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## Technological progress, firm selection, and unemployment

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## Abstract

In the standard search-matching model, the effect of an increase in the productivity growth rate on the unemployment rate is quantitatively much smaller than that found in the data. This paper revisits this issue by considering the selection effect, through which an increase in the rate of disembodied technological progress induces firms with low productivity levels to exit and increases the average productivity. With this effect, one percent-point increase in the rate of technological progress decreases the unemployment rate by 0.28 percent, which is about 40 times as strong as the effect in the corresponding model without the selection effect.

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# 1 Introduction

It is widely believed that a slowdown in the productivity growth rate is a major source of persistent high unemployment. Indeed, the empirical evidence suggests that the productivity growth rate and the unemployment rate are negatively correlated. Theory confirms the evidence. In the standard search-matching model of unemployment, an increase (decrease) in the growth rate of disembodied technological progress increases (decreases) the value of job creation and as a result decreases (increases) the unemployment rate, known as the capitalization effect.

While the standard model can replicate this negative correlation, it fails to replicate the *magnitude* of this effect. In the textbook model, one percent-point increase in the productivity growth rate decreases the unemployment rate by only 0.007 percent. In the data, however, one percent-point increase in the productivity growth rate decreases the unemployment rate by 0.25–0.71 percent (Blanchard and Wolfers, 2000). An important challenge in the literature is to fill this gap.

In this paper, I revisit this important issue by focusing on the *selection effect*. The selection effect has been the central issue in the international trade literature, in which firms with heterogeneous productivity levels self-select into trade firms, domestic firms, and exit firms (Melitz, 2003; Felbermayr *et al.*, 2011). The novelty of this paper is to introduce the selection effect into the growth-unemployment literature. To be concrete, I develop a search-matching model with monopolistic competition and heterogeneous firms similar to Felbermayr and Prat (2011). The key new assumption is that each firm's productivity is the product of the level of disembodied technology common to all firms and an idiosyncratic component that is unique to each firm.

Consider a scenario in which the rate of technological progress increases from 2 percent to 3 percent. An increase in the rate of technological progress decreases the unemployment rate through *two* effects. One is the conventional capitalization effect, through which the value of job creation increases for all firms. The other is the selection effect. A higher rate of technological progress induces more firms to enter the market and create jobs, which reduces the vacancy-filling rate they face and reduces their profit prospects. Since more firms enter and draw their productivity levels, there are more high-productivity firms. This induces the marginal firms at the 2 percent growth rate to exit. In the new equilibrium, there are more firms, and the firms are on average more productive.

Using the model, I study the quantitative impact of disembodied technological progress on the unemployment rate, and I find a strong effect. With the selection effect, one percent-point increase in the rate of technological progress decreases the unemployment rate by 0.28 percent while the corresponding model without the selection effect implies that the impact is 0.007 percent. The effect is reduced to 0.14 when the calibration target for the unemployment benefit is reduced to an alternative level and when the elasticity of substitution parameter is increased to an alternative level. However, the effect is still much greater than the one typically reported in the literature, which is below 0.01. Thus, the main result reported in this paper is robust.

# 2 The Model

The structure of the model is closely related to Felbermayr and Prat (2011). The economy consists of a continuum of homogeneous individuals with a unit mass and a continuum of monopolistically competitive firms. Each firm produces a differentiated final consumption good. As in Melitz (2003) and Felbermayr and Prat (2011), each firm has a distinct productivity level. Total measure of operating firms, $n$, is determined as part of equilibrium by entry and exit of these firms. Time is discrete and horizon is infinite. All agents are risk neutral and discount the future at the common rate $r$.

Each firm has a linear production technology and output in period $t$ is given by $\varphi a_t l_t$, where $l_t$ is the labor input. The novel assumption in this paper is that the level of productivity $\varphi a_t$ consists of *two* components. The first component, $\varphi$, captures the idiosyncratic part of productivity that is unique to each firm. The second component, $a_t$, is the level of *disembodied* technology that is common to all firms. I assume that this component grows exogenously at rate $g > 0$. Each operating firm must pay a fixed operation cost, wages, and vacancy costs.

The key ingredient of the model is entry and exit of heterogeneous firms in a growing economy. Each potential firm pays a once-and-for-all entry cost at the time of entry and draws its productivity $\varphi$ from distribution function $F(\varphi)$ with density $f(\varphi)$. I assume that while there is disembodied technological progress, the distribution function is constant over time. Throughout, I assume a Pareto distribution, $F(\varphi) = 1 - (\varphi_{\min}/\varphi)^{\alpha}$.

The labor market is frictional. The aggregate number of matches made is determined by the standard constant-returns-to-scale matching function $m(u_t, V_t)$, where $u_t$ denotes the measure of the unemployed and $V_t$ denotes the aggregate vacancies posted by all firms. From the matching function, the vacancy filling rate is given by $m(u_t, V_t)/V_t = q(\theta_t)$, where $\theta_t = V_t/u_t$. Similarly, the job finding rate is given by $m(u_t, V_t)/u_t = \theta_t q(\theta_t)$. Throughout, I specify the matching function as $m(u, V) = m_0 u^{\eta} V^{1-\eta}$.

