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Habit formation, growth, and Ramsey's conjecture

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Abstract

This note deals with long-run distribution of wealth (i.e., ownership pattern of physical capital) in an endogenously growing economy populated by different types of households with habit-forming consumption. We show a counter-intuitive result where the most impatient household (with the highest subjective discount rate) could eventually own “almost all” of the economy's capital, as long as it has the strongest consumption habit. Furthermore, we consider whether patience becomes the sole determinant of eventual wealth distribution in a perpetually growing economy with habit formation, proposing a slight modification to Ramsey's conjecture.

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1. Introduction

Habit formation has recently received extensive research interest from economists. In macroeconomic theory, Carroll *et al.* (1997, 2000) and Shieh *et al.* (2000), among others, develop endogenous growth models that include habit formation related to consumption. By extending these studies, which are less concerned with distributive issues, this note clarifies how the differences with regard to both consumption habits and subjective discount rates among households affect the long-run wealth distribution as well as aggregate output growth.

Regarding the long-run wealth distribution of an economy, Ramsey (1928) conjectures that the most *patient* class of households, who have the *lowest* subjective discount rate, would own the *entire* (physical) wealth of the economy. Becker (1980) confirms Ramsey’s conjecture using a neoclassical growth model in discrete time whereas Mitra and Sorger (2013) do so with a continuous-time specification. However, others show that Ramsey’s conjecture does not hold in some cases. For example, Drugeon and Wigniolle (2017) prove, by examining a neoclassical growth model in which each household with temptation motive must bear self-control costs, that the most patient household could *not* necessarily own all capital, even in the long run. Nakamura (2014) states that in a perpetually growing economy with “AK” technology, the elasticity of intertemporal substitution would be a more important determinant of long-run wealth distribution than patience. Specifically, he reveals that the most *impatient* class of households with the *largest* elasticity of intertemporal substitution could eventually hold “almost all” of the wealth in an economy with sustained growth.¹

This note contributes to the related literature by presenting a simple endogenous growth model with “internal” formation of consumption habits, in which the eventual wealth distribution becomes extremely uneven. Specifically, we show a counter-intuitive result where, all other things being equal, households with the *strongest* consumption habit could eventually hold “almost all” of the economy’s physical wealth, even though they are the most *impatient* (i.e., they have the *highest* subjective discount rate).² This result is in sharp contrast to Ramsey’s conjecture. Furthermore, we consider whether patience becomes the sole determinant of eventual wealth distribution in our model, slightly modifying Ramsey’s conjecture.

The remainder of this note proceeds as follows. In Section 2, we describe the analytical

¹Tsukahara (2016) provides an extension of Nakamura (2014) in which each household prefers to hold capital as well as engage in consumption, and shows that in an economy with sustained growth, the most *impatient* class of households could eventually own the “entire” capital (not “almost all” the capital).

²In contrast to our study, Díaz *et al.* (2003) calibrate a stochastic neoclassical growth model and find that a persistent consumption habit increases precautionary saving and reduces wealth inequality.

model, in which we extend Nakamura (2014) to include habit formation. We analyze the model in Section 3, and state our results on eventual wealth distribution in Section 4. Section 5 provides the concluding remarks. Mathematical details are explained in the appendices.

2. Model

We denote time by t , which goes from 0 to $+\infty$. Suppose a closed economy that always includes two types of households, indexed by $i \in \{H, L\}$. There is no population growth, and the total population of households is normalized to unity. Let $\lambda \in (0, 1)$ denote the share of H -type households in the population. Hence, the share of L -type households is $1 - \lambda$. For analytical ease, we assume that λ is constant through time. We ignore government activities.

For household i , let $k_i(t)$ and $c_i(t)$ denote capital holdings and consumption at time t , respectively. The flow budget equation of household i is given by

$$\dot{k}_i(t) := \frac{dk_i(t)}{dt} = r(t)k_i(t) - c_i(t), \quad (1)$$

where $r(t)$ represents the (real) rate of return on capital at time t . We denote the initial value of $k_i(t)$, which is exogenously given, by $\bar{k}_i > 0$. As in Nakamura (2014), we do not consider wage income explicitly, for analytical simplicity.

