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A flexible nested logit model

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Abstract

This paper develops simple but flexible nested logit. The basic idea is to introduce heterogeneity in the key parameter driving substitution patterns in the nested logit model: the correlation between utilities. By doing so the model generates a flexible demand system, overcoming an undesirable property of the classic nested logit. It is also relatively easy to estimate and compute, properties that could prove useful to researchers and practitioners trying to avoid the operational costs (i.e. numerical difficulties) of the general Random Coefficient model.

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1.Introduction

Demand estimation has been an important element in many industrial organizations studies. Indeed, after determining the preference parameters it is possible to address issues such as elasticity measurement, consumer surplus and market power. Typical studies in the field use aggregate data and discrete-choice models to uncover demand parameters. Discrete-choice models, indeed, have become a frequent choice in the demand literature, especially due to the reduction of parameter space and the ability to accommodate consumer heterogeneity, overcoming the limitations of competing models to deal with markets with many varieties (dimensionality course) , such as the Almost Ideal Demand System (Deaton and Muellbauer, 1980) .

The nested logit (NL) and the random-coefficient logit (RC) are the two most popular options within the discrete choice toolbox[1]. The former is simple to estimate, as it can be framed into basic econometric models. Indeed, it is linear in the parameters and (most importantly) in the error term, allowing for estimation using standard OLS (or 2SLS to correct for endogeneity). However, the simplicity comes at a cost: flexibility, a criterion to evaluate demand models in which the NL does not perform so well. For instance, it implies by construction equal cross price elasticities within nests, a property that may be undesirable in many applications. The latter model (RC), developed by Berry, Levinsohn and Pakes (1995), BLP henceforth, overcomes many of the nested logit´s limitations. By introducing consumer heterogeneity in the preferences for characteristics, the model is able to produce a flexible demand system without fixing a priori some pattern of substitution among goods. However, given the high level of non-linearity in the error term, the econometric model becomes somewhat complex and full of practical difficulties, usually stemming from numerical problems the researcher usually faces. In fact, an active area of research is to improve the practical use and reliability of this model[2]. For instance, Knittel and Metaxoglou (2014) show multiple numerical challenges researches usually face when estimating RC models: convergence to points where the first- and second-order conditions fail, convergence to multiple locally optimal points and other convergence difficulties. Their findings also indicate that economically interesting variables, such as price elasticities, consumer welfare variation and changes in profits following merger simulations vary significantly for different numerical setups. The main advance in this direction can be found in Dubé et al.(2012), who propose rewriting the optimization problem as a constrained one. The new approach, known as MPEC (mathematical program with equilibrium constraints), reduces significantly the computational burden and overcomes additional numerical difficulties of the tradition RC estimation method proposed by BLP. However, according to Dubé et al.(2012), for models with a few markets and a large number of products the numerical the advantages of MPEC are not significant. Therefore, despite MPEC´s important contribution to the literature, numerical problems are still present.

This paper develops a model that captures the advantages of each of the two competing discrete choice models (simplicity of the NL and the flexibility of the RC), providing then an alternative choice that can prove to be adequate in settings in which the limitations of NL and RC are severe or costly to overcome.

[1] A full description of the nested logit and the random coefficient logit can be found in Berry (1994) and Berry, Levinsohn and Pakes (1995), respectively.

[2] See Dube et al. (2012), Judd and Skrainka (2011) and Skrainka (2011),

2. Model

This section presents the flexible nested logit- FNL[3]. Consumers rank products according to their characteristics and prices. There are *N+1* choices in the market, *N* inside goods and one reference good (or outside good).

Consumer *i* chooses brand *j*, given price p_j, a K-dimensional row vector of observed characteristics (x_j), an unobserved characteristic (denoted by the scalar ξ_j), and unobserved idiosyncratic preferences v_{ij}, according to the following indirect utility function:

(1) $$u_{ij} = \alpha p_j + x_j \beta + \xi_j + v_{ij}$$

The parameter α is a scalar representing price disutility and β is K-dimensional column vector of coefficients.

The last term (v_{ij}), in turn , is decomposed into:

(2) $$v_{ij} = \zeta_{ig} + (1-\sigma_i)\varepsilon_{ij}$$

The first term ζ_{ig} represents the effect of shocks that affects all products within a given nest g and its distribution depends on σ_i, a parameter that measures the correlation between the levels of utilities for goods within the same nest (Berry,1994). Note that σ_i varies across consumers and can be statistically modeled by a probability distribution with parameters θ and support [0,1]. This is the distinction between the FNL and the standard NL. In the latter, σ is the same for all consumers, which imposes a relatively inflexible elasticity matrix. By introducing heterogeneity in this parameter, this approach (FNL) improves the flexibility of the elasticity matrix, as it does not impose equal cross price elasticities within goods within a nest. The other component (ε_{ij}), is an i.i.d. extreme value random variable.