Timing is as follows. At the beginning of each period, there is a disembodied technological progress. Given the new technology level, each firm and its employees bargain over the wage rate $W_t$ and produce a differentiated consumption good. The firm posts $v_t$ units of vacancies, each of which is filled with probability $q(\theta_t)$. Separations occur after production so that $\lambda l_t$ of the employees become unemployed in the next period. In addition, each firm faces an exit shock with probability $\delta$ at the end of each period.

Throughout, I focus on the *balanced growth equilibrium* in which all values grow at the same rate. To ensure the existence of a balanced growth equilibrium, I follow the literature to assume that the vacancy cost, the unemployment benefit, the fixed cost of production, and the entry cost all grow at rate $g$. Specifically, I assume that the unit vacancy cost is $a_t c$, the unemployment benefit is $a_t z$, the fixed cost of production is $a_t I$, and the entry cost is $a_t K$, where $c$, $z$, $I$, and $K$ are parameters (Mortensen and Pissarides, 1998). In addition, I assume $g < r$.

In this economy, individuals are either employed or unemployed. The employed earns the wage rate $W_t$ and the unemployed earns the unemployment benefit $a_t z \equiv Z_t$. In each period, individual $j$ chooses the demand for each consumption good $Q_j(\omega)$ so as to maximize the following utility function

$$\left\{ n^{-\frac{1}{\sigma}} \int_{\omega \in \Omega} [Q_j(\omega)]^{\frac{\sigma-1}{\sigma}} d\omega \right\}^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

subject to the budget constraint $\int_{\omega\in\Omega} Q_j(\omega) p(\omega) d\omega = PY_j$, where $p(\omega)$ denotes the price of variety $\omega$, $\Omega$ is the set of product varieties, $\sigma > 1$ denotes the elasticity of substitution between any two varieties, $P = \{n^{-1} \int_{\omega\in\Omega} [p(\omega)]^{1-\sigma} d\omega\}^{1/(1-\sigma)}$ is the price index, and $Y_j$ is income.[1] Evidently, $Y_j = W$ for the employed and $Y_j = Z$ for the unemployed.

[1] I follow Felbermayr and Prat (2011) to discount the level of utility by $n^{1/(1-\sigma)}$.

The aggregate demand for variety $\omega$ is given by $\int Q_j(\omega)\,dj = [p(\omega)/P]^{-\sigma} Y/n$, which is decreasing in $p(\omega)$, where $Y = \int Y_j dj$. Given the demand schedule, each monopolistic firm sets its price $p(\omega)$. However, the firm's problem is dynamic because of search frictions. As in the model of Felbermayr and Prat (2011), the firm optimally chooses the level of job vacancies to influence the level of employment in the *next* period. This in turn influences the level of production and the price.

Now consider a firm with productivity $\varphi$. The inverse demand function for the firm is $p(\varphi)/P = (n\varphi al/Y)^{-1/\sigma}$ and therefore the revenue for the firm is $[p(\varphi)/P]\varphi al = (\varphi al)^{(\sigma-1)/\sigma}(n/Y)^{-1/\sigma} \equiv R(l;\varphi,a)$. Thus, the flow profit for the firm in each period is given by $\pi(l;\varphi,a) = R(l;\varphi,a) - W(l;\varphi,a)l - acv - aI$, where $W(l;\varphi,a)$ is the wage function to be determined. The value of an operating firm with idiosyncratic productivity $\varphi$ when the level of technology is $a$ satisfies

$$J(l;\varphi,a) = \max_v \left[\pi(l;\varphi,a) + \frac{1-\delta}{1+r}J(l';\varphi,a')\right] \tag{2}$$

subject to $l' = (1-\lambda)l + q(\theta)v$, where $a' = (1+g)a$. The first-order condition and the envelope condition imply

$$\frac{ac}{q(\theta)} = \frac{1-\delta}{1+r}\frac{\partial J(l';\varphi,a')}{\partial l'}, \tag{3}$$

$$\frac{\partial J(l;\varphi,a)}{\partial l} = (\varphi a)^{\frac{\sigma-1}{\sigma}}\left(\frac{n}{Y}\right)^{-\frac{1}{\sigma}}\frac{\sigma-1}{\sigma}l^{-\frac{1}{\sigma}} - W'(l)l - W(l) + \frac{(1-\lambda)\,ac}{q\,(\theta)}. \tag{4}$$

For a worker, the value of being employed by a firm with productivity $\varphi$ when the level of technology is $a$ is given by

$$E(l;\varphi,a) = W(l;\varphi,a) + \frac{1}{1+r}\left[(1-s)E(l';\varphi,a') + sU(a')\right], \tag{5}$$

and the value of being unemployed is given by

$$U(a) = az + \frac{1}{1+r}\left[\theta q(\theta)\int_{\varphi^*}^{\infty} E(l',\varphi,a')\mu(\varphi)d\varphi + (1-\theta q(\theta))U(a')\right], \tag{6}$$

where $s = \lambda + \delta - \lambda\delta$ is the probability that either an exogenous separation occurs or the firm is hit by an exit shock, and $\mu(\varphi)$ is the equilibrium density of firms.

