Environmental R&D, imperfectly competitive recycling market, and recycled content standards

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**Abstract**

In this paper, we construct a model consisting of an upstream monopoly recycler and downstream oligopolistic firms to examine the economic and environmental impacts of a recycled content standard (RCS). In our model, final goods firms undertake environmental research and development (ER&D) for the purpose of improving green design of the goods. Using this framework with a linear demand curve, we show that a stricter RCS reduces both the output of final goods and the degree of green design, even though it decreases the price of recycled materials. The profits of the recycler and the final goods firms also decrease. Besides, the economy improves its recycling ratio and can curtail the total amount of waste if the degree of the green design is less than a half.
1. Introduction

In order to curtail waste, many countries have put in practice the 3Rs of reduce–reuse–recycle. For example, under the 2001 Home Appliances Recycling Law, the Japanese home electronics industry is required to maintain a recycling rate of at least 50%, while the US state of California requires that manufacturers of polyethylene terephthalate packaging must meet a minimum of 10% recycled content to stimulate the use of recycled materials. In addition to the implementation of these recycling measures, the firms potentially affected have also improved the eco-friendly (green) design of their products through environmental research and development (ER&D).

In this paper, we construct a model consisting of an upstream monopoly recycler and downstream oligopolistic firms undertaking ER&D for the purpose of analyzing the economic and environmental impacts of a recycled content standard (RCS). RCSs require firms to use a certain percentage of recycled materials as an input, with the intention of reducing waste by stimulating their use. The existing literature analyzing the use of RCSs includes Palmer and Walls (1997), Higashida and Jinji (2006), and Iida (2011). However, none of these studies considers how the strengthening of an RCS affects ER&D for promoting the recycling of final products. To do this, we adopt a model including ER&D in downstream firms à la Katsoulacos and Xepapadeas (1996), Chiou and Hu (2001), and Tsai et al. (2015). Through applying this type’s model, we reveal the relationship between the RCS and the downstream firms’ decisions on green design via ER&D.

One distinctive feature of our model is the assumption of an imperfectly competitive market for recycled materials. As argued by Eichner (2005), Sugeta and Shinkuma (2012, 2014), and Dubois and Eyckmans (2015), some recycling markets reach the stage where only one or few recyclers operate and these determine the price of recycled materials. However, these studies do not examine the impact of an RCS. In this paper, we assume a monopoly recycler and downstream oligopolistic firms as a first step, and then confirm the impact of a stricter RCS on not only ER&D but also the price of recycled materials. Moreover, we explain

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1 Eichner (2005), Tsai et al. (2013), Sugeta and Shinkuma (2014), and Dubois and Eyckmans (2015) consider the case where the government imposes the target on recycling rates.
2 Katsoulacos and Xepapadeas (1996) show that when polluting duopoly firms undertake ER&D and there are spillovers between firms, the optimal emission tax is lower than the marginal damage. Chiu and Hu (2001) investigate ER&D competition (or cooperation) through environmental research joint ventures under emission taxes. Tsai et al. (2015) examine the relationship between the optimal environmental tax and tariff in the situation where only a home firm can undertake ER&D. Moreover, it is possible for upstream eco-industry firms outside the polluting sector to undertake ER&D. For example, Greker and Rosendahl (2008) and Nimubona and Benchekroun (2015) analyze cost-reducing ER&D in the oligopolistic eco-industry.
3 For example, the Japanese markets for home appliances are imperfectly competitive and thus mainly consist of only some representative firms, i.e. Panasonic, Sony, etc. Accordingly, to examine this relationship, we suppose that the final goods market is oligopolistic.
the impact on profits and the total amount of waste through the change in these variables.

Using this framework with a linear demand curve, we show that a stricter RCS reduces both the output of final goods and the degree of green design, even though it decreases the price of recycled materials. The profits of the recycler and final goods firms also decrease. Besides, the economy improves its recycling ratio and can curtail the total amount of waste if the degree of the green design is less than a half.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 considers the economic and environmental impacts of a stricter RCS. Section 4 includes a brief discussion of the optimal RCS. Section 5 provides some concluding remarks.

