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A calibration of the output elasticity of public capital

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Abstract

The empirical literature is far from having reached a consensus on the output elasticity of public capital. This lack of consensus continues to hinder quantitative general equilibrium analysis and fiscal policy study. Therefore, this paper aims to clarify the order of magnitude of this elasticity by performing a calibration for 14 European Union (EU) countries in the period 1980-2014. To do this, a growth model is developed based on the theoretical and empirical literature on public capital. The calibrated elasticity ranges between 0.05 and 0.13. These results are in line with the panel estimates for 6 EU countries in the period 1969-2002 by Creel and Poilon (2008).

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1. Introduction

Despite a great deal of effort being devoted to estimating the contribution of public capital to aggregate output, the empirical literature has still not reached a consensus on this subject. Indeed, a comprehensive survey carried out by Bom and Ligthart (2014), involving 68 studies and 578 estimates,[1] shows econometric estimates of the output elasticity of public capital ranging from -1.7 to 2.04. These results not only reflect country differences, but even studies on the same country obtain quite different values. For instance, for the case of the American economy the estimates range from -0.491 to 0.56. The authors find an average elasticity of 0.106, though this figure is not representative of the wide range of estimates.

It has also been recognized that some econometric studies, and especially earlier ones, present methodological shortcomings, such as spurious correlation, reverse causation, small sample problems or endogeneity of regressors (Romp and de Haan, 2007; Bom and Ligthart, 2014). These deficiencies cast doubt on the reliability of estimates. Moreover, these studies usually rely on the so-called production function approach to estimate the output elasticity of public capital. Though it seems a natural way to do it, this approach fails to consider the relationships among relevant macroeconomic aggregates (Bom and Ligthart, 2014). These omissions may well yield biased estimates due to flawed model specification.

This paper aims to clarify the order of magnitude of the output elasticity of public capital. This is a relevant task, since this elasticity is a crucial piece of information for quantitative general equilibrium analysis and fiscal policy study. As an alternative to econometric estimation, a calibration of the output elasticity of public capital for 14 European Union (EU) countries during the period 1980-2014 is performed, using the structural equations of a theoretical model. Two data sets are used in the analysis: the series of private and public capital and investment constructed by the International Monetary Fund (IMF, 2017) and the Penn World Table version 9.0 (PWT 9.0).

The approach followed in this paper poses the question about the type of model that could suitably represent the accumulation of public capital. In this respect, the model constructed here is based on the results of theoretical and empirical literature on this topic. Indeed, a strand of the theoretical literature on endogenous growth considers public capital as the only engine of sustained growth (e.g. Barro, 1990; Fisher and Turnovsky, 1998). Nonetheless, the comprehensive report about public infrastructure by the World Bank (1994) concludes that, though public investment is essential for good economic performance, it could hardly be considered as a sufficient condition for sustained growth.[2] This view is supported by Sala-i-Martin's (1997) empirical results, which show no impact of public investment on long-run growth. Another important aspect refers to whether public capital should be treated as a factor input in the production function, or as a factor influencing the total factor productivity (TFP). As noted by Duggal, Saltzman and Klein, (1999), the former approach seems unreasonable, since private firms are unable to calculate the marginal cost of public capital. Moreover, the existence of perfect competition requires the production function to exhibit constant return to scale in private

[1] These authors reviewed studies based on the production function approach spanning the 1983-2008 period. They also carried out a meta-regression analysis and found the presence of publication bias, which implies that those studies that obtain significant and large estimates are more prone to be published.

[2] Along these lines, some endogenous growth models include human capital investment besides public capital accumulation (e.g. Glomm and Ravikumar, 1997; Agénor, 2011).

factor inputs. Congestion is another important aspect to be addressed. The services from public capital are not a pure public good, as there is a certain degree of rivalry in their use (e.g. World Bank, 1994; Romp and de Haan, 2007). This is why general equilibrium models usually assume that public capital is subject to congestion (e.g. Fisher and Turnovsky, 1998; Seung and Kraybill, 2001; Alonso-Carrera, Freire-Serén and Manzano, 2009).