I follow Felbermayr and Prat (2011) to assume that the wage rate is determined by intra-firm bargaining. Specifically, the wage rate is determined so that

$$(1-\beta)\left[E(l;\varphi,a) - U(a)\right] = \beta\frac{\partial J(l;\varphi,a)}{\partial l}, \tag{7}$$

where $\beta$ is the exogenous bargaining power of the worker.

**Proposition 1** *The wage rate satisfies*

$$W\,(l;\varphi,a) = \beta\frac{\sigma-1}{\sigma-\beta}\frac{R(l;\varphi,a)}{l} + (1-\beta)\left[az + \frac{\beta}{1-\beta}\frac{ac\theta}{1-\delta}\right]. \tag{8}$$

**Proof.** In the appendix. ■

Use $(\varphi a l)^{(\sigma-1)/\sigma}(n/Y)^{-1/\sigma} = R(l;\varphi,a)$ and (8) to rewrite (4) as

$$\frac{\partial J(l;\varphi,a)}{\partial l} = (1-\beta)\frac{\sigma-1}{\sigma-\beta}\frac{R(l;\varphi,a)}{l} + (1-\lambda)\frac{ac}{q(\theta)} - (1-\beta)\left[az + \frac{\beta}{1-\beta}\frac{ac\theta}{1-\delta}\right]. \quad (9)$$

Note that (3) and (9) jointly give the job-creation condition and imply that the revenue per worker $R(l;\varphi,a)/l$ is the same for all $\varphi$, from which $l(\varphi_1)/l(\varphi_2) = (\varphi_1/\varphi_2)^{\sigma-1}$ and $p(\varphi_1)/p(\varphi_2) = (\varphi_1/\varphi_2)^{-1}$.

There is a large pool of potential entrants. As in Melitz (2003) and Felbermayr and Prat (2011), each entrant pays the sunk entry cost $a_t K$ to realize its type $\varphi$, and then decides whether to exit the market. As a result, entry decision is characterized by the two standard conditions. Namely, the free entry condition and the zero cutoff profit condition:

$$aK = \int_{\varphi^*}^{\infty} J(0,\varphi,a) f(\varphi) d\varphi, \quad (10)$$

$$J(0,\varphi^*,a) = 0, \quad (11)$$

where $\varphi^*$ denotes the cutoff productivity level and $J(0,\varphi,a)$ is the value of entry for a type-$\varphi$ firm:

$$J(0,\varphi,a) = -ac\frac{l'}{q(\theta)} - aI + \frac{1-\delta}{1+r}J(l';\varphi,a'). \quad (12)$$

Here, the linear vacancy cost implies that each entrant creates a mass of vacancies to operate with $l'$ employees in the next period.

In what follows, I focus on a *balanced growth equilibrium*, in which all values grow at the same rate as $a$. Let $\tilde{\varphi}$ denote the average productivity, which is defined by $p(\tilde{\varphi}) = P$. As in the literature, $\varphi^*$ pins down the level of $\tilde{\varphi}$ such that $\tilde{\varphi}(\varphi^*) = [\int_{\varphi^*}^{\infty} \varphi^{\sigma-1}\mu(\varphi)d\varphi]^{1/(\sigma-1)}$. In the case of Pareto, this expression implies $(\tilde{\varphi}/\varphi^*)^{\sigma-1} = \alpha/(\alpha+1-\sigma)$.

**Proposition 2** *A balanced growth equilibrium is defined by a pair $(\theta,\varphi^*)$ that satisfies*

$$\frac{\Lambda^{-1}-(1-\lambda)}{1-\beta}\frac{c}{q(\theta)} + z + \frac{\beta}{1-\beta}\frac{c\theta}{1-\delta} = \frac{\sigma-1}{\sigma-\beta}\tilde{\varphi}(\varphi^*), \quad (13)$$

$$\frac{1-\Lambda}{1-F(\varphi^*)}\frac{K}{I} = \left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1, \quad (14)$$

*where* $\Lambda = (1-\delta)(1+g)/(1+r)$.

**Proof.** In the appendix ■

In the case of Pareto, (14) has a closed-form solution,

$$\varphi^* = \left(\frac{I/K}{1-\Lambda}\frac{\sigma-1}{\alpha+1-\sigma}\right)^{1/\alpha}\varphi_{\min}. \quad (15)$$

Given the equilibrium level of $\theta$, the equilibrium unemployment rate $u$ is derived from the fact that the flows in and out of employment must equate, $u\theta q(\theta) = (1-u)s$, from which $u = s/[\theta q(\theta)+s]$. Evidently, $u$ decreases with $\theta$. Similarly, since the aggregate employment satisfies $nl(\tilde{\varphi}) = 1-u$, the equilibrium mass of firms is given by $n = (1-u)/l(\tilde{\varphi})$, where $l(\tilde{\varphi})$ denotes the equilibrium level of employment for the firm with

Table 1: Parameter Values

| Variable | Symbol | Value |
|---|---|---|
| Interest rate | $r$ | 0.05 |
| Rate of technological progress | $g$ | 0.02 |
| Probability of being unemployed | $s/12$ | 0.036 |
| Probability of firm destruction | $\delta$ | 0.087 |
| Elasticity of matching function | $\eta$ | 0.5 |
| Worker's bargaining power | $\beta$ | 0.5 |
| Scale of matching function | $m_0$ | 8.40 |
| Flow value of unemployment | $z$ | 0.58 |
| Cost of posting a vacancy | $c$ | 0.28 |
| Elasticity of substitution among differentiated goods | $\sigma$ | 3.5 |
| Shape of firm-specific productivity distribution | $\alpha$ | 2.65 |
| Minimum level of firm-specific productivity | $\varphi_{\min}$ | 0.05 |
| Entry cost | $K$ | 0.24 |
| Flow fixed cost | $I$ | 0.18 |

the average productivity $\tilde{\varphi}$. In what follows, I denote the wage rate in the balanced growth equilibrium as $W/a \equiv w$.