2. Model
There are two markets in this economy: an upstream monopoly market for recycled materials and a downstream oligopolistic market for final goods. There are $n$ firms in the downstream market and these firms engage in Cournot competition. Let $x_i, (i = 1, \cdots, n)$ be the output of $i$th downstream firm, and $p$ denotes the price of the final goods. The inverse demand function is then $p(X) = a - bX, (a, b > 0)$, where $X$ represents the aggregate output of final goods.

We express the degree of firm $i$’s green design through ER&D as $e_i$. The observed total amount of waste ($E$) is then written as $E = \sum_{i=1}^{n}(1-e_i)x_i$. In the following analysis, we define the recycling ratio ($\alpha$) as the ratio of the amounts recovered as recycled materials to the recyclable materials included in the final goods. In other words, it denotes the ratio by amounts of the output of recycled materials to the input of recyclable resources in the recycling sector. That is, when we express the output of recycled materials as $y$, $\alpha$ is written as $\alpha = y / \sum_{i=1}^{n}e_i x_i$. If the goods are not recycled, they cause environmental damage. The individual consumer treats $E$ as a public bad that does not affect individual actions.

We assume that the government of the country implements two environmental policies: (I) an RCS ($\mu \in (0,1)$) to stimulate the usage of recycled materials; and (II) an environmental tax ($t$) to reduce waste products by encouraging the final goods producer to promote ER&D, and thus improve green design.

2.1 Profit maximization of final goods firms
The final goods firms are subject to the environmental tax if their product cannot be recycled. These firms use one unit of a mix of recycled and virgin materials to produce one unit of the

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4 We assume that final goods wasted at time $T-1$ are recycled as recycled materials at time $T$. However, as in Higashida and Jinji (2006), we focus only on the steady-state equilibrium.

5 Note that our definition for “recycling ratio” is different from one of Eichner (2005), Sugeta and Shinkuma (2012, 2014), and Dubois and Eyckmans (2015). However, as shown in Dubois and Eyckmans (2015), the production process of recycled materials also generates residuals. E.g., for home appliance recycling, the residuals of scraps still remain even if a part of them is used in thermal recycle. Our recycling ratio describes this situation. By doing so, we reveal the impact of a RCS on the degree of green design through ER&D.
The import of virgin materials is from the rest of the world with the price \((w)\) fixed at the world level. Like Higashida and Jinji (2006), we assume that the price of virgin materials is lower than that of recycled materials \((r)\), that is, \(r > w\).\(^6\) In this case, the marginal cost of \(i\)th final goods firm \((c_i)\) is as follows:\(^7\)
\[
c_i(e_i, r, \mu, t) = \mu r + (1 - \mu)w + t(1 - e_i).
\] (1)
Furthermore, to achieve better green design, the downstream firms invest in ER&D. We assume the ER&D cost is a quadratic function of the design.

In the following analysis, we assume that each downstream firm cannot exercise any monopsony power in the recycling market. That is, the firm takes the price of recycled materials as given. In this case, the profit of the firm \(i (\pi_i)\) is as follows:
\[
\max_{x_i} \pi_i(X, e_i, r, \mu, t) = \{a - b(\sum_{i=1}^{n} x_i) - c_i(e_i, r, \mu, t)\}x_i - ke_i^2 / 2,
\] (2)
where \(k > 0\) is an investment efficiency parameter. Hereafter, we suppose that the downstream firms are symmetric. By maximizing firm \(i\)'s profit and considering firms’ symmetricity, we obtain the output level of a downstream firm in Cournot–Nash equilibrium:
\[
x(e, r, \mu, t) = (a - c) / (n+1)b.
\] (3)