Hereafter, we use a dot ($\dot{\cdot}$) over the variable $x(t)$ to denote the first derivative of $x(t)$ with respect to time; that is, $\dot{x}(t) := dx(t)/dt$. The time argument, t , is often suppressed for brevity.

Let z_i denote the *habit stock* of household i . We assume that the habit formation of each household follows an *internal* (or *inward-looking*) process; specifically,

$$\dot{z}_i = \alpha_i(c_i - z_i), \quad (2)$$

where $\alpha_i > 0$ is hereafter referred to as the *adjustment speed of habit*.³ The initial value of z_i , denoted by $\bar{z}_i > 0$, is exogenously given.

Suppose that all households live infinitely and that household i chooses the time path of $c_i(t)$ to maximize its dynastic utility,

$$U_i := \int_0^{+\infty} u_i(c_i, z_i) \exp(-\rho_i t) dt, \quad (3)$$

³It would be interesting to assume the following *external* (or *outward-looking*) habit formation process instead of the internal (or inward-looking) process; specifically, $\dot{z}_i = \alpha_i(c - z_i)$, where $\alpha_i > 0$ and $c := \lambda c_H + (1 - \lambda)c_L$ represents the entire economy's average consumption.

subject to (1) and (2), taking \bar{k}_i and \bar{z}_i as given. In (3), $\rho_i > 0$ denotes the subjective discount rate of household i . Without loss of generality, we assume the following:

Assumption 1. $\rho_H \geq \rho_L$.

Following Carroll *et al.* (1997, 2000) and Díaz *et al.* (2003), we also specify the instantaneous utility function of household i as the “multiplicative” form⁴:

$$u_i(c_i, z_i) := \frac{(c_i z_i^{-\gamma_i})^{1-\sigma_i}}{1-\sigma_i}, \quad (4)$$

where $\gamma_i \in (0, 1)$ reflects the influence of habit on utility and $\sigma_i > 0$ denotes the inverse of the elasticity of intertemporal substitution, provided that z_i is given. Regarding (4), we should note that $u_i(c_i, z_i)$ is *not* jointly concave with respect to (c_i, z_i) and, thus, the usual first-order conditions for the interior optimum might not apply under the internal habit formation assumption.⁵ To cope with this technical difficulty, we follow previous studies (e.g., Gómez [2008, 2010a]) in which the same specification as (4) is employed, and assume

Assumption 2. $\sigma_i > 1$ for each i ,

which is empirically plausible (see, e.g., Havranek *et al.* [2015]).

Let $y(t)$ and $k(t)$ denote the aggregate output at time t and the aggregate capital available at time t , respectively. To describe the aggregate production of the economy as simply as possible, we assume “AK” production technology as $y = Ak$, where $A > 0$ remains constant over time.⁶

Suppose that capital does not depreciate.⁷ It may be noted that in a perfectly competitive capital market, the rate of return on capital will be equal to the marginal productivity of capital. Therefore, we have $r(t) = A$ for each t . We note that $k = \lambda k_H + (1 - \lambda)k_L$.

3. Analysis

3.1. Balanced Growth

The equilibrium dynamics of (c_i, k_i, z_i) resulting from the optimization behavior of household $i \in \{H, L\}$ is described jointly by differential equations (17) to (19), derived in Appendix A.

⁴Another commonly-used specification is the “subtractive” one, such as $u_i(c_i, z_i) := v_i(c_i - \gamma_i z_i)$, where $v_i(\cdot)$ is a strictly concave function. See, e.g., Gómez (2010b).

⁵Hiraguchi (2008) explores this problem in detail.

⁶Carroll *et al.* (1997, 2000) and Shieh *et al.* (2000) do the same.

⁷As long as we assume that the capital stock depreciates at a constant rate, $\delta \in (0, A)$, the results of the present analysis cannot change essentially, provided that A is replaced by $(A - \delta)$.