# 3 Quantitative Analysis

The main result of this paper is that the model with the selection effect outperforms the corresponding model without it in generating the realistic impact of technological progress on unemployment. In this section, I describe how the model parameters are chosen and present the quantitative results. The details are presented in the appendix.

The model is calibrated to the US economy, and its time period is chosen to be one year. I follow the standard calibration procedure whenever possible. The baseline parameter values are summarized in Table 1.

I choose the annual interest rate to be $r = 0.05$ and the rate of technological progress to be $g = 0.02$. I follow Pissarides (2009) to set the monthly job separation rate as $s/12 = 0.036$. The annual firm destruction rate $\delta = 0.087$ is computed as the sample mean over the 1977-2014 period from the Business Dynamics Statistics.[2] I follow Petrongolo and Pissarides (2001) to set the elasticity in the matching function to be $\eta = 0.5$. As in the literature, I set the exogenous bargaining power to satisfy $\beta = \eta$.

According to Ebell and Haefke (2009), the entry cost in the US in 1997 equals 0.6 months of per-capita income, and the entry cost in 1978 amounts to 5.2 months of per-capita income. This suggests that there is a large variation in the entry cost over time. Here, I simply use the mean value of these estimates: $K = [(0.6 + 5.2)/2] \times 1/12 = 0.24$.

I obtain the scale parameter of the matching function, $m_0 = 8.40$, the flow value of unemployment, $z = 0.58$, and the cost of posting a vacancy, $c = 0.28$, from the three targets: $\theta = 0.72$ (Pissarides, 2009), $z/w = 0.71$ (Hall and Milgrom, 2008) and $\theta q(\theta) = 0.594$ (Pissarides, 2009). The implied recruitment cost $c/q(\theta)$ from these parameters is 14.0 percent of the quarterly wage, which is consistent with Elsby and Michaels (2013).

[2] The firm destruction rate in each year is computed as Firmdeath_Firms divided by Firms. These series are obtained from https://www.census.gov/ces/dataproducts/bds/data_firm.html

