, u:

$$u = \int_0^1 \ln x(z) dz, \tag{1}$$

where we assume the logarithmic utility function to simplify the following analysis. The budget condition is written as follows:

$$\int_0^1 p(z)x(z)dz \leq I. \tag{2}$$

The inverse demand function is derived from Equations (1) and (2) as follows:

$$p(z) = \frac{1}{\lambda x(z)}, \tag{3}$$

where λ is the marginal utility of income. In accordance with Kamei (2014), we set $\lambda = 1$. Hence, $\lambda = 1/I$ is obtained when we choose the logarithmic utility function and the result will simplify the following analysis.

2.2 Production

2.2.1 Cournot competition

In the following discussion, the production sector z has $n(z)$ firms that compete *à* la Cournot in sector z. From the assumption of the continuum of the sector, the oligopolistic firms cannot influence macroeconomic variables, such as labor wage, w, and total income, I, and hence these firms rationally ignore those macroeconomic variables. Additionally, $y(z)$ is the output of each firm in sector z, and all firms in each sector are assumed to be identical. Therefore, the total output of sector z is denoted by $x(z) = n(z)y(z)$. In this section the number of firms is exogenous. In section 4, we investigate the free entry case to analyze the reallocation effect of trade liberalization, which affects firm employment and firm productivity. The profit for each firm in sector z is defined as follows:

$$\pi(z) = p(z)y(z) - \tilde{c}(z) - wF, \tag{4}$$

where $\tilde{c}(z)$ is the total variable cost under the optimal number of teams in the firms in sector z to be explained below and $F(> 0)$ is the parameter of fixed entry cost.

Hence, the profit maximizing condition is derived from Equations (3) and (4) as follows:

$$\frac{\partial \pi(z)}{\partial y(z)} = 0 \Leftrightarrow \frac{n(z) - 1}{n(z)^2 y(z)} = \frac{\partial \tilde{c}(z)}{\partial y(z)}, \tag{5}$$

where the LHS and RHS of Equation (5) represent each firm's marginal revenue and marginal cost, respectively.

2.2.2 Division of labor under vertical specialization

Here, we define production costs as follows. Following the descriptions of Chaney and Ossa (2013) and Kamei (2014), we construct a model of the division of labor under vertical specialization. To produce a final good, each firm conducts a number of tasks in sequence. We define the output of raw materials as an early task in the task sequence. The length of the segment of all tasks is normalized to 2, which is defined as a production line. If the firm conducts tasks from 0 to $\omega_1 \in [0, 2]$, an intermediate good ω_1 is produced. Next, the firms perform tasks $\omega > \omega_1$ to obtain the final good. Similarly, if the firm conducts the tasks from ω_1 to $\omega_2 \in [\omega_1, 2]$, then the intermediate good ω_2 is produced, and so on. Finally, when a final task 2 is finished, which implies the completion of an iteration of an arranged task, the firm obtains one unit of the final good.[3)]

To produce the final goods, firms must organize teams that are assigned tasks. The number of teams in each firm in sector z is denoted as $t(z)$. To acquire a core competence $q \in [0, 2]$ of a team in the production line, each firm requires $f > 0$ units of labor before the teams perform their tasks.

Following Chaney and Ossa (2013), the labor requirements for each team to produce one unit of intermediate goods ω_2 are defined as follows:

$$l(\omega_1, \omega_2) = \frac{1}{2} \int_{\omega_1}^{\omega_2} |q - \omega|^{\gamma} d\omega, \tag{6}$$

where $\gamma > 0$ is a efficiency parameter of the division of labor. All teams are symmetric, which implies γ and f are identical across teams.

