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On parental care and home production

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Abstract

I build a simple model on parental care and home production to consider an application of the Alchian-Allen theorem in time allocation problems. This theorem predicts that an increase in a parent's wage should lower the relative demand of parental care time to housework time. However, empirical studies find the opposite result. My model shows that the complementarity between parental time and education spending can explain this paradox.

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1 Introduction

Parents spend a large amount of time caring and playing with their young children. Compared with other non-market work, people find that time with children is more enjoyable and rewarding[1]. This time input increases children's human capital. Landry et al. (2003) and Belsy et al. (2007) suggest that parental time helps children develop better communication skills and more curiosity in learning. Given this unique role of child care, its separation from other forms of home production in modeling is warranted[2].

Child care also differs from other household chores in its cost. When a parent is watching her children, child care expenses are avoided. So, the opportunity cost of time with children, being the forgone wage minus child care spending, is lower than the cost of home production, being the forgone wage. Given this price difference, time allocation problems provide a potential application of the Alchian-Allen theorem. The theorem states that a one unit increase in the prices of two substitute goods will result in a higher relative demand for the pricier good (see Alchian and Allen (1964)). This is because the same change in two prices causes the expensive good to become relatively less expensive and thus more desirable. A common example is that when a gasoline tax is imposed, the relative demand for premium gasoline increases due to a decrease in its relative price.

According to this theorem, as a parent's wage increases, home production becomes relatively less costly and thus more attractive. Therefore, one should expect the wage effect on home production/childcare ratio to be positive. However, empirical evidence shows the opposite relationship. Kimmel and Connelly (2007) use data from 2003 and 2004 American Time Use Survey (ATUS) to estimate the wage effect on time spent in leisure, childcare, home production and employment. After controlling for the number of children in different age groups, child care prices, and some demographic variables, they show that one percent increase in wage, on average, increases childcare time by 62 minutes and decreases home production time by 73 minutes on weekdays. Guryan et al. (2008) use 2003-2006 ATUS data and find similar results with a focus on schooling. They show that more years of schooling result in more time with children and less time in home production.

Why does the Alchian-Allen theorem fail in this application? First, the theorem only

[1] See Krueger et al. (2009).

[2] I will use the term "home production" to refer to all non-market work other than child care.

addresses the substitution effect of a price change. If a good has an unusual income effect (i.e. an inferior good), the theorem result may not hold. Minagawa and Upmann (2013) demonstrate this point by deriving the ordinary demand functions of leisure and child care. Second, the theorem result is obtained from a two-good framework. Gould and Segall (1969) show that in a case of more than two goods with income compensation, the Alchian-Allen result will not be reached unambiguously. I focus on the second case and show that even without an unusual income effect, the Alchian-Allen result may not hold. To demonstrate this, I build a simple model of time allocation and derive the conditions under which the Alchian-Allen result will not hold.

In my model, parents allocate one unit of time among work, home production and child care. Their labor income is spent on consumption, housework and education purchase. Spending on housework can be seen as investing in modern machines or hiring labor to do chores. Such spending is a substitute for home production time. Education purchase includes buying children educational toys and paying for private lessons. The relationship between such spending and parental time is key to my result. Both Youderian (2017) and Vural and Zhu (2013) model children's human capital production and calibrate the substitutability parameter between parental time and goods investment. Their results imply that education expenditure is complementary to time input. My model predicts that if this complementarity is strong enough, the relative compensated demand of home production time will decrease, as opposed to increase as predicted by the Alchian-Allen theorem.

2 The model

Consider a model where parents choose time spent on work, home production and child care which are denoted by t_w, t_h, and t_c, respectively[3]. Home production time does not bring value directly, but the outcome of this time input, such as a tidy house and cooked meals, increases utility. This outcome can also be purchased in the market and the quantity of this purchase is denoted by q_h. Let the output of home production be $H = f(t_h, q_h)$,where function f is increasing and concave in t_h and q_h. I allow a relative substitutability between time and goods input in home production. A homeowner is happy with a clean house whether

[3] I abstract leisure from the model for simplicity and focus on the question at hand.

she cleaned it herself or outsourced the work. This results in $f_{t_h q_h} < 0$ and $f_{q_h t_h} < 0$. Similarly, time and goods investment in children form their human capital $C = g(t_c, q_c)$, where function g is increasing and concave in t_c and q_c. Nordblom (2003) argues that compared to formal education, parental interaction provides different, yet complementary, skills to children. Youderian (2018) and Zhu and Vural (2013) both build and specifically estimate a model of children's human capital and their calibration results also imply a complementarity between the two inputs. Therefore, I assume $g_{t_c q_c} > 0$ and $g_{q_c t_c} > 0$.

Parents spend their labor income outsourcing housework q_h, investing in children's education q_c, and consuming a composite good q_z. They derive utility from completed housework, children's human capital, and consumption. Their preferences can be expressed by a utility function $u(H, C, q_z)$.