Given those equations for household i , we focus on a balanced growth path along which c_i, k_i and z_i grow at the same rate for each i . We obtain the following:

$$\xi_i^* := \left(\frac{c_i}{z_i} \right)_{\text{BGP}} = 1 + \frac{A - \rho_i}{\alpha_i[(1 - \gamma_i)\sigma_i + \gamma_i]}, \quad (5)$$

$$\eta_i^* := \left(\frac{z_i}{k_i} \right)_{\text{BGP}} = \frac{\alpha_i[A(1 - \gamma_i)(\sigma_i - 1) + \rho_i]}{\alpha_i[(1 - \gamma_i)\sigma_i + \gamma_i] + A - \rho_i}, \quad (6)$$

$$\chi_i^* := \left(\frac{c_i}{k_i} \right)_{\text{BGP}} = \frac{A(1 - \gamma_i)(\sigma_i - 1) + \rho_i}{(1 - \gamma_i)\sigma_i + \gamma_i} (= \xi_i^* \cdot \eta_i^*), \quad (7)$$

where the suffix ‘‘BGP’’ refers to the balanced growth path. Note that χ_i^* is independent of the adjustment speed of habit, α_i .

The growth rate of k_i on the balanced growth path is given by

$$g_i^* := \frac{\dot{k}_i}{k_i} = A - \chi_i^* = \frac{A - \rho_i}{(1 - \gamma_i)\sigma_i + \gamma_i}.$$

Because $(1 - \gamma_i)\sigma_i + \gamma_i > 0$, the following assumption ensures $g_i^* > 0$.

Assumption 3. $A > \rho_i$ for each i .

On the balanced growth path, the aggregate capital, k , grows at the rate of

$$g^* := \frac{\dot{k}}{k} = A - s_H \chi_H^* - s_L \chi_L^*,$$

where $s_H := \lambda k_H / k$ and $s_L := (1 - \lambda)k_L / k$ are the capital share of households H and L , respectively. Additionally, $s_H + s_L = 1$. Note that g^* is not always constant because g^* depends on the wealth distribution, s_i , which evolves over time.

3.2. Dynamics of Wealth Distribution

We can verify easily that s_H evolves according to

$$\dot{s}_H = (\chi_H^* - \chi_L^*)s_H(s_H - 1), \quad (8)$$

on the balanced growth path. We denote an exogenous initial value of s_H by \bar{s}_H . $\bar{s}_L (= 1 - \bar{s}_H)$ means the initial value of s_L .

Suppose $\bar{s}_H \in (0, 1)$ initially. If and only if $\chi_H^* < \chi_L^*$, then s_H increases and converges to 1. In this case, g^* approaches $A - \chi_H^*$. By contrast, if and only if $\chi_H^* > \chi_L^*$, then s_H decreases and converges to 0. In this case, g^* approaches $A - \chi_L^*$.

We should note that household i will *not* hold all of the capital of this economy, even though $s_i \rightarrow 1$ ($s_j \rightarrow 0$); that is, household i will be much wealthier than household j in the long run, where $i, j \in \{H, L\}$ and $i \neq j$. The reason is straightforward: As long as Assumption 3 is satisfied, both types of households keep accumulating private capital (i.e., $g_H^* > 0$ and $g_L^* > 0$).

We prepare the following to state our main results:

Definition 1. Household $i \in \{H, L\}$ eventually owns *almost all* of the economy's capital, if and only if $s_i \rightarrow 1$ as $t \rightarrow +\infty$ under Assumption 3.

4. Main Results

First, we show the following:

Proposition 1. (a) We have $s_H \rightarrow 1$ (or $s_L \rightarrow 0$), irrespective of $\bar{s}_i \in (0, 1)$ given initially, if and only if

$$\frac{\varepsilon_H}{\varepsilon_L} > \frac{A - \rho_L}{A - \rho_H}, \quad (9)$$

where $\varepsilon_i := 1/[(1 - \gamma_i)\sigma_i + \gamma_i]$, which Carroll *et al.* (2000) refer to as the “infinite-horizon inter-temporal elasticity of substitution” of household $i \in \{H, L\}$.

(b) We have $s_L \rightarrow 1$ (or $s_H \rightarrow 0$) irrespective of the initial value of $\bar{s}_i \in (0, 1)$ if and only if (9) holds with the reverse inequality.

Proof. See Appendix B. ||

Note that Proposition 1 contains Nakamura's (2014) results as a special case in which $\gamma_H = \gamma_L = 0$.

Corollary 1. For each $i \in \{H, L\}$, the adjustment speed of habit, α_i , is irrelevant for eventual wealth distribution.