Accordingly, a model is calibrated in which there is exogenous growth, so the economy cannot grow sustainably by simply accumulating public capital. Moreover, public capital influences TFP and is subject to congestion. At this point, it should be noted that the calibration requires the assumption that the government implements an optimal fiscal policy, which gives rise to optimal provision. To elucidate whether or not actual public capital is optimal, we must rely on a particular model and, of course, a correct model that represents an economy is unknown.[3] However, it could argue that, in mature democracies, governments have incentives to provide the required public capital to guarantee the optimal economic and social functioning. Any deviation, either a shortage or an excess of public capital, would be punished in elections. Thus, while deviations may occur in the short-medium term, one would expect public capital to be near its optimum level in the long-run.

The remainder of the paper is organized as follows. Section 2 presents the model. Sections 3 and 4 solve for the social optimum and the optimal fiscal policy, respectively. Section 5 performs the calibration. Lastly, Section 6 concludes.

2. The model

In each period $t = 0,1,2,...$, the economy is inhabited by a continuum of measure one of identical households, which amounts to the labor force. Thus, the variables are expressed in per capita terms.

The economy only produces a final good, Y_t, with a similar technology to that in Alonso-Carrera, Freire-Serén and Manzano (2009):

$$Y_t = A\, K_t^{\alpha} \left(1+\theta\right)^{(1-\alpha)t} \left(\frac{G_t}{K_t}\right)^{\varphi}, \quad A > 0, \quad \theta \geq 0, \quad \alpha \in \left(0,1\right), \quad \varphi < \alpha. \tag{1}$$

The technology exhibits constant returns to scale in private factor inputs, capital K_t and labor, where the capital and labor shares in income are equal to α and $1-\alpha$, respectively. It is worth noting that this assumption allows the existence of perfect competition. The stock of public capital, G_t, influences the TFP. Moreover, public capital is divided by private capital of the economy, which stands for a congestion externality. The output elasticity of public capital is captured by parameter φ. The accumulation of public capital is not enough to allow for sustained growth in the long-run. This result is achieved

[3] Using an endogenous growth model in which public capital is the engine of growth, Aschauer (2000) and Kamps (2005) attempted to elucidate whether there is a shortage or an excess of public capital. However, their results were not based on the optimal fiscal policy, but on the growth-maximizing policy. In this respect, it could be argued that governments do not seek to maximize economic growth, but social welfare, since the former objective would result in capital over-accumulation. Aschauer (2000) found a shortage of public capital in the United States, while Kamps (2005) obtained that there was neither shortage nor excess in the EU countries.

because of the presence of labor-augmenting technological progress that grows at the exogenous rate θ.

The aggregate good is devoted to consumption, C_t, and gross investment in private and public capitals, I_t^K and I_t^G, respectively. Thus, the feasibility constraint of the economy is:

$$C_t + I_t^K + I_t^G \leq A\, K_t^{\alpha} \left(1+\theta\right)^{(1-\alpha)t} \left(\frac{G_t}{K_t}\right)^{\varphi}. \tag{2}$$

Moreover, private and public capitals accumulate as:

$$K_{t+1} \leq I_t^K + \left(1-\delta_K\right) K_t, \tag{3}$$

$$G_{t+1} \leq I_t^G + \left(1-\delta_G\right) G_t, \tag{4}$$

where private and public capitals depreciate at the rates $\delta_K > 0$ and $\delta_G > 0$, respectively.

Lastly, discounted utility takes the standard form:

$$\sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\sigma}-1}{1-\sigma}, \quad \beta > 0, \quad \sigma > 0, \tag{5}$$

3. The Social Optimum

The social planner chooses C_t, I_t^K, I_t^G, K_{t+1} and G_{t+1} to maximize (5), subject to (2), (3), (4) and the initial capital endowments $K_0 > 0$ and $G_0 > 0$.