Let w be the wage, p_h and p_c be the price of housework purchase and children's education respectively, and the price of a composite good be normalized to 1. When parents are not spending time with their children, they need to pay for child care at a market price of w_c. I focus on interior solutions and therefore assume $w_c < w$. Assuming total time endowment is $T = 1$, a parent's optimization problem is

$$\max_{t_w, t_h, t_c, q_w, q_c, q_z} u(H, C, q_z)$$

$$\begin{aligned}
p_h q_h + p_c q_c + q_z + (t_h + t_w) w_c &= t_w w \\
t_w + t_h + t_c &= 1 \\
H &= f(t_h, q_h) \\
C &= g(t_c, q_c).
\end{aligned}$$

Combining the income and time constraints simplifies to $p_h q_h + p_c q_c + q_z + w t_h + (w - w_c) t_c = w - w_c$. This expression illustrates that time spent on housework and child care has a price, similar to consumption goods. The price of doing housework is the forgone wage $p_{t_h} = w$, while the price of spending time with children is $p_{t_c} = w - w_c$, as the parent avoids paying for child care. Let's define the total endowment $e(w, w_c) = w - w_c$ and rewrite the budget constraint $p_h q_h + p_c q_c + q_z + p_{t_h} t_h + p_{t_c} t_c = e$.

My focus is on the compensated demand for t_h and t_c, so I write their compensated (Hicksian) demand functions $t'_h(p_h, p_c, p_{t_h}, p_{t_c}, u')$ and $t'_c(p_h, p_c, p_{t_h}, p_{t_c}, u')$. These are the solutions to the expenditure minimization problem

$$\min_{t_h, t_c, q_w, q_c, q_z} p_h q_h + p_c q_c + q_z + p_{t_h} t_h + p_{t_c} t_c$$

$$\begin{aligned} u(H, C, q_z) &\geq u' \\ H &= f(t_h, q_h) \\ C &= g(t_c, q_c). \end{aligned}$$

The model yields the following result.

Proposition: *The Alchian-Allen result does not hold if education purchase is a strong complement to child care time.*

The change in relative compensated demand for home production relative to child care resulting from one unit change in their prices is

$$\frac{\partial \frac{t_h'}{t_c'}}{\partial w} = \frac{t_h'}{t_c'} \frac{1}{w - w_c} [(\varepsilon'_{t_h t_h} - \varepsilon'_{t_c t_h})(\frac{w - w_c}{w} - 1) + (\varepsilon'_{t_c q_z} - \varepsilon'_{t_h q_z}) + (\varepsilon'_{t_c q_h} - \varepsilon'_{t_h q_h}) + (\varepsilon'_{t_c q_c} - \varepsilon'_{t_h q_c})] \quad (1)$$

where ε'_{jk} is the compensated price elasticity where $j \in (t_h, t_c)$ and $k \in (t_h, q_z, q_h, q_c)$. See proof of equation (1) in the appendix.

Proof: For the Alchian-Allen result to hold, the expression above needs to be positive. Let's discuss each of the four components in the brackets. As long as home production is not a perfect complement to child care, $\varepsilon'_{t_h t_h} - \varepsilon'_{t_c t_h} < 0$ and thus $(\varepsilon'_{t_h t_h} - \varepsilon'_{t_c t_h})(\frac{w - w_c}{w} - 1) > 0$. The sign of $\varepsilon'_{t_c q_z} - \varepsilon'_{t_h q_z}$ is ambiguous, depending on the substitutability between child care and consumption relative to the substitutability between home production and consumption. Similarly, $\varepsilon'_{t_c q_h} - \varepsilon'_{t_h q_h}$ has an ambiguous sign. The last term in equation (1) $\varepsilon'_{t_c q_c} - \varepsilon'_{t_h q_c}$ is the key. Based our discussion on the human capital accumulation process in childhood, I consider education spending and parental time as complements. This allows $\varepsilon'_{t_c q_c} < 0$ which leads to $\varepsilon'_{t_c q_c} - \varepsilon'_{t_h q_c} < 0$.

The sign of $\frac{\partial \frac{t_h'}{t_c'}}{\partial w}$ relies on the magnitude of $\varepsilon'_{t_c q_c} - \varepsilon'_{t_h q_c}$ relative to the other three terms. If education purchase is a sufficiently strong complement to parental time, i.e. $\varepsilon'_{t_c q_c} - \varepsilon'_{t_h q_c}$ is sufficiently negative, $\frac{\partial \frac{t_h'}{t_c'}}{\partial w} < 0$. This implies that as wage goes up, relative demand for home production time to child care time decreases. This is consistent with empirical evidence from

Guryan et al. (2008)[4].