Proof. Neither α_H nor α_L appears in (7) or (8). ||

The next proposition clarifies how the strength of the consumption habit, γ_i , affects the long-run wealth distribution:

Proposition 2. Suppose that $\gamma_H \neq \gamma_L$ and $\sigma_H = \sigma_L (= \sigma > 1)$. Even though $\rho_H > \rho_L$, household H could eventually own almost all of the economy's capital, irrespective of the initial wealth distribution (\bar{s}_H, \bar{s}_L) , as long as $\gamma_H > \gamma_L$.

Proof. From Proposition 1(a), household H eventually owns almost all of the capital for any initial wealth distribution if and only if (9) holds. For (9) to hold under $\rho_H \geq \rho_L$,

the following should be satisfied:

$$\frac{\varepsilon_H}{\varepsilon_L} > 1, \text{ or } (1 - \gamma_L)\sigma_L + \gamma_L > (1 - \gamma_H)\sigma_H + \gamma_H.$$

When $\sigma_H = \sigma_L = \sigma$, we can rewrite this inequality as $(\sigma - 1)(\gamma_H - \gamma_L) > 0$. Because we assume $\sigma > 1$, we obtain $\gamma_H > \gamma_L$. Therefore, the claim follows. \parallel

Note that as long as Assumption 3 and $\sigma > 1$ are satisfied, the savings rate of a household i on the balanced growth path, given by $(A - \rho_i)/[A\{\sigma - (\sigma - 1)\gamma_i\}]$, strictly increases with the strength of habit, γ_i . In a perpetually growing economy with $\sigma > 1$, the stronger the household's habit (i.e., the larger γ_i is), the greater is the elasticity of intertemporal substitution, defined as $\varepsilon_i(= 1/[(1 - \gamma_i)\sigma_i + \gamma_i])$, and thus the more willing would the household be to postpone its consumption to the future. Eventually, this can give the household with the *strongest* habit ownership of almost all of the economy's capital, even if that household has the highest subjective discount rate.

Recall here that Ramsey's conjecture (in the original sense) is that only the most patient household eventually owns the *entire* capital of the economy. We should note that Ramsey's conjecture cannot hold in our model as long as we have Assumption 3: Under Assumption 3, the impatient (H) and patient (L) households can keep accumulating private capital (i.e., $g_i^* > 0$ for each i). Furthermore, Proposition 1(a) states that the most *impatient* could become the wealthiest eventually. That is, as Nakamura (2014) notes, patience is not always a crucial determinant of long-run wealth distribution in the economy with perpetual growth.

In the rest of this section, we change the analytical viewpoint slightly. Specifically, we consider whether patience becomes the sole determinant of the eventual wealth distribution in our model. For this purpose, we propose the following:

Definition 2. Our modified version of Ramsey's conjecture states that a household would eventually own *almost all* of the economy's capital if and only if that household is the most patient: $\rho_H > \rho_L \Leftrightarrow (s_L \rightarrow 1 \text{ as } t \rightarrow +\infty)$.

According to Definition 2, a household becomes much wealthier than the others over time if and only if that household is the most patient. Note that the modified version of Ramsey's conjecture allows a household other than the most patient one to keep accumulating private capital, which differs from Ramsey's original conjecture that households besides the most patient would eventually own no private capital.

Proposition 3. Suppose that $A > \rho_i$ for each i (Assumption 3). If $\varepsilon_H = \varepsilon_L$, then the modified version of Ramsey's conjecture is true, irrespective of the initial wealth

distribution, (\bar{s}_H, \bar{s}_L) .

Proof. According to Proposition 1(b), we have $s_L \rightarrow 1$ as $t \rightarrow +\infty$ for any initial $\bar{s}_L \in (0, 1)$, if and only if

$$\frac{\varepsilon_H}{\varepsilon_L} < \frac{A - \rho_L}{A - \rho_H}.$$

If $\varepsilon_H = \varepsilon_L$, then the above condition reduces to $\rho_H > \rho_L$ under Assumption 3. Therefore, Proposition 3 follows. \parallel

Proposition 3 states that if $\varepsilon_H = \varepsilon_L$, then the most patient class of households necessarily holds almost all of the economy's capital in the long run. Note that $\varepsilon_H = \varepsilon_L$ can hold, even though $\sigma_H \neq \sigma_L$ and $\gamma_H \neq \gamma_L$, by the definition of ε_i . Evidently, if $\gamma_H = \gamma_L$ (resp. $\sigma_H = \sigma_L$), we have $\varepsilon_H = \varepsilon_L$ only when $\sigma_H = \sigma_L$ (resp. $\gamma_H = \gamma_L$).