By substituting (3) into (2), we obtain the following profit function. Each downstream firm then maximizes the function with respect to the green design:
\[
\max_e \pi(e, r, \mu, t) = (a - c)^2 / (n+1)^2b - ke^2 / 2.
\] (4)
Differentiating (4) with respect to \(e\), from the first-order condition, the degree of the design is expressed as follows:
\[
\bar{e}(r, \mu, t) = 2t\{\phi - \mu(r - w)\} / A,
\] (5)
where \(\phi \equiv a - t - w\) represents the margin without a RCS, and \(A \equiv (n+1)^2b k - 2t^2 > 0\) because the second-order condition for (4) is \(\partial^2 \pi / \partial e^2 = 2t^2 - (n+1)^2bk < 0\). Then, substituting (5) into (3), we obtain the output which is evaluated at \(\bar{e}\):
\[
\bar{x}(r, \mu, t) = (n+1)k\{\phi - \mu(r - w)\} / A.
\] (6)

From (5) and (6), the degree of green design and the production of final goods depend on (I) the margin without a RCS \((\phi)\) and (II) the additional cost of the RCS \((\mu(r - w))\). Noting \(r > w\) by assumption, we find that \(\phi > \mu(r - w) > 0\) because (6) needs to be positive. In this case, the degree of the green design is also positive.

### 2.2 The recycled materials balance condition

As mentioned above, each downstream firm uses one unit of a mixture of recycled and virgin materials to produce one unit of the final good \(x\). Because the RCS establishes this minimum

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\(^6\) As in Higashida and Jinji (2006), if \(\mu=0\) the monopoly recycler cannot operate because recycled materials are not demanded. That is, with a RCS, the downstream firms are obligated to purchase relatively expensive recycled materials. Alternatively, Sugeta and Shinkuma (2012, 2014) assume that the price of recycled materials supplied by the monopoly recycler is lower than that of virgin materials \((w>r)\) without a RCS.

\(^7\) In our model, the price of virgin materials is exogenous. Hence, we omit “\(w\)” in the corresponding function.
mixture ratio, the derived demand for recycled materials becomes $\mu nx$. The recycled materials balance is then subject to

$$\mu nx = y(= aenx). \tag{7}$$

From (7), the recycling ratio is determined by $\alpha = \mu / e$ in our model. Note that if $\alpha = 1$, the level of the RCS and the degree of green design are identical as long as the balance condition for recycled materials is satisfied. Considering this point, we focus on the case of $\alpha < 1$ in the following analysis.

From (6), the materials balance is written as $\mu n\bar{x} = y$. Hence, we can express the inverse demand function for recycled materials as follows:

$$\bar{r}(y, \mu, t) = (\phi + \mu w) / \mu - Ay / Nk\mu^2, \tag{8}$$

where $N \equiv n(n+1)$.

### 2.3 Profit maximization of the monopoly recycler

Here, we consider the profit maximization of a monopoly recycler. In the following analysis, we assume that the marginal cost ($c^R$) of the recycler is constant. Then the profit maximization of the recycler is expressed as

$$\max_{y} \pi^R(y, \mu, t) = (\bar{r}(y, \mu, t) - c^R)y. \tag{9}$$

Plugging (8) into (9), from the first-order condition, we obtain the recycled materials’ output:

$$y^*(\mu, t) = Nk\mu(\phi + \mu(w - c^R)) / 2A. \tag{10}$$

From (10), we find that $\phi + \mu(w - c^R) > 0$ if the output level of the materials is positive at the equilibrium. Sugeta and Shinkuma (2012, 2014) refer to $c^R - w$ as the production efficiency of the upstream recycler’s facility relative to the price of virgin materials. As we assume $r > w$, $r \geq c^R > w$ in our model. Moreover, substituting (10) into (8), we derive the equilibrium price of the materials as follows:

$$r^*(\mu, t) = \{\phi + \mu(w + c^R)\} / 2\mu. \tag{11}$$

Finally, substituting (11) into (5) and (6), we obtain the output of final goods and the degree of green design at the equilibrium:

$$x^*(\mu, t) = (n+1)k(\phi + \mu(w - c^R)) / 2A, \quad e^*(\mu, t) = t(\phi + \mu(w - c^R)) / A. \tag{12}$$

Because $A > 0$, both the output and the degree of the green design are also positive if the upstream recycler earns positive profits.