The first order conditions of the problem evaluated in the long-run equilibrium yield:

$$R^{SO} \equiv \underbrace{\left(\alpha-\varphi\right)\left(\frac{\tilde{K}}{\tilde{Y}}\right)^{-1} - \delta_K}_{\substack{\text{Net marginal}\\ \text{return on}\\ \text{private capital}}} = \underbrace{\varphi\left(\frac{\tilde{G}}{\tilde{Y}}\right)^{-1} - \delta_G}_{\substack{\text{Net marginal}\\ \text{return on}\\ \text{public capital}}}, \tag{6}$$

$$\frac{\left(1+\theta\right)^{\sigma}}{\beta} - 1 = \left(\alpha-\varphi\right)\left(\frac{\tilde{K}}{\tilde{Y}}\right)^{-1} - \delta_K, \tag{7}$$

$$\underbrace{1-\frac{\tilde{C}}{\tilde{Y}}}_{\text{Saving rate}} = \underbrace{\frac{\tilde{I}^K}{\tilde{Y}}}_{\substack{\text{Private}\\ \text{investment}\\ \text{rate}}} + \underbrace{\frac{\tilde{I}^G}{\tilde{Y}}}_{\substack{\text{Public}\\ \text{investment}\\ \text{rate}}}, \tag{8}$$

$$\theta + \delta_K = \frac{\tilde{I}^K}{\tilde{Y}}\left(\frac{\tilde{K}}{\tilde{Y}}\right)^{-1}, \tag{9}$$

$$\theta + \delta_G = \frac{\tilde{I}^G}{\tilde{Y}}\left(\frac{\tilde{G}}{\tilde{Y}}\right)^{-1}, \tag{10}$$

where the wiggle symbol denotes detrended variables in the social optimum (SO), i.e. $\tilde{Y} \equiv Y_t^{SO}(1+\theta)^{-t}$ and so on, that become constant in the long-run as the economy grows at rate θ. Equation (6) states that the net marginal returns on private and public capital, R^{SO}, must be equalized. Equation (7) is the Euler equation of consumption. The distribution of the saving rate between private and public investment rates is reflected in equation (8), while equations (9) and (10) come from the accumulation of private and public capitals, respectively.

4. Competitive Equilibrium and the Optimal Fiscal Policy

In a competitive equilibrium, firms take the term $(G_t/K_t)^{\varphi}$ in (1) as given, since public capital is a congested public good. As in Marrero and Novales (2011), the social optimum can be decentralized using an income tax τY_t, with rate $\tau \in (-1,1)$, and a lump-sum tax $T_t = \upsilon Y_t$, $\upsilon \in (-1,1)$, such that $I_t^G = (\tau + \upsilon) Y_t$. Thus, the variable $x \equiv \tau + \upsilon$ represents the public investment rate or government size. Note that a government that seeks to maximize social welfare must optimally finance public investment and also fix the congestion externality.

The competitive equilibrium yields the following conditions in the long-run:

$$\frac{(1+\theta)^{\sigma}}{\beta} - 1 = \underbrace{(1-\tau)\alpha\left(\frac{\bar{K}}{\bar{Y}}\right)^{-1} - \delta_K}_{\text{Interest rate } (r)}, \tag{11}$$

$$\theta + \delta_K = \underbrace{\left(1 - \frac{\bar{C}}{\bar{Y}} - x\right)}_{\frac{\bar{I}^K}{\bar{Y}}}\left(\frac{\bar{K}}{\bar{Y}}\right)^{-1}, \tag{12}$$

$$\theta + \delta_G = x\left(\frac{\bar{G}}{\bar{Y}}\right)^{-1}, \tag{13}$$

where detrended variables in the competitive equilibrium (CE) are denoted with a bar, i.e. $\bar{Y} \equiv Y_t^{CE}(1+\theta)^{-t}$ and so on. The Euler equation of consumption is given by (11), and (12) and (13) come from the accumulation of private and public capitals, respectively. Comparing (6) through (10) with (11) through (13), the optimal fiscal policy turns out to be $\tau^{SO} = \varphi/\alpha$ and $x^{SO} = (\theta + \delta_G)\tilde{G}/\tilde{Y}$; thus $\upsilon^{SO} = x^{SO} - \varphi/\alpha$. The tax rate is always positive, since it is aimed at fixing the congestion externality. By contrast, υ^{OGP} is positive (negative) if revenues from the proportional income tax fall short (exceed) of the required revenues for financing the optimal public investment. No lump-sum tax would be required when revenues from income tax allow an exact financing of the optimal public investment.