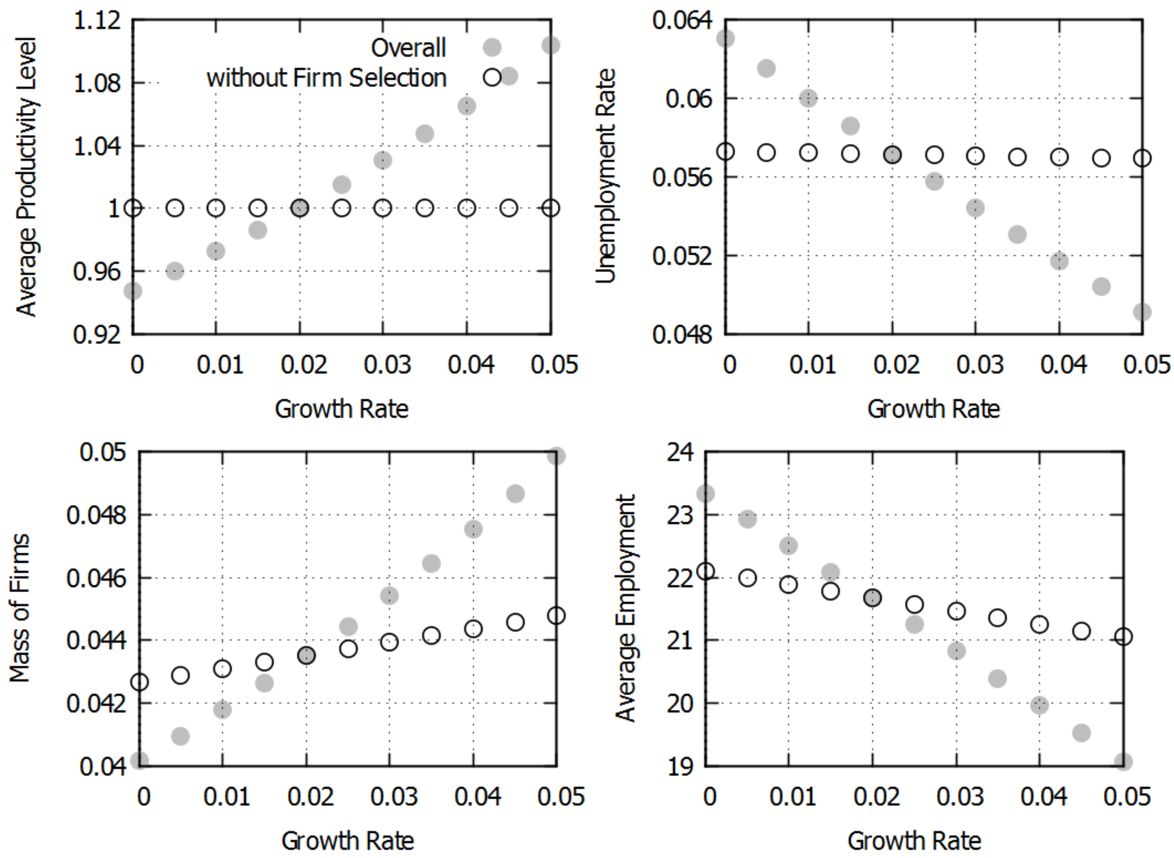


Figure 1: Impacts of productivity growth

It is well known that the estimate for the elasticity of substitution $\sigma$ has a range. Rather than choosing an arbitrary value, I target the markup to pin down $\sigma$. Specifically, I choose $\sigma = 3.5$ so that the implied markup is $(\sigma - \beta)/(\sigma - 1) = 1.2$, which is consistent with the estimates by Martins *et al.* (1996) and by Christopoulou and Vermeulen (2008).

According to Axtell (2001), the size distribution for US firms is approximately Zipf. I follow Axtell (2001) and set the shape parameter for the model's size distribution function to be $\alpha/(\sigma - 1) = 1.06$, from which I obtain $\alpha = 2.65$. To pin down the minimum productivity level $\varphi_{\min}$, I normalize the average productivity level $\tilde{\varphi}$ to one, from which I obtain $\varphi_{\min} = 0.10$.

The flow fixed cost, $I = 0.18$, is chosen to set the average firm size to be $l(\varphi) = 21.67$, which is the sample mean for the 1977-2014 period in the Business Dynamics Statistics.[3]

The model solutions for the wage rate, the unemployment rate, the cutoff productivity level, the mass of firms, and the aggregate vacancies are $w = 0.82$, $u = 0.057$, $\varphi^* = 0.32$, $n = 0.044$, and $V = 0.04$.[4]

Figure 1 presents the main results of this paper. I compute the average productivity level, the unemployment rate, the mass of firms, and the average firm size for different levels of the rate of technological progress. To illustrate the role of the selection effect, in each panel I present the result from the corresponding model *without* the selection effect,

[3]The average firm size in each year is computed as Emp divided by Firms. These series are obtained from https://www.census.gov/ces/dataproducts/bds/data_firm.html

[4]I used wxMaxima 16.04.2 to obtain the quantitative results. All codes are available upon request.

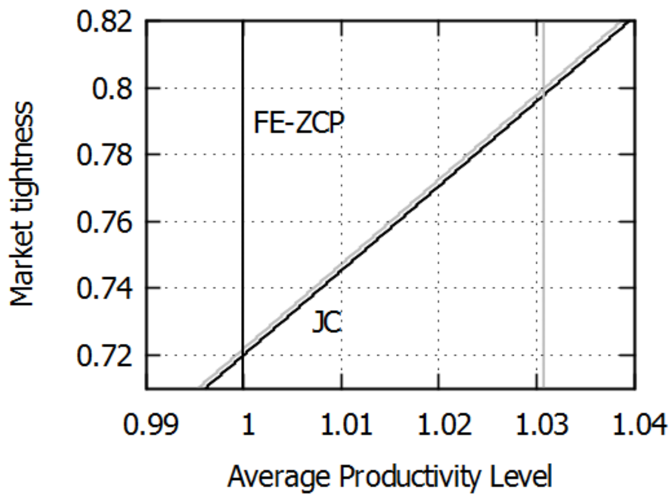


Figure 2: Balanced growth equilibria under $g = 0.02$(black) and $g = 0.03$(gray)

Table 2: Sensitivity to $z/w$

| $z/w$ | $z$ | $c$ | $\frac{c/q(\theta)}{\text{Quarterly wage}}$ | $du/dg$ |
|---|---|---|---|---|
| 0.71 (baseline) | 0.58 | 0.28 | 0.14 | $-0.28$ |
| 0.4 | 0.32 | 0.57 | 0.28 | $-0.14$ |

in which all firms have the same productivity $\varphi = 1$.

The four panels in Figure 1 clearly show that the impacts of an increase in the rate of technological progress on these variables are much greater in the model with the selection effect than without. In particular, the model with the selection effect implies $du/dg = -0.282$ at $g = 0.02$, while the model without the selection effect implies $du/dg = -0.007$.[5] This result is within the range of estimates by Blanchard and Wolfers (2000) and greater than those obtained by Pissarides and Vallanti (2007) and Miyamoto and Takahashi (2011).

Figure 2 illustrates why the model with the selection effect generates a strong quantitative effect. The upward-sloping curve, labeled JC, shows the equilibrium relationship defined by (13), and its shifts reflect the conventional capitalization effect. On the other hand, the vertical line, labeled FE-ZCP, shows the equilibrium relationship defined by (14), and its shifts reflect the selection effect. The solid black lines represent the equilibrium under $g = 0.02$ while the solid gray lines correspond to the same model under $g = 0.03$.