### 3. Comparative statics of a RCS

#### 3.1 The effect on the price of recycled materials and the degree of green design

We first check the impact on the price of recycled materials. Differentiating (11) with respect to $\mu$, we obtain:

$$\frac{\partial r^*}{\partial \mu} = -\phi / 2\mu^2 < 0. \tag{13}$$

8 For simplicity, we summarize the cost involving the recovery process for scrap goods as the constant marginal cost of a recycling firm. As carefully analyzed in Palmer et al. (1997) and Kaffine (2014), scrap prices, which are determined from the market-clearing condition for scrap goods, play an important role as a determinant of waste and recycling policy costs.
When the parameter $a$ is sufficiently large, the size of the final goods market is also large, and the demand for recycled materials is then brisk. However, the price of recycled materials can fall, even though the demand for the materials increases, directly.

The RCS reduces the demand for recycled materials via a decrease in the output of final goods, whereas it increases the demand, directly. Hence, considering recycled materials balance given in (7), we find that from (10), the impact of the RCS on the output of recycled materials is unclear, then it depends on the sign of $\phi + 2\mu(w - c^R)$. Nevertheless, the RCS can fall the price of materials. From (8), when the government strengthens the RCS, the demand curve for recycled materials becomes flatter. In other word, the demand for the materials becomes more elastic with respect to the price. It means that the monopoly recycler’s markup falls to a low level. Hence, the price can decrease after the tightening of the RCS.

Next, we can confirm that from (12), the effect of a RCS on the output of final goods and the green design become

$$\frac{\partial x^*}{\partial \mu} = (n + 1)k(w - c^R)/2A < 0, \quad \frac{\partial e^*}{\partial \mu} = t(w - c^R)/A < 0.$$  

(14)

The equilibrium marginal cost of a downstream firm is written as (A1) in Appendix 1. From the equation, we find that a stricter RCS increases the marginal cost of the final goods firm even though the price of recycled materials decreases. This implies that the direct cost-push effect of the RCS outweighs its price-reducing effect. Hence, each final goods firm reduces both the output and the degree of green design because the RCS intensifies its cost disadvantage.

### 3.2 The effect on profits

Here, we investigate the impact of a stricter RCS on the profits of the downstream and upstream firms. We also provide specific equilibrium equations for the profits and the total amount of waste in Appendix 1. First, the equilibrium profit of a downstream firm is written as (A3) in Appendix 1. We then show that the effect of a RCS on the firm’s profit is

$$\frac{\partial \pi^*}{\partial \mu} = k(w - c^R)[\phi + \mu(w - c^R)]/2A < 0.$$  

(15)

That is, each final goods firm loses from a stricter RCS because it increases each firm’s marginal cost.

Next, the equilibrium profit of the upstream monopoly recycler is written as (A4) in Appendix 1. From this equation, we have the following:

$$\frac{\partial \pi^{r*}}{\partial \mu} = Nk(w - c^R)[\phi + \mu(w - c^R)]/2A < 0.$$  

(16)

A stricter RCS decreases the price of recycled materials. This accounts for a reduction in the recycler’s revenue, and thus the recycler’s profit. On the other hand, noting that the impact of the RCS on the output of the materials is indeterminate, the recycler’s revenue may swell through increased production. However, (16) means that the former effect exceeds the later effect even if the RCS increases the output of recycled materials.