5. Calibration

The analysis assumes that the government implements an optimal fiscal policy. As previously mentioned, this assumption can be justified by arguing that governments have long-run incentives to provide the stock of public capital that guarantees optimal economic and social functioning.

The calibration of the model is performed for 14 EU countries during the period 1980-2014. The analysis uses series of GDP, public and private capitals, and public and private investment from the IMF (2017). The population and the labor share from the PWT 9.0 are also used.[4] It must be mentioned that the database of the IMF (2017) includes statistical information from 1960. However, in the calibration it is assumed that countries are in their long-run equilibrium, which is not true for some countries before 1980.[5] For this reason, the calibration is performed from 1980 onwards. Though a long time period must be considered to capture the long-term trend, thus avoiding business cycle fluctuations, it is convenient to break down the entire period into the two sub-periods 1980-1997 and 1998-2014, in order to check the robustness of the results. The results corresponding to 1980-2014 and the two sub-periods are summarized in Tables 1, 2 and 3, respectively.

For each country, growth rate θ is set equal to the average annual growth rate of GDP per capita. The capital share in income α is computed as one minus the annual average labor share in GDP. The parameter δ_G is calibrated introducing the growth rate, θ, the average annual investment rate of public capital in GDP, $\tilde{I}^G/\tilde{Y}$, and the annual average ratio of public capital over GDP, $\tilde{G}/\tilde{Y}$, in equation (10). Likewise, the value for δ_K is obtained substituting θ, the average annual investment rate of private capital in GDP, $\tilde{I}^K/\tilde{Y}$, and the annual average ratio of private capital over GDP, $\tilde{K}/\tilde{Y}$, in equation (9). The output elasticity of public capital φ is then obtained introducing δ_G, δ_K, α, $\tilde{G}/\tilde{Y}$ and $\tilde{K}/\tilde{Y}$ in equation (6). These values allow the implied optimal tax, $\tau^{SO} = \varphi/\alpha$, to be computed. The data and calibrated parameter values allow computing the optimal net marginal return on capital accumulation, R^{SO}, in equation (6).

It should be noted that parameters β and σ in the preferences play no role in the calibration of φ, and can be adjusted to fulfill equation (11). Therefore, no sensitive analysis is required, since all relevant parameter values to calibrate the output elasticity of public capital can be obtained from actual data.

Lastly, the calibration is also performed considering the country average of θ, α, $\tilde{Y}/\tilde{G}$, $\tilde{Y}/\tilde{K}$, $\tilde{I}^G/\tilde{Y}$ and $\tilde{I}^K/\tilde{Y}$. The results from this exercise appear in the bottom row of the tables.

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[4] Data from the PWT 9.0 were retrieved from http://www.rug.nl/ggdc/productivity/pwt/. For a complete description of the PWT 9.0, see Feenstra, Inklaar and Timmer (2015).

[5] As stated by Alonso-Carrera, Freire-Serén and Manzano (2009) for the case of Spain.

Table 1. Calibration of the Model for 14 EU Countries, 1980-2014.