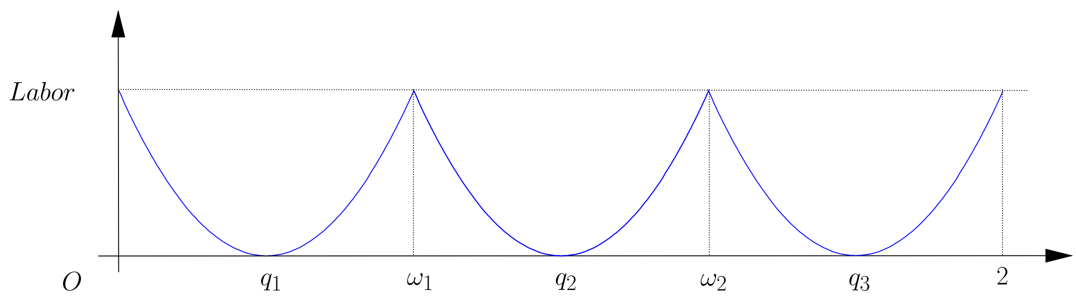

Figure 1: Labor requirement of each task: t=3 case

3) The description of the production process resembles that given by Dixit and Grossman (1982)

A team with a smaller range of tasks leads to a smaller labor requirement to produce a final good. Thus, if firms are rational, the core competences of teams are placed at equal intervals on the product line. For example, Figure 1 depicts the labor requirement at $t = 3$. The horizontal axis represents the tasks from 0 to 2, and the vertical axis represents the labor requirement of each task except the fixed input. The firm then divides its product line into three segments because it involves three teams.

From Equation (6), the firm's total variable cost, $c(z)$, is derived as follows:

$$c(z) = w\left(t(z)f + t(z)y(z)\int_0^{\frac{1}{t(z)}} \omega^\gamma d\omega\right) \tag{7}$$

$$= w\left(t(z)f + \frac{y(z)t(z)^{-\gamma}}{1+\gamma}\right). \tag{8}$$

From Equation (8), the firm cannot reduce production cost by organizing an infinite number of teams because there is a team fixed cost, f, for organizing a team. Thus, a firm in sector z derives the optimal number of teams $\tilde{t}(z)$, which minimizes $c(z)$ given $y(z)$:

$$\frac{\partial c(z)}{\partial t(z)} = 0 \Leftrightarrow \tilde{t}(z) = \left[\frac{\gamma}{\gamma+1}\frac{y(z)}{f}\right]^{\frac{1}{\gamma+1}}. \tag{9}$$

From Equation (9), if $\gamma = 0$, then $\tilde{t}(z) = 0$.

Substituting Equation (9) into Equation (8), the total variable cost under the optimal number of teams, $\tilde{c}(z)$, is derived as follows:

$$\tilde{c}(z) = wy(z)^{\frac{1}{\gamma+1}} f^{\frac{\gamma}{\gamma+1}} \left(\frac{1+\gamma}{\gamma}\right)^{\frac{\gamma}{1+\gamma}}. \tag{10}$$

We partially differentiate Equation (10) by $y(z)$ and obtain the marginal cost as follows:

$$\frac{\partial \tilde{c}(z)}{\partial y(z)} = \frac{1}{1+\gamma} wy(z)^{\frac{-\gamma}{\gamma+1}} f^{\frac{\gamma}{\gamma+1}} \left(\frac{1+\gamma}{\gamma}\right)^{\frac{\gamma}{1+\gamma}}. \tag{11}$$

From Equation (11), we also derive $\frac{\partial \tilde{c}(z)^2}{\partial^2 y(z)} < 0$.

2.3 The autarkic equilibrium

In this section, we consider the autarkic economy in an equilibrium. From Equations (5) and (11), we obtain

$$y(z) = \left[\frac{n(z)-1}{n(z)^2} \cdot \frac{\gamma^{\frac{\gamma}{1+\gamma}}(1+\gamma)^{\frac{1}{1+\gamma}}}{wf^{\frac{\gamma}{1+\gamma}}}\right]^{1+\gamma}. \tag{12}$$

The total number of workers is L. Hence, the labor market-clearing condition is derived as follows:

$$L = \int_0^1 n(z)\left[y(z)^{\frac{1}{1+\gamma}} f^{\frac{\gamma}{1+\gamma}} \left(\frac{1+\gamma}{\gamma}\right)^{\frac{\gamma}{1+\gamma}} + F\right] dz. \tag{13}$$