### 3.3 The effect on the total amount of waste

Here, we focus on the environmental impact of a stricter RCS. To start, we confirm the impact on the recycling ratio. Note that the equilibrium recycling ratio is determined by $\alpha^* = \mu/e^*$. 
Considering (14), the effect of a RCS on the recycling ratio is represented as
\[ \frac{\partial \alpha}{\partial \mu} = \left\{ \frac{\mu A - \mu (w - c^R)}{(\mu^*)^2} \right\} A > 0. \] (17)
A stricter RCS decreases the degree of green design. Noting that e\!*\! of wasted goods is directed to the recycling process, this decrease is accompanied by a reduction in the supply of scrap goods for recycling. That is, the RCS improves the recycling ratio not only by an increase in the numerator (\(\mu\)), but also via a decrease in the denominator (\(\mu^*\)).

At the equilibrium, the total amount of waste is represented as (A5) in Appendix 1. Thus, the impact of the RCS on total waste becomes
\[ \frac{\partial E^*}{\partial \mu} = Nk(1 - 2e^*)(w - c^R) / 2A. \] (18)
Considering \( w < c^R \), we find that the sign of \( \frac{\partial E^*}{\partial \mu} \) depends on the degree of green design. The RCS decreases the output of polluting downstream firms and reduces the degree of the design. That is, both the scale and technique effects have a negative impact on total waste. Consequently, total waste may decrease if \( 1/2 < e^* \).

Hence, we summarize the results for the comparative statics in this section as follows:

**Proposition:** (I) A stricter RCS decreases both the output and degree of green design of oligopolistic final goods firms, even though it reduces the price of recycled materials. (II) The profits of final goods firms and the monopoly recycler clearly decrease. (III) The recycling ratio improves. The total amount of waste may decrease if the degree of green design is less than a half.

In this paper, we investigate only a RCS for recycling. However, both the upstream and downstream markets are imperfectly competitive in our model. In addition, there is a negative externality resulting from waste products. As analogized from previous studies, e.g., Eichner (2005), this suggests that a combination of policy instruments is required to reduce waste, and then to correct distortions arising from imperfect competition.

**4. A note on the optimal RCS**

In this section, we consider the optimal RCS. Here we assume that environmental tax revenue is transferred in a lump-sum fashion to domestic consumers. In this case, the welfare of the country at the equilibrium (\(W^*\)) is expressed as
\[ W^*(t, \mu) = CS^*(t, \mu) + \pi^*(t, \mu) + \pi^R(t, \mu) + (t - \Omega)E^*(t, \mu), \] (19)
where \(CS^*\) is the consumer surplus at the equilibrium and \(\Omega\) is the social valuation of the environmental damage associated with the waste products.

Hence, the first-order condition for welfare maximization with regard to an RCS is written as follows:
\[ \frac{\partial W^*}{\partial \mu} = k(w - c^R)[\{\phi + \mu(w - c^R)\}B + (t - \Omega)2NA] / 4A^2 = 0, \] (20)
where \(B\) is defined as
\[ B \equiv n(n + 1)^2bk + 2(1 + N)A - 4tN(t - \Omega). \] (21)

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\footnote{See Appendix 2 for details of the calculation.}
In addition, the second-order condition is
\[ \frac{\partial^2 W^*}{\partial \mu^2} = k(w - c^R)^2 B / 4A^2. \] (22)
As shown in (22), the second-order derivative with respect to \(\mu\) may be negative or positive depending on the sign of \(B\), where \(B\) consists of the marginal social damage, the environmental tax, and the investment efficiency parameter. Hence, we confirm the optimal RCS for two cases.