	Calibration targets from actual data						Calibrated parameter values				
	θ (%)	α	$\tilde{G}/\tilde{Y}$	$\tilde{K}/\tilde{Y}$	$\tilde{I}^G/\tilde{Y}$ (%)	$\tilde{I}^K/\tilde{Y}$ (%)	δ_G	δ_K	φ	τ^{SO}	R^{SO} (%)
Austria	1.66710	0.38988	0.79973	2.2655	3.7264	21.804	0.029924	0.079573	0.072377	0.18564	6.0577
Belgium	1.42622	0.37033	0.58948	2.3995	3.2180	22.688	0.040328	0.080293	0.054123	0.14615	5.1488
Denmark	1.36817	0.35779	1.26015	1.7282	4.2380	18.232	0.019949	0.091815	0.098501	0.27531	5.8217
Finland	1.64641	0.38529	0.76634	2.3650	4.8224	21.383	0.046463	0.073953	0.078381	0.20344	5.5817
France	1.27309	0.36299	0.78015	1.9239	4.4482	18.493	0.044286	0.083395	0.083021	0.22871	6.2131
Germany	1.62237	0.35289	0.62739	1.8898	2.5906	17.205	0.025069	0.074816	0.064521	0.18284	7.7772
Greece	0.45291	0.47481	0.47813	1.3921	3.5524	13.682	0.069769	0.093758	0.112850	0.23768	16.6253
Ireland	3.35393	0.51100	0.57804	2.1923	4.2271	24.563	0.039590	0.078507	0.088821	0.17382	11.4070
Italy	0.96908	0.44345	0.59493	2.3188	3.3408	20.650	0.046463	0.079361	0.074968	0.16906	7.9548
Netherlands	1.51972	0.35679	0.76121	1.8560	3.8584	17.522	0.035490	0.079211	0.080171	0.22470	6.9831
Portugal	1.63642	0.36229	0.64704	2.0556	4.2251	19.985	0.048935	0.080858	0.071026	0.19605	6.0836
Spain	1.59850	0.36680	0.58273	1.9585	4.0929	21.592	0.054252	0.094265	0.066144	0.18032	5.9255
Sweden	1.61144	0.44826	0.86449	1.8414	4.4107	17.166	0.034906	0.077108	0.118382	0.26409	10.2033
United Kingdom	1.85618	0.38456	0.62183	1.7703	2.6698	17.123	0.024373	0.078163	0.075213	0.19558	9.6581
Country average	1.5715	0.39765	0.71083	1.9969	3.8158	19.435	0.037965	0.081610	0.078074	0.19634	7.8426

Source: IMF (2017), PWT 9.0 and author's calculations.

Note: The calibration targets from the data are annual averages for the period 1980-2014.

Table 2. Calibration of the Model for 14 EU Countries, 1980-1997.

	Calibration targets from actual data						Calibrated parameter values				
	θ (%)	α	$\tilde{G}/\tilde{Y}$	$\tilde{K}/\tilde{Y}$	$\tilde{I}^G/\tilde{Y}$ (%)	$\tilde{I}^K/\tilde{Y}$ (%)	δ_G	δ_K	φ	τ^{SO}	R^{SO} (%)
Austria	1.94174	0.36477	0.91797	2.4128	4.4053	21.779	0.028573	0.070847	0.072422	0.19854	5.0321
Belgium	1.75476	0.36378	0.64777	2.5396	3.7056	21.327	0.039657	0.066433	0.060113	0.16524	5.3142
Denmark	2.09334	0.35407	1.52871	1.6382	4.5850	16.007	0.009059	0.076777	0.117361	0.33147	6.7712
Finland	1.79905	0.35863	0.78750	2.7239	5.2298	22.003	0.048420	0.062789	0.071651	0.19979	4.2566
France	1.54162	0.34791	0.79414	2.0515	4.6158	17.921	0.042707	0.071938	0.080355	0.23097	5.8477
Germany	1.84342	0.32861	0.71828	2.0223	3.0551	17.276	0.024099	0.066992	0.063392	0.19291	6.4155
Greece	0.52320	0.47245	0.38805	1.3879	2.7744	13.211	0.066265	0.089961	0.096047	0.20330	18.1249
Ireland	4.23185	0.50685	0.66642	2.5410	4.2667	24.301	0.021705	0.053319	0.088622	0.17485	11.1276
Italy	1.90279	0.41608	0.57662	2.4533	3.2562	20.548	0.037443	0.064728	0.066444	0.15969	7.7787
Netherlands	1.82159	0.32026	0.84411	2.0262	3.6553	17.391	0.025088	0.067614	0.068844	0.21496	5.6470
Portugal	2.63143	0.35609	0.59203	2.1107	4.0703	19.492	0.042437	0.066035	0.067091	0.18841	7.0888
Spain	2.18586	0.35732	0.55550	1.9243	3.9528	19.381	0.049299	0.078856	0.067300	0.18835	7.1855
Sweden	1.39316	0.43732	0.94775	2.0561	4.6870	16.578	0.035522	0.066699	0.117755	0.26927	8.8725
United Kingdom	2.31699	0.39279	0.72624	1.9414	2.7032	18.273	0.014052	0.070954	0.076860	0.19568	9.1781
Country average	1.9986	0.38407	0.76365	2.1307	3.9259	18.964	0.031423	0.069017	0.082502	0.21481	7.2519

Source: IMF (2017), PWT 9.0 and author's calculations.