Initially, the equilibrium $\theta$ is at the target level, $\theta = 0.72$. When $g = 0.03$, $\theta$ increases to $\theta = 0.80$. The key observation is that, without the selection effect, the vertical FE-ZCP line does not shift. As the figure shows, in this case the JC curve makes an upward shift only slightly, and $\theta$ increases to $\theta = 0.722$. With the selection effect, the vertical FE-ZCP line makes a right-ward shift. This significantly increases $\theta$ and the effect is enhanced if the slope of the JC curve is steep. Thus, given the slope of the JC curve, the strong quantitative results found in this model come mostly from the selection effect.

To assess the robustness of the main results, here I present some sensitivity analyses.

[5] I also calibrate the textbook Pissarides (2000) model with the same targets and find that $du/dg = -0.007$. Pissarides and Vallanti (2007) show that $du/dg = -0.02$ in their model.

Table 3: Sensitivity to $\sigma$

| Markup | $\sigma$ | $\alpha$ | $z$ | $c$ | $du/dg$ |
|---|---|---|---|---|---|
| 1.1 | 6 | 5.3 | 0.63 | 0.31 | −0.14 |
| 1.2 (baseline) | 3.5 | 2.65 | 0.58 | 0.28 | −0.28 |
| 1.3 | 2.7 | 1.80 | 0.54 | 0.26 | −0.41 |

In the baseline case, I choose the target for $z/w$ to be 0.71 since the implied vacancy cost is plausible under this target value. However, it is informative to report the result under the alternative target, $z/w = 0.4$, since this value is often used in the literature (Shimer, 2005). Table 2 presents the result. When the target for $z/w$ is 0.4, the impact of an increase in the rate of technological progress becomes $du/dg = -0.14$, which is a half of the baseline case. However, this is still 20 times as strong as the effect obtained from the model without the selection effect.

It is also informative to report the results under alternative values of entry cost $K$ and fixed operation cost $I$. In the baseline case, I choose $K = [(0.6 + 5.2)/2] \times 1/12$ to target the average of the estimates of the entry cost in 1997 and 1978. An alternative target is $K = 0.6 \times 1/12$, as in Felbermayr and Prat (2011). Under my calibration procedure, any change in the value of $K/I$ changes the value of $\varphi_{\min}$, leaving the values of $c$ and $z$ unchanged. As a result, the value of $du/dg$ is the same as in the baseline case.

Finally, Table 3 reports the results under different targets for the markup. In the baseline case, the implied value of $\sigma$ is 3.5, which is somewhat below the conventional range of the empirical estimate in the trade literature, which is in between 5 to 10 (Anderson and Wincoop, 2004). The implied value for $\sigma$ is 6 when the target markup is 1.1. In this case, the impact of an increase in the rate of technological progress becomes $du/dg = -0.14$. While reduced, the effect is still 20 times as strong as the effect obtained from the model without the selection effect. If the target markup is chosen to be 1.3, then the model generates a significantly strong effect, $du/dg = -0.41$.

# 4 Conclusion

An important challenge in the theory of unemployment is to quantitatively understand the impact of productivity growth on unemployment. In this paper, I focused on the role of the selection effect (Melitz, 2003; Felbermayr and Prat, 2011) and found a strong quantitative effect. I also showed that the result is robust to changes in calibration targets. Thus, I conclude that firm heterogeneity is an essential ingredient for the study of growth and unemployment.

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# Appendix

## A Proof of Proposition 1

Use (7) and (3) to rewrite (5) as

$$E(l;\varphi,a) = W(l;\varphi,a) + \frac{\beta}{1-\beta}\frac{1-s}{1-\delta}\frac{ac}{q(\theta)} + \frac{1}{1+r}U(a'). \tag{16}$$

Substitute (4) and (16) into (7) to obtain an ordinary differential equation, $W'(l;\varphi,a) + \beta^{-1}W(l;\varphi,a) = Al^{-1/\sigma} + B$, where $A$ and $B$ are terms independent of $l$. I follow Kudoh *et al.* (2017) to solve it as

$$W(l;\varphi,a) = \frac{\sigma}{\sigma-\beta}\beta A l^{-1/\sigma} + \beta B = \beta\frac{\sigma-1}{\sigma-\beta}\frac{R(l;\varphi,a)}{l} - (1-\beta)\left[\frac{1}{1+r}U(a') - U(a)\right]. \tag{17}$$

From (6), (7), and (3),

$$\begin{aligned}\frac{1}{1+r}U(a') - U(a) &= -az - \frac{\theta q(\theta)}{1+r}\int_{\varphi^*}^{\infty}\left[E(l',\varphi,a') - U(a')\right]\mu(\varphi)d\varphi \\ &= -az - \frac{\theta q(\theta)}{1+r}\frac{\beta}{1-\beta}\frac{\partial J(l';\varphi,a')}{\partial l'} = -az - \frac{\theta q(\theta)}{1+r}\frac{\beta}{1-\beta}\frac{1+r}{1-\delta}\frac{ac}{q(\theta)}.\end{aligned}$$

Thus,

$$W(l;\varphi,a) = \beta\frac{\sigma-1}{\sigma-\beta}\frac{R(l;\varphi,a)}{l} + (1-\beta)\left[az + \frac{\beta}{1-\beta}\frac{ac\theta}{1-\delta}\right].$$