For expositional purposes, it is convenient to rewrite the utility function as

(2) $$u_{ij} = \delta_j + v_{ij}$$

$\delta_j = \alpha p_j + x_j \beta + \xi_j$ represents the mean utility of product *j* obtained from price and characteristics. The utility derived form the consumption of the outside good can be normalized to zero u_{i0}=0. Then, following standard manipulation of the nested logit, the probability of individual *i* choosing good *j* (s_{ij}) in a given nest g takes the familiar logit form

(3) $$s_{ij/g}(p, \delta(\alpha, \beta, X, \xi), \sigma_i) = \frac{e^{\delta_j/(1-\sigma_i)}}{D_{g_i}}$$

where $D_{g_i} = \sum_{j \in J_g} e^{\delta_j/(1-\sigma_i)}$.

[3] A model that closely related to the FNL is one developed by Venkataraman and Kadiyali (2005). However their model is based on a Generalized Extreme Value model with more parameters than the FNL.

In turn, the probability of group g being chosen by consumer i is:

(4) $$s_{ig} = \frac{D_g^{(1-\sigma_i)}}{\sum_g D_g^{(1-\sigma_i)}}$$

Then, the probability that consumer i chooses product j is given by

(5) $$s_{ij} = \frac{e^{\delta_j/(1-\sigma_i)}}{D_{g_i}^{\sigma_i} \sum_g D_{g_i}^{1-\sigma_i}}$$

The scalar s_{ij} is the conditional market share of product j, i.e. the market share that would prevail if all individuals had the same σ. In the FNL this is not true, therefore it is necessary to aggregate to the product level in order to take the model to the (available) data (shares). This is done by calculating the unconditional probability of product j being chosen, which is given by:

(6) $$s_j(\delta(\alpha,\beta,X,p,\xi),\theta) = \int s_{ij}(\delta(\alpha,\beta,X,p,\xi),\sigma_i)\,dF_\sigma$$

The theoretical market share of product j (s_j) is a function of the parameters of the σ_i distribution (F_σ represents its cumulative distribution) and the *N+1*-dimensional vector δ, which collects all δ_j*'s*. Notice that, by definition, δ is an implicit function of α, β, X (a matrix containing all observed characteristics of all products in the market) and ξ, a vector that collects all ξ_j´s.

One can then proceed as described in Berry (1994) and BLP, who propose an algorithm with a nested fixed point to minimize a GMM objective function. Although this equation is still non-linear in the error terms (ξ_j´s), it avoids most numerical problems documented in the literature as it is a minimization problem that is easy to control since it has only a one dimensional random component to integrate out[4].

3. Monte Carlo Results

To illustrate the model, we conduct standard Monte-Carlo experiments to study the performance of the estimation algorithm presented in the previous section in retrieving the true parameters from an artificial data set.

[4] This is so if one exploits the theoretical bounds of sigma (between zero and one) and assumes that its distribution has only a one dimensional parameter. One example is the triangular distribution. Other distributions can be used.

We follow the standard assumption that marginal costs of product j are constant and given by

(7) $$c_j = \gamma_1 W_1 + \gamma_2 W_2 + \gamma_3 x_j + \omega_j$$

where x_j is a one dimensional vector of exogenous characteristic and *W1* and *W2* are cost shifters that do not affect consumers preferences. The three variables are drawn independently from a N(1,1).

We also assume that that unobserved cost and demand shocks are correlated and are drawn from the following bivariate normal distribution with mean zero and variance 1 for both variables and covariance 0.3.

Since our focus is on the demand side, we adopt the assumption that competitive markets. Therefore the supply side can be simply described by p=c. Then, endogenous prices are equal to marginal costs and are generated by the following specification

(8) $$p_j = \gamma_1 W_1 + \gamma_2 W_2 + \gamma_3 x_j + \omega_j$$

We estimate the model using GMM with moments generated by the exogenous characteristics X_1 and cost shifters (instruments) W_1, W_2.

Table1 - Monte-Carlo studies

	True	Bias	St. err.	RMSE**
J=25,M=4				
	1	-0.041019	0.106793	0.1144
	-1	-0.028953	0.04622	0.05454
	0.5	-0.029406	0.106212	0.110208
J=25,M=8				
	1	-0.01268	0.07494	0.076013
	-1	-0.016115	0.04114	0.04419
	0.5	-0.01422	0.08835	0.089492
J=25,M=12				
	1	-0.00601371	0.0651906	0.065467
	-1	-0.0153505	0.03875	0.04168
	0.5	-0.00975362	0.0802232	0.080814

*J= Number of products; M= Number of markets.**Root mean square error

Table 1 above presents the Monte-Carlo results obtained from panel data sets with 25 products and different number of markets M. For each experiment we use 150 replications. The results indicate that the estimators are consistent, since all the biases are small even at relatively small sample sizes (for instance, the scenario with 200 observations - 25 five products and 8 markets). Also, Table 1 makes clear that the estimates converge properly since as the sample size increases the relevant statistics (biases, standard errors and consequently the RMSEs) get closer to zero.

4. Conclusion

This paper fully develops a simple but flexible nested logit by introducing heterogeneity in the key parameter driving substitution patterns: the correlation between utilities. By doing so the model generates a flexible demand system, overcoming an undesirable property of the classic nested logit. It is also relatively easy to estimate and compute, properties that could prove useful to researchers and practitioners trying to avoid the operational costs (i.e. numerical difficulties) of the general Random Coefficient model. Monte-Carlo experiments also show that the estimates converge properly since biases, standard errors and consequently the RMSEs get closer to zero as the sample size increases.

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