Note: The calibration targets from the data are annual averages for the period 1980-1997.

Table 3. Calibration of the Model for 14 EU Countries, 1998-2014.

	Calibration targets from actual data						Calibrated parameter values				
	θ (%)	α	$\tilde{G}/\tilde{Y}$	$\tilde{K}/\tilde{Y}$	$\tilde{I}^G/\tilde{Y}$ (%)	$\tilde{I}^K/\tilde{Y}$ (%)	δ_G	δ_K	φ	τ^{SO}	R^{SO} (%)
Austria	1.268294	0.41646	0.67454	2.1095	3.0075	21.830	0.031902	0.090803	0.070801	0.17001	7.3060
Belgium	1.053485	0.37726	0.52776	2.2511	2.7017	24.129	0.040656	0.096652	0.047709	0.12646	4.9742
Denmark	0.576967	0.36172	0.97579	1.8235	3.8706	20.588	0.033896	0.107133	0.079540	0.21989	4.7617
Finland	1.268396	0.41351	0.74393	1.9850	4.3909	20.727	0.046340	0.091735	0.088163	0.21321	7.2170
France	0.872767	0.37896	0.76534	1.7886	4.2707	19.099	0.047074	0.098050	0.086238	0.22757	6.5606
Germany	1.379749	0.37860	0.53115	1.7496	2.0988	17.130	0.025717	0.084113	0.064375	0.17004	9.5482
Greece	0.205598	0.47730	0.57352	1.3965	4.3762	14.181	0.074248	0.099487	0.128690	0.26962	15.0138
Ireland	2.200135	0.51539	0.48445	1.8230	4.1852	24.841	0.064390	0.114263	0.089117	0.17291	11.9565
Italy	-0.05728	0.47243	0.61433	2.1764	3.4303	20.757	0.056411	0.095945	0.085056	0.18004	8.2043
Netherlands	1.055911	0.39547	0.67344	1.6758	4.0733	17.661	0.049927	0.094833	0.091796	0.23212	8.6382
Portugal	0.421589	0.36885	0.70530	1.9972	4.3891	20.506	0.058015	0.098459	0.075181	0.20383	4.8580
Spain	0.836732	0.37685	0.61157	1.9946	4.2413	23.933	0.060983	0.111623	0.064731	0.17177	4.4860
Sweden	1.689718	0.45984	0.77633	1.6142	4.1181	17.789	0.036149	0.093308	0.119372	0.25959	11.7616
United Kingdom	1.307352	0.37584	0.51127	1.5891	2.6344	15.905	0.038454	0.087017	0.072702	0.19344	10.3746
Country average	1.0057	0.41203	0.65491	1.8553	3.6992	19.934	0.046427	0.097388	0.073390	0.17812	8.5142

Source: IMF (2017), PWT 9.0 and author's calculations.

Note: The calibration targets from the data are annual averages for the period 1998-2014.

Extreme figures regarding the calibration targets on the left hand side of the tables deserve to be commented upon, since they will affect the calibration results.

Actual data for the sub-period 1998-2014 reflect the effects of the downturn that followed the financial crisis of 2008. Italy even exhibited a negative average annual growth rate (-0.057%) during this period, and the country average (1%) is much lower than in Table 1 (1.57%) and Table 2 (1.99%). Considering the three periods, the Irish annual growth rate is considerably higher than the country average, while the opposite occurs for Greece. Labor and capital shares in income agree with the standard values computed in the literature (Gollin, 2002). Greece and Ireland present the highest capital shares in the three periods under analysis.

The ratio of public capital over GDP significantly varies among the countries. Denmark stands out in terms of a high ratio, which is well-above unity in Tables 1 and 2 and 0.97 in Table 3. The ratio of private capital over GDP ranges from 1.59 for the United Kingdom (Table 3) to 2.72 for Finland (Table 2), with the exception of the Greek economy, which presents the lowest ratio (around 1.4) in the three periods. The percentage of public investment over GDP in the tables takes quite low values, going from 2.1% for Germany (Table 3) to 5.2% for Finland (Table 2), which agrees with previous results in the literature (Kamps, 2006). Lastly, the country average of the percentage of private investment over GDP takes values around 20%. Greece constitutes an exception, with fairly low percentages ranging from 13.2% (Table 2) to 14.18% (Table 3). The Irish presents the highest percentage, which is above 24% in the three tables.