Therefore, we derive the total output of each firm in autarkic equilibrium $y^A(z)$ as follows:

$$y^A(z) = \left[\frac{L/n(z) - F}{f^{\frac{\gamma}{1+\gamma}}\left(\frac{1+\gamma}{\gamma}\right)^{\frac{\gamma}{1+\gamma}}}\right]^{1+\gamma}. \tag{14}$$

Additionally, using Equation (14), the total output in autarky is obtained as follows:

$$n(z)y^A(z) = \left[\frac{(L - Fn(z))n(z)^{\frac{-\gamma}{1+\gamma}}}{f^{\frac{\gamma}{1+\gamma}}\left(\frac{1+\gamma}{\gamma}\right)^{\frac{\gamma}{1+\gamma}}}\right]^{1+\gamma}. \tag{15}$$

In the autarky equilibrium, the optimal number of teams in each firm is derived as

$$\tilde{t}^A(z) = \frac{\gamma}{1+\gamma}\frac{L/n(z) - F}{f}. \tag{16}$$

The above expression of the optimal number of teams, which is interpreted as firm productivity, will be useful in understanding the effect of trade liberalization in the following section.

3 The Open Economy

In the open economy, the world is divided into two symmetric countries: home country and foreign country. In terms of assumptions regarding the symmetric countries, we focus on analysis of the home country in the following discussion.

In this section, we investigate how trade liberalization affects the optimal number of teams, firm productivity and welfare. In the open economy, total consumption of good z in each country is comprised $x^T(z) = 2n(z)y^T(z)$. Additionally, the labor market-clearing condition in the open economy is derived as follows:

$$L = \int_0^1 n(z)\left[2^{\frac{1}{1+\gamma}} y^T(z)^{\frac{1}{1+\gamma}} f^{\frac{\gamma}{1+\gamma}} \left(\frac{1+\gamma}{\gamma}\right)^{\frac{\gamma}{1+\gamma}} + F\right] dz. \tag{17}$$

In addition, in the open economy, the each firm produces $2y^T(z)$, which is derived in an equilibrium as follows:

$$2y^T(z) = \left[\frac{L/n(z) - F}{f^{\frac{\gamma}{1+\gamma}} \left(\frac{1+\gamma}{\gamma}\right)^{\frac{\gamma}{1+\gamma}}}\right]^{1+\gamma} \tag{18}$$

Moreover, we derive the optimal number of teams and total output in sector z in equilibrium from equation (18) as follows:

$$\tilde{t}^T(z) = \frac{\gamma}{1+\gamma} \frac{L/n(z) - F}{f}, \tag{19}$$

$$2n(z)y^T(z) = \left[\frac{(L - Fn(z))n(z)^{\frac{-\gamma}{1+\gamma}}}{f^{\frac{\gamma}{1+\gamma}} \left(\frac{1+\gamma}{\gamma}\right)^{\frac{\gamma}{1+\gamma}}}\right]^{1+\gamma}. \tag{20}$$

From Equations (14), (15), (16), (18), (19) and (20), we know that if the number of firms is assumed to be exogenous, then $2y^T = y^A$, $x^T = x^A$ and $\tilde{t}^T(z) = \tilde{t}^A(z)$. We immediately obtain the following proposition:

Proposition 1. *If the number of firms is exogenous, trade liberalization does not affect the optimal number of teams, total output and, therefore, welfare in each country.*

Comparing the autarky and open economy, if the number of firms is exogenous, trade liberalization does not affect the total output and the optimal number of teams in each firm. Hence, trade liberalization does not affect firm productivity in each sector.