The intuition of this proposition is simple. As the relative price of child care increases, the demand for other goods (i.e. education spending) increases. Since education spending complements time input, the marginal return to care time increases due to the higher level of education spending. This effect works against the Alchian-Allen result. If the complementarity is strong enough, the change in relative demand will be pushed to the opposite direction.

3 Conclusion

I consider an application of the Alchian-Allen theorem on a time allocation problem. The theorem predicts a different wage effect on the relative demand of home production than what is found in the data. To reconcile this inconsistency, I introduce to the model a recent finding about how time and goods input interact in producing human capital. Calibration results in Youderian (2017) and Vural and Zhu (2013) show that parental time and education spending are complements, and this relationship can explain why the Alchian-Allen result does not hold. My paper provides theoretical validation to this complementarity and highlights its importance in modeling parental time allocation.

4 Appendix

In this appendix, I show the proof of equation (1).

$$\frac{\partial \frac{t_h'}{t_c'}}{\partial w} = \frac{1}{t_c'^2}\left(\frac{\partial t_h'}{\partial w} t_c' - \frac{\partial t_c'}{\partial w} t_h'\right). \tag{2}$$

Given $t_h'(p_h, p_c, p_{t_h}, p_{t_c}, u'), t_c'(p_h, p_c, p_{t_h}, p_{t_c}, u'), p_{t_h} = w$ and $p_{t_c} = w - w_c$, we have $\frac{\partial t_h'}{\partial w} = \frac{\partial t_h'}{\partial p_{t_h}} \frac{\partial p_{t_h}}{\partial w} + \frac{\partial t_h'}{\partial p_{t_c}} \frac{\partial p_{t_c}}{\partial w} = \frac{\partial t_h'}{\partial p_{t_h}} + \frac{\partial t_h'}{\partial p_{t_c}}$. Similarly, $\frac{\partial t_c'}{\partial w} = \frac{\partial t_c'}{\partial p_{t_h}} + \frac{\partial t_c'}{\partial p_{t_c}}$. Rewrite equation (2)

$$\frac{\partial \frac{t_h'}{t_c'}}{\partial w} = \frac{1}{t_c'^2}\left(\left(\frac{\partial t_h'}{\partial p_{t_h}} + \frac{\partial t_h'}{\partial p_{t_c}}\right) t_c' - \left(\frac{\partial t_c'}{\partial p_{t_h}} + \frac{\partial t_c'}{\partial p_{t_c}}\right) t_h'\right). \tag{3}$$

[4] I acknowledge that the results of my model are not the only possible explanation for the empirical findings. My purpose is to show that these findings can be produced by the inapplicability of the Alchian-Allen theorem apart from income effect.

Use the expressions of price elasticity, such as $\varepsilon'_{t_h t_h} = \frac{\partial t'_h}{\partial p_{t_h}} \frac{p_{t_h}}{t'_h}$, to rewrite equation (3)

$$\frac{\partial \frac{t'_h}{t'_c}}{\partial w} = \frac{t'_h}{t'_c} \frac{1}{p_{t_c}} ((\varepsilon'_{t_h t_h} - \varepsilon'_{t_c t_h}) \frac{p_{t_c}}{p_{t_h}} + (\varepsilon'_{t_h t_c} - \varepsilon'_{t_c t_c}))$$

Plugging in $p_{t_h} = w$ and $p_{t_c} = w - w_c$

$$\frac{\partial \frac{t'_h}{t'_c}}{\partial w} = \frac{t'_h}{t'_c} \frac{1}{w - w_c} ((\varepsilon'_{t_h t_h} - \varepsilon'_{t_c t_h}) \frac{w - w_c}{w} + (\varepsilon'_{t_h t_c} - \varepsilon'_{t_c t_c})).$$

To better discuss the intuition of this equation, we use $\varepsilon'_{t_h t_c} + \varepsilon'_{t_h t_h} + \varepsilon'_{t_h q_h} + \varepsilon'_{t_h q_c} + \varepsilon'_{t_h q_z} = 0$ and $\varepsilon'_{t_c t_c} + \varepsilon'_{t_c t_h} + \varepsilon'_{t_c q_h} + \varepsilon'_{t_c q_c} + \varepsilon'_{t_c q_z} = 0$ to substitute $\varepsilon'_{t_h t_c}$ and $\varepsilon'_{t_c t_c}$

$$\frac{\partial \frac{t'_h}{t'_c}}{\partial w} = \frac{t'_h}{t'_c} \frac{1}{w - w_c} ((\varepsilon'_{t_h t_h} - \varepsilon'_{t_c t_h})(\frac{w - w_c}{w} - 1) + (\varepsilon'_{t_c q_z} - \varepsilon'_{t_h q_z}) + (\varepsilon'_{t_c q_h} - \varepsilon'_{t_h q_h}) + (\varepsilon'_{t_c q_c} - \varepsilon'_{t_h q_c})).$$

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