This look at the actual data shows a behavior of the Greek economy outside the trend in the sample of countries.

The calibration of the depreciation rates yields country averages ranging from 3.1% to 4.6% and from 6.9% to 9.7% for public and private capital, respectively. As expected, δ_G turns out to be lower than δ_K because the ratio of public investment over GDP is much lower than the ratio of private investment over GDP.

For the period 1980-2014, the calibrated output elasticity of public capital φ ranges from 0.0541 for Belgium to 0.1183 for Sweden, with a country average of 0.078. As previously mentioned, the split off of the time period 1980-2014 into two sub-periods allows checking the robustness of the results. The figures for the sub-period 1980-1997 yield a minimum value of 0.0601 (Belgium) and a maximum value of 0.1177 (Sweden), with a country average of 0.0825. Lastly, the calibrated elasticity in the sub-period 1998-2014 ranges between 0.0477 for Belgium and 0.1286 for Greece, being the country average of 0.0733. Thus, from these exercises we can infer an order of magnitude of the output elasticity of public capital between 0.05 and 0.13. At this point, it is pertinent to compare the results with econometric estimates of this elasticity. Creel and Poilon (2008) is the closest empirical study to the present analysis. These authors estimate a panel for 6 EU countries (Austria, Belgium, Germany, France, Italy and Netherlands) in the period 1969-2002, using the production function approach. This is the first study on the role of public capital in EU countries using a panel. The authors performed a well-crafted panel analysis to avoid methodological shortcomings that would yield inaccurate results. Importantly, stationarity of variables was controlled. An estimate of 0.14 was obtained for the entire period. However, since a change of regime was observed in the data, the estimation was performed in the sub-periods 1969-1985 and 1986-2002, which yielded elasticities of 0.1 and 0.05, respectively. These estimates are in line with the calibrated elasticities in Tables 1 through 3.

The values for the optimal tax rate τ^{SO} are higher than the optimal public investment rate x^{SO}, which means that the revenues from income tax exceed the required revenues for financing the optimal public investment. Thus, the government grants lump-sum subsidies to households, i.e. $\upsilon^{SO} = x^{SO} - \varphi/\alpha < 0$.

The implied net marginal return on capital accumulation shown in the last column of the tables significantly varies among the countries. Leaving Greece aside, the rate of return R^{SO} ranges from around 5% to around 12% in the three periods considered. Greece presents an overly high rate of return above 15%, which is due to the quite low ratio of private capital over GDP in this economy. The country average for this variable is 7.84%, 7.25% and 8.51% in Tables 1, 2 and 3, respectively. When the Greek rate of return is omitted, the country average turns out to be 7.29%, 6.96% and 7.74% in the respective tables. In the literature on economic growth, the rate of return on capital accumulation has been measured through the annual real rate of return on the stock market (Kongsamut, Rebelo and Xie, 2001). For instance, Siegel (2002) reports a long-run real rate of return of around 7% for the American economy. Our estimates are compatible with this figure.

6. Conclusion

The wide range of econometric estimates of the output elasticity of public capital, ranging from large negative to positive values, is certainly an obstacle for quantitative general equilibrium analysis and fiscal policy study.

The present paper has made a contribution to clarify the order of magnitude of this elasticity. Unlike the usual production function approach followed by econometric studies, this study has performed a calibration of the structural equations of a dynamic model with public capital accumulation. The construction of the model has been based on theoretical and empirical results on this topic. A sample of 14 EU countries spanning the 1980-2014 period has been used.

The calibrated values for the output elasticity of public capital range from 0.05 to 0.13. These figures agree with the panel estimates for 6 EU countries by Creel and Poilon (2008), which is the closest study to the analysis carried out here.

In light of the lack of consensus on the output elasticity of public capital in the econometric literature, calibration analysis seems to provide a new avenue for future research on this topic. Further exploration is certainly needed in this direction, involving, for instance, several production sectors, disaggregated public capital, capital adjustment costs or a strategy to allow for non-optimal solutions.

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