## B Proof of Proposition 2

The average productivity $\tilde{\varphi}$ is defined by $p(\tilde{\varphi}) = P$. This implies $R(l;\tilde{\varphi},a)/l = \tilde{\varphi}a$. Thus, (9) becomes

$$\frac{\partial J(l;\varphi,a)}{\partial l} = a\left[(1-\beta)\frac{\sigma-1}{\sigma-\beta}\tilde{\varphi} + (1-\lambda)\frac{c}{q(\theta)}\right] - (1-\beta)a\left[z + \frac{\beta}{1-\beta}\frac{c\theta}{1-\delta}\right]. \tag{18}$$

Thus, (3) and (18) imply that in any balanced growth equilibrium,

$$\frac{ac}{q(\theta)} = \frac{1-\delta}{1+r}J'(l';\varphi,a') = \frac{1-\delta}{1+r}a'\left[(1-\beta)\frac{\sigma-1}{\sigma-\beta}\tilde{\varphi} + (1-\lambda)\frac{c}{q(\theta)} - (1-\beta)z - \frac{\beta c\theta}{1-\delta}\right],$$

from which

$$\frac{c}{q(\theta)} = \frac{1-\delta}{1+r}(1+g)\left[(1-\beta)\frac{\sigma-1}{\sigma-\beta}\tilde{\varphi} + (1-\lambda)\frac{c}{q(\theta)} - (1-\beta)z - \frac{\beta c\theta}{1-\delta}\right],$$

and finally

$$\frac{\Lambda^{-1} - (1-\lambda)}{1-\beta}\frac{c}{q(\theta)} + \left[z + \frac{\beta}{1-\beta}\frac{c\theta}{1-\delta}\right] = \frac{\sigma-1}{\sigma-\beta}\tilde{\varphi}. \tag{19}$$

This obtains (13).

Now consider the wage function (8). Use (19) to rewrite the second term to obtain

$$W(l;\varphi,a)=\beta\frac{\sigma-1}{\sigma-\beta}\tilde{\varphi}a+(1-\beta)a\left[\frac{\sigma-1}{\sigma-\beta}\tilde{\varphi}-\frac{\Lambda^{-1}-(1-\lambda)}{1-\beta}\frac{c}{q(\theta)}\right].$$

Thus, in any BGE,

$$\begin{aligned}\pi(l;\varphi,a)&=\tilde{\varphi}al-\beta\frac{\sigma-1}{\sigma-\beta}\tilde{\varphi}al-(1-\beta)\frac{\sigma-1}{\sigma-\beta}\tilde{\varphi}al+\frac{\left[\Lambda^{-1}-(1-\lambda)\right]ac}{q(\theta)}l-\frac{ac\lambda}{q(\theta)}l-aI\\&=\left[1-\frac{\sigma-1}{\sigma-\beta}\right]\tilde{\varphi}al+\frac{\left[\Lambda^{-1}-1\right]ac}{q(\theta)}l-aI\\&=\frac{1-\beta}{\sigma-\beta}\tilde{\varphi}al(\varphi)+\frac{\left[\Lambda^{-1}-1\right]ac}{q(\theta)}l(\varphi)-aI\equiv\pi(\varphi)a.\end{aligned}$$

Thus, the value of a firm in any BGE satisfies

$$J(l;\varphi,a)=\pi(\varphi)a+\frac{1-\delta}{1+r}J(l';\varphi,a')\Leftrightarrow\frac{J(l;\varphi,a)}{a}=\pi(\varphi)+\frac{1-\delta}{1+r}\frac{a'}{a}\frac{J(l';\varphi,a')}{a'},$$

from which I define $\tilde{J}(\varphi)\equiv J(l;\varphi,a)/a$ recursively as $\tilde{J}(\varphi)=\pi(\varphi)+\Lambda\tilde{J}(\varphi)$, from which $\tilde{J}(\varphi)=\pi(\varphi)/(1-\Lambda)$. Note that $\tilde{J}(\varphi)$ is independent of $a$. In any BGE, the value of an operating firm of type $\varphi$ is $J(l;\varphi,a)=\pi(\varphi)/(1-\Lambda)a$. Substitute this into (12) to obtain the value of entry for a type-$\varphi$ firm in a BGE as:

$$\begin{aligned}J(0,\varphi,a)&=-\frac{acl}{q(\theta)}-aI+\frac{1-\delta}{1+r}\frac{\pi(\varphi)}{1-\Lambda}a'=\left[-\frac{cl}{q(\theta)}-I+\frac{\Lambda}{1-\Lambda}\pi(\varphi)\right]a\\&=\left[-\frac{cl}{q(\theta)}-I+\frac{\Lambda}{1-\Lambda}\left[\frac{1-\beta}{\sigma-\beta}\tilde{\varphi}l(\varphi)+\frac{\left[\Lambda^{-1}-1\right]c}{q(\theta)}l(\varphi)-I\right]\right]a\\&=\left[\frac{\Lambda}{1-\Lambda}\frac{1-\beta}{\sigma-\beta}\tilde{\varphi}l(\varphi)-\frac{1}{1-\Lambda}I\right]a.\end{aligned}$$

Thus, $J(0,\varphi^{*},a)=0$ implies

$$\frac{1-\beta}{\sigma-\beta}\tilde{\varphi}l(\varphi^{*})=\frac{I}{\Lambda}.\tag{20}$$