Corollary 2. Suppose that both σ_H and σ_L are sufficiently closer to 1. Then, the modified version of Ramsey's conjecture approximately holds, irrespective of whether consumption is habit-forming or not.

Proof. Recall that $\varepsilon_i := 1/[(1 - \gamma_i)\sigma_i + \gamma_i]$. If both σ_H and σ_L are sufficiently close to 1, then ε_H and ε_L are almost equal for any (γ_H, γ_L) . Therefore, the claim follows from Proposition 3. \parallel

5. Concluding Remarks

In this note, we present an endogenous growth model with two types of households and habit-forming consumption. Using this model, we consider how the differences with regard to both consumption habits and subjective discount rates among households affect long-run wealth distribution of a perpetually growing economy.

We find the following. First, in our model, the eventual distribution of wealth would become extremely uneven (Proposition 1). Second, the consumption habit enhances the accumulation of private wealth. Therefore, the most *impatient* household could eventually own almost all of the economy's physical wealth, as long as it has the *strongest* consumption habit (Proposition 2). Furthermore, we consider when patience becomes the sole determinant of wealth inequality, slightly modifying Ramsey's conjecture (Definition 2 and Proposition 3).

Finally, our model is sufficiently simple for diverse extensions. For example, we could extend the present model to include elastic labor supply, following Gómez (2015), among others. It would be also promising to construct an agent-based model with heterogeneous households as an extension of our model for simulating long-run wealth distribution in growing economies.

Appendix A

We derive equations (5) to (7) in the main text following Gómez (2008, 2010a).

The current-value Hamiltonian for household i 's maximization problem is

$$\mathcal{H}_i := \frac{(c_i z_i^{-\gamma_i})^{1-\sigma_i}}{1-\sigma_i} + \mu_i(Ak_i - c_i) + \nu_i \alpha_i (c_i - z_i),$$

where μ_i and ν_i are co-state variables associated with (1) and (2), respectively.

The first-order conditions for an interior optimum are

$$\frac{\partial \mathcal{H}_i}{\partial c_i} = 0 \Leftrightarrow c_i^{-\sigma_i} z_i^{-(1-\sigma_i)\gamma_i} = \mu_i - \alpha_i \nu_i \quad (10)$$

$$\dot{\mu}_i = \rho_i \mu_i - \frac{\partial \mathcal{H}_i}{\partial k_i} \Leftrightarrow \dot{\mu}_i = (\rho_i - A)\mu_i \quad (11)$$

$$\dot{\nu}_i = \rho_i \nu_i - \frac{\partial \mathcal{H}_i}{\partial z_i} \Leftrightarrow \dot{\nu}_i = (\rho_i + \alpha_i)\nu_i + \gamma_i c_i^{1-\sigma_i} z_i^{-[(1-\sigma_i)\gamma_i+1]} \quad (12)$$

The transversality conditions are

$$\begin{aligned} \lim_{t \rightarrow +\infty} \mu_i k_i \exp(-\rho_i t) &= 0, \\ \lim_{t \rightarrow +\infty} \nu_i z_i \exp(-\rho_i t) &= 0. \end{aligned}$$

Define $\zeta_i := -\nu_i/\mu_i$. Solving this and (10) simultaneously, we have

$$\mu_i = [c_i^{-\sigma_i} z_i^{-(1-\sigma_i)\gamma_i}] (1 + \alpha_i \zeta_i)^{-1}, \quad (13)$$

$$\nu_i = -[c_i^{-\sigma_i} z_i^{(1-\sigma_i)\gamma_i}] \zeta_i (1 + \alpha_i \zeta_i)^{-1}. \quad (14)$$