\textbf{Case 1:} \( t \leq \Omega \) or \( t - \Omega < \{ n(n + 1)^2 bk + 2(1 + N)A \} / 4tN \) when \( t > \Omega \); that is, \( B > 0 \)

In this case, it is clear that \( \frac{\partial^2 W^*}{\partial \mu^2} > 0 \). Hence, the optimal RCS is infinitely close to 1. (I) If the first-order derivative with respect to \(\mu\) is positive when \(\mu \to 0\), or (II) even if the derivative is negative when \(\mu \to 0\), as long as the welfare level is positive when \(\mu \to 1\), and then the welfare level is higher than that in the case of \(\mu \to 0\). Noting that an RCS decreases the output of final goods, almost all waste being recycled is the preferred measure of this economy if the marginal social damage is sufficiently high. However, we find that if \( t = \Omega \), \( \frac{\partial W^*}{\partial \mu} = kB(w - c^R)\{\phi + \mu(w - c^R)\} / 4A^2 < 0 \) will definitely hold because \( B > 0 \).

\textbf{Case 2:} \( t - \Omega > \{ n(n + 1)^2 bk + 2(1 + N)A \} / 4tN \) when \( t > \Omega \); that is, \( B < 0 \)

In the case where the marginal damage is considerably smaller than the environmental tax level, we may obtain \( \frac{\partial^2 W^*}{\partial \mu^2} < 0 \). In such a case, from (20), we can derive the optimal RCS as follows:
\[ \mu^*(t) = -(\phi B + (t - \Omega)2NA) / (w - c^R)B. \] (23)
In (A8) of Appendix 2, we provide the detailed calculation for (23). Noting \( w < c^R \) and \( B < 0 \), \( \mu^*(t) \) can be located in the interval \((0,1)\) only if the term \( (n + 1)^2 bk < 2t(a - w) \), then \( t - \Omega > -\phi(n(n + 1)^2 bk + 2(1 + N)A) / 2N\{(n + 1)^2 bk - 2t(a - w)\} \). In other words, if the demand parameter \(a\) is not substantially large and the environmental tax level is not sufficiently high relative to the marginal damage, the optimal RCS can be infinitely close to zero, even in Case 2.

\[ \text{5. Concluding remarks} \]
We analyzed the economic and environmental effects of an RCS using a model including ER&D. In our model, a stricter RCS reduces both the output of final goods and the degree of green design, even though it decreases the price of recycled materials. The profits of the recycler and the final goods firms also decrease. Regarding the environmental impact of an RCS, the recycling ratio in this economy improves, and the total amount of waste can be curtailed if the ER&D level is less than a half in the case of a linear demand curve.

Further research is required to address the following points. First, as discussed, we consider only an RCS as a policy instrument for recycling. In fact, there are several recycling policies apart from an RCS (for example, recycling rate targets and recycling subsidies). Hence, we need to explore the impact of other recycling policies in this model including ER&D. Second, we could investigate the real-world trade in recycled materials and whether the promotion of international resource circulation improves the global environment. Hence, a future research task would be to incorporate foreign recycling into this model.
Appendix 1: Specific equilibrium equations

Here, we provide specific equilibrium equations with respect to the marginal cost of downstream firms, its profits, and its total waste. First, considering (11) and (12) at the equilibrium, the marginal cost of the firms is

\[ c^⋆(\mu, t) = \mu r^⋆(\mu, t) + (1 - \mu)w + t(1 - e^⋆(\mu, t)) \]
\[ = [(A - 2r^2)a + (A + 2r^2)\{w + t - \mu(w - c^k)\}] / 2A. \]  

(A1)

Hence, the impact of an RCS on \( c^⋆ \) is as follows:

\[ \partial c^⋆ / \partial \mu = -(A + 2r^2)(w - c^k) / 2A > 0. \]  

(A2)

Considering the inverse demand function and (A1), the profit of the downstream firm is written as follows:

\[ \pi^⋆(\mu, t) = (pr^⋆(\mu, t) - c^⋆(\mu, t))x^⋆(\mu, t) - ke^{22}(\mu, t) / 2 \]
\[ = k(\phi + \mu(w - c^k))^2 / 4A. \]  

(A3)

The profit of the monopoly recycler is analogously calculated. Substituting (10) and (11) into (9), this is expressed as

\[ \pi^R^⋆(\mu, t) = (r^⋆(\mu, t) - c^\mu(\mu, t))y^⋆(\mu, t) - ke^{22}(\mu, t) / 2 \]
\[ = Nk(\phi + \mu(w - c^k))^2 / 4A. \]  

(A4)

Note that \( \phi > \mu(c^k - w) \) if \( r > c^k \), then the recycler obtains the positive profit.