The free entry condition implies

$$\begin{aligned}K&=\int_{\varphi^{*}}^{\infty}\left[\frac{\Lambda}{1-\Lambda}\frac{1-\beta}{\sigma-\beta}\tilde{\varphi}l(\varphi)-\frac{1}{1-\Lambda}I\right]f(\varphi)d\varphi\\&=\frac{\Lambda}{1-\Lambda}\frac{1-\beta}{\sigma-\beta}\tilde{\varphi}\int_{\varphi^{*}}^{\infty}l(\varphi)f(\varphi)d\varphi-\frac{I}{1-\Lambda}\int_{\varphi^{*}}^{\infty}f(\varphi)d\varphi\\&=\frac{\Lambda}{1-\Lambda}\frac{1-\beta}{\sigma-\beta}\tilde{\varphi}\left[1-F(\varphi^{*})\right]l(\tilde{\varphi})-\frac{I}{1-\Lambda}\left[1-F(\varphi^{*})\right]\\&=\frac{\left[1-F(\varphi^{*})\right]}{1-\Lambda}\left[\Lambda\frac{1-\beta}{\sigma-\beta}\tilde{\varphi}l(\tilde{\varphi})-I\right].\end{aligned}$$

Substitute (20) into the above to obtain

$$\frac{K}{1-F\left(\varphi^{*}\right)}=\frac{I}{1-\Lambda}\left[\frac{l\left(\tilde{\varphi}\right)}{l\left(\varphi^{*}\right)}-1\right]=\frac{I}{1-\Lambda}\left[\left(\frac{\tilde{\varphi}}{\varphi^{*}}\right)^{\sigma-1}-1\right],$$

where I have used the fact $l(\varphi_1)/l(\varphi_2)=(\varphi_1/\varphi_2)^{\sigma-1}$. Finally, since $F$ is Pareto, $F(\varphi)=1-(\varphi_{\min}/\varphi)^{\alpha}$ and $(\tilde{\varphi}/\varphi^{*})^{\sigma-1}=\alpha/(\alpha+1-\sigma)$. Thus,

$$\frac{K}{(\varphi_{\min}/\varphi^{*})^{\alpha}}=\frac{I}{1-\Lambda}\left[\left(\frac{\tilde{\varphi}}{\varphi^{*}}\right)^{\sigma-1}-1\right]=\frac{I}{1-\Lambda}\frac{\sigma-1}{\alpha+1-\sigma}.$$

This is (14).

# C Calibration Details

The parameter values for $r$, $g$, $s$, $\delta$, $\eta$, $\beta$, and $K$ are exogenously given. With $\beta=0.5$, $\sigma$ is the solution to $(\sigma-0.5)/(\sigma-1)=1.2$, which equates the markup in the model to its target value. Thus, I obtain $\sigma=3.5$. The value of $\alpha$ is calculated as $\alpha=[\alpha/(\sigma-1)]\times(\sigma-1)=1.06\times(3.5-1)$, where $\alpha/(\sigma-1)$ is the the shape parameter of the firm size distribution, which is 1.06 from Axtell (2001). With $\eta=0.5$, I obtain the value of $m_0$ as $m_0=m_0\theta^{1-\eta}\times\theta^{\eta-1}=0.594\times 12\times 0.72^{0.5-1}$, where $m_0\theta^{1-\eta}=0.594\times 12$ and $\theta=0.72$. The parameter values of $z$ and $c$ are given by solving the wage equation and (13):

$$w=\left(\frac{z}{w}\right)^{-1}z=z+\frac{\beta}{1-\beta}\frac{\theta}{1-\delta}c+\frac{\beta}{1-\beta}\frac{c}{q(\theta)}[\Lambda^{-1}-1+\lambda],$$

$$\left(\frac{z}{w}\right)^{-1}z=\frac{\sigma-1}{\sigma-\beta}\tilde{\varphi}-\frac{c}{q(\theta)}[\Lambda^{-1}-1+\lambda].$$

where $z/w=0.71$ and $\tilde{\varphi}=1$. To obtain the value of $I$, use $l(\varphi_1)/l(\varphi_2)=(\varphi_1/\varphi_2)^{\sigma-1}$ and $(\tilde{\varphi}/\varphi^{*})^{\sigma-1}=\alpha/(\alpha+1-\sigma)$ to rewrite(20) as

$$\frac{I}{\Lambda}=\frac{1-\beta}{\sigma-\beta}\tilde{\varphi}l\left(\varphi^{*}\right)=\frac{1-\beta}{\sigma-\beta}\tilde{\varphi}\left(\frac{\varphi^{*}}{\tilde{\varphi}}\right)^{\sigma-1}l(\tilde{\varphi})=\frac{1-\beta}{\sigma-\beta}\frac{\alpha+1-\sigma}{\alpha}\tilde{\varphi}l(\tilde{\varphi}),$$

where $\tilde{\varphi}=1$ and the target value of $l(\tilde{\varphi})$ is given as 21.76. Finally, given $I$ and $K$, the value of $\varphi_{\min}$ is given by the solution to (15) and $(\tilde{\varphi}/\varphi^{*})^{\sigma-1}=\alpha/(\alpha+1-\sigma)$.