4 Free Entry and Exit

From now on, we consider free entry and exit of firms whereby each firm enters the market until $\pi(z) = 0$. From $\pi(z) = 0$, total income I equals total wages: $I = \int_0^1 [n(z) \cdot 0] dz + wL = wL$. Additionally, we obtain $w = 1/L$ because the assumption $\lambda = 1$ derives $I = 1$. Using Equations (12), (18), (13), (17) and $w = 1/L$, we derive the numbers of firms in the autarky,

$n^A(z)$, and in the open economy, $n^T(z)$, as follows;

$$n^A(z) = \frac{-L\gamma + \sqrt{L^2\gamma^2 + 4FL(1+\gamma)}}{2F} > 0. \tag{21}$$

$$n^T(z) = \frac{-L\gamma + \sqrt{L^2\gamma^2 + \frac{4FL(1+\gamma)}{2}}}{2F} > 0. \tag{22}$$

From Equations (21) and (22), we state the following proposition:

Proposition 2. *Trade liberalization reduces the number of firms.*

Proof. From Equations (21) and (22), we immediately obtain $n^A(z) - n^T(z) > 0$. ■

We explain the intuition of Proposition 2. Trade liberalization implies an increase in the number of foreign rivals, which reduces the profits of each firm, and, conversely, increases firm profits in the export markets. From $n^A(z) - n^T(z) > 0$, we know that the former effect exceeds the latter effect. Consequently, trade liberalization reduces both firm profits and firm number.

Moreover, using Equations (16), (19) and Proposition 2, we derive the following proposition:

Proposition 3. *Trade liberalization increases the optimal number of teams, and hence improves firm productivity.*

Proof. From Equations (16), (19) and $n^A(z) - n^T(z) > 0$, we immediately obtain $t^T(z) - t^A(z) > 0$. ■

From Proposition 2, we know that trade liberalization reduces the number of firms. Surviving firms then acquire labor previously employed in exiting firms. This increase in employment promotes a deeper division of labor, and so increases firm productivity. Next, we obtain the following proposition:

Proposition 4. *Trade liberalization increases total output in each sector and improves welfare.*

Proof. From Equations (15), (20), (21) and (22), the difference between the total output for each sector under the autarky and open economy is

$$x^T(z) - x^A(z) = \frac{1}{f^{1+\gamma}\left(\frac{1+\gamma}{\gamma}\right)^{\gamma}} \left[\underbrace{\frac{L - Fn^T(z)}{n^T(z)^{\frac{\gamma}{1+\gamma}}} - \frac{L - Fn^A(z)}{n^A(z)^{\frac{\gamma}{1+\gamma}}}}_{+} \right]^{1+\gamma} > 0, \tag{23}$$

$$\Leftrightarrow u\left(x^T(z)\right) - u\left(x^A(z)\right) > 0. \tag{24}$$

■

We explain the intuition of Proposition 4. From Proposition 3, we know that trade liberalization increases firm productivity. Therefore, an increase in firm productivity promotes an increase in total output and a decrease in prices, thus improving welfare.

5 Conclusion

To suggest a new interpretation between trade liberalization and firm productivity, we merge an oligopolistic competition in a general equilibrium with the division of labor under vertical specialization. The results are summarized as follows. Trade liberalization decreases the number of firms in each country. The surviving firms then acquire labor previously employed by the exiting firms. Hence, the surviving firms promote a deeper division of labor and increased firm productivity. Finally, trade liberalization improves welfare.

While our model suggests a new interpretation of trade induced firm productivity, some problems with the model still need to be solved. First, although non-traded sectors exist in the real world, we assume the existence of traded-sectors only. Generally, trade liberalization directly affects traded-sectors, $z \in [0, \tilde{z}]$. However, trade liberalization also indirectly affects non-traded sectors, $z \in [\tilde{z}, 1]$, because trade liberalization affects the income of consumers in a general equilibrium framework. Moreover, when some non-traded sectors become traded ones, $\Delta\tilde{z} > 0$, then we can investigate how the partial trade liberalization affects the firm productivity of the traded, $z \in [0, \tilde{z} + \Delta\tilde{z}]$, and non-traded sectors, $z \in [\tilde{z} + \Delta\tilde{z}, 1]$, respectively.

Second, we can consider a more general case where the efficiency parameters of division of labor, γ or f, are heterogeneous between countries. Trade liberalization would then cause sector productivity differentials between countries, which implies that trade liberalization *a posteriori* generates differentials in comparative advantage between countries.

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