Finally, noting (12), total waste at the equilibrium is

\[ E^⋆(\mu, t) = (1 - e^⋆(\mu, t))nx^⋆(\mu, t) \]
\[ = Nk(\phi + \mu(w - c^k))[A - t(\phi + \mu(w - c^k))] / 2A^2. \]  

(A5)

Hence, considering \( e^⋆ = t(\phi + \mu(w - c^k)) / A \) from (12), we can derive (18) regarding the effect of a RCS on the total amount of waste is written as follows:

\[ \partial E^⋆ / \partial \mu = n\{(1 - e^⋆)(\partial c^⋆ / \partial \mu) - x^⋆(\partial e^⋆ / \partial \mu)\} \]
\[ = n(w - c^k)((1 - e^⋆)(n + 1)k - 2tx^⋆) / 2A \]
\[ = Nk(w - c^k)[1 - 2t(\phi + \mu(w - c^k)) / A] / 2A \]
\[ = Nk(1 - 2e^⋆)(w - c^k) / 2A. \]

Appendix 2: Welfare effect of a RCS

From (19), the welfare effect of a stricter RCS is represented as follows:

\[ \partial W^⋆ / \partial \mu = \partial CS^⋆ / \partial \mu + \partial \pi^⋆ / \partial \mu + \partial \pi^R^⋆ / \partial \mu + (t - \Omega)\partial E^⋆ / \partial \mu, \]  

(A6)

where the partial derivative of consumer surplus is given as

\[ \partial CS^⋆ / \partial \mu = -x^⋆(\partial pr^⋆ / \partial \mu) \]
\[ = n(n + 1)^2bk^2(w - c^k)(\phi + \mu(w - c^k)) / 4A^2. \]  

(A7)

Hereafter, we define \( \Phi = \phi + \mu(w - c^k) = a - t - w + \mu(w - c^k) \). Then, substituting (15), (16), (18), and (A7) into (A6), we can obtain (20):
Thus, we define \( B \equiv n(n+1)^2bk + 2(1+N)A - 4tN(t-\Omega) \) in (21).

Moreover, by differentiating (20) with respect to \( \mu \), the second-order partial derivative is derived as follows:

\[
\frac{\partial^2 W^*}{\partial \mu^2} = k(w-c^k)^2 B / 4A^2.
\]

This equation corresponds to (22).

If \( B < 0 \), the second-order condition is satisfied. In this case, the optimal RCS may be located in the interval \((0,1)\). Actually, solving (20) with respect to \( \mu \), and considering \( A \equiv (n+1)^2bk - 2t^2 > 0 \) and \( \Phi \equiv \phi + \mu(w-c^k) > 0 \), the optimal RCS is expressed as follows:

\[
\mu^* = -[\phi B + (t - \Omega)2NA] / (w-c^k)B
\]

\[
= -[(t - \Omega)2NA + \phi(n(n+1)^2bk + 2(1+N)A - 4tN(t-\Omega))] / (w-c^k)B
\]

\[
= -[\phi(n(n+1)^2bk + 2(1+N)A) + (t - \Omega)2N(A - 2t\phi)] / (w-c^k)B
\]

\[ (A8) \]

\[
= -[\phi(n(n+1)^2bk + 2(1+N)A)] / (w-c^k)B
\]

This equation corresponds to (23). Noting \( w < c^k \), \( \mu^* > 0 \) only if \( (n+1)^2bk < 2t(a-w) \), and then \( t-\Omega > -\phi(n(n+1)^2bk + 2(1+N)A) / 2N((n+1)^2bk - 2t(a-w)) \).

References