Differentiating (10) with respect to time, t , yields

$$-\sigma_i c_i^{-\sigma_i} z_i^{-(1-\sigma_i)\gamma_i} \dot{c}_i - (1 - \sigma_i) \gamma_i c_i^{-\sigma_i} z_i^{-[(1-\sigma_i)\gamma_i+1]} \dot{z}_i = \dot{\mu}_i - \alpha_i \dot{\nu}_i. \quad (15)$$

Substituting (11) and (12) into (15) to eliminate $\dot{\mu}_i$ and $\dot{\nu}_i$, and using (13) and (14), we have

$$\dot{c}_i = \frac{c_i}{\sigma_i} \left[\frac{A + \alpha_i}{1 + \alpha_i \zeta_i} + \gamma_i \alpha_i \sigma_i \cdot \frac{c_i}{z_i} - \rho_i + \alpha_i \{(1 - \sigma_i)\gamma_i - 1\} \right]. \quad (16)$$

Define $\xi_i := c_i/z_i$ and $\eta_i := z_i/k_i$. Using (1), (2), and (16), we can obtain a system of differential equations:

$$\dot{\xi}_i = \frac{\xi_i}{\sigma_i} \left[\frac{A + \alpha_i}{1 + \alpha_i \zeta_i} - \alpha_i \xi_i \sigma_i (1 - \gamma_i) - \rho_i - \alpha_i (1 - \gamma_i) (1 - \sigma_i) \right], \quad (17)$$

$$\dot{\eta}_i = -\eta_i [A + \alpha_i - \xi_i (\eta_i + \alpha_i)], \quad (18)$$

$$\dot{\zeta}_i = [A + \alpha_i (1 - \gamma_i \xi_i)] \zeta_i - \gamma_i \xi_i. \quad (19)$$

We find immediately that this system has a trivial solution $(\xi_i, \eta_i, \zeta_i) = (0, 0, 0)$, which we should discard. Focusing only on the non-trivial stationary point such that $\dot{\xi}_i = \dot{\eta}_i = \dot{\zeta}_i = 0$ and $(\xi_i, \eta_i, \zeta_i) \neq (0, 0, 0)$, we have

$$\xi_i^* = 1 + \frac{A - \rho_i}{\alpha_i[(1 - \gamma_i)\sigma_i + \gamma_i]}, \quad (5)$$

$$\eta_i^* = \frac{\alpha_i[A(1 - \gamma_i)(\sigma_i - 1) + \rho_i]}{\alpha_i[(1 - \gamma_i)\sigma_i + \gamma_i] + A - \rho_i}, \quad (6)$$

$$\zeta_i^* = \frac{\gamma_i[(A - \rho_i) + \alpha_i\{(1 - \gamma_i)\sigma_i + \gamma_i\}]}{(1 - \gamma_i)[A\sigma_i + \alpha_i\{(1 - \gamma_i)\sigma_i + \gamma_i\}] + \gamma_i\rho_i}.$$

We easily obtain equation (7) in the main text using (5) and (6).

Gómez (2008, 2010a) proves that the stationary point of this system, (ξ^*, η^*, ζ^*) , is locally saddle-point stable.

Appendix B

By defining $1/\varepsilon_i := (1 - \gamma_i)\sigma_i + \gamma_i$, we rewrite (7) as

$$\chi_i^* = (1 - \varepsilon_i)A + \varepsilon_i\rho_i.$$

Subtracting χ_L^* from χ_H^* yields

$$\begin{aligned} \chi_H^* - \chi_L^* &= (1 - \varepsilon_H)A + \varepsilon_H\rho_H - (1 - \varepsilon_L)A - \varepsilon_L\rho_L \\ &= \varepsilon_L \left\{ \underbrace{\left(1 - \frac{\varepsilon_H}{\varepsilon_L}\right)A + \frac{\varepsilon_H}{\varepsilon_L}\rho_H - \rho_L}_{=: \Omega} \right\}. \end{aligned}$$

Because $\varepsilon_L > 0$, we have

$$\chi_H^* \lesseqgtr \chi_L^* \Leftrightarrow \Omega \lesseqgtr 0 \Leftrightarrow \frac{A - \rho_L}{A - \rho_H} \lesseqgtr \frac{\varepsilon_H}{\varepsilon_L}.$$

From this and (8), Proposition 1 follows.

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