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Testing the Fisher Effect in the US

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Abstract

To apply Quantile Unit Root test and Quantile Cointegration test, this paper revisits the classical Fisher hypothesis. Due to the lower power of conventional unit root tests and Engle-Granger cointegration test, these two newly proposed econometric models shed similar light from different angles. The Quantile Cointegration test indicates that the real interest rate is stationary, which is in line with Fisher Effects. Besides, the empirical results also show asymmetric performance in the mean-reverting process. Likewise, the Quantile Cointegration test reports full Fisher Effects in the upper quantiles, and Fisher puzzles in the lower quantiles by using nominal interest rate and inflation rate. These findings have meaningful economic implications for the US monetary policy authorities. Specifically, the monetary policy authority should pay attention to these asymmetries when making monetary policies, especially avoiding the negative effects of tight monetary policy on mild inflation. Besides, under the condition of hyperinflation, the interest rate would play a one to one role in curbing the irrational inflation.

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1. Introduction

The Fisher hypothesis (Fisher, 1930) indicates the long-run consistent co-movement between the nominal interest rate and the expected rate of inflation. In line with the full Fisher Effects, the real interest rate will remain stable in response to the implementation of monetary policy, which means the monetary super-neutrality without illusion. Thus, testing whether the Fisher Effects hold on a nation significantly matters for the US monetary policy makers.

After combing the existing studies, we find two kinds of empirical methods to test the Fisher Effects. The first strand focuses on investigating the stationary property of real interest rate. Rose (1988) makes use of various unit root tests to investigate the stationarity of the real interest rate. However, Rose fails to believe the Fisher Effects. King et al. (1991) use ADF test to survey the stationarity of the real interest rate for the US. Gali (1992) revisits the Fisher Effects by employing the similar econometric model and reveals the real interest rate is non-stationary. Malliaropulos (2000) strongly supports Fisher Effects in both medium and long-run. Million (2003) points out that there is no stochastic trend for interest rate in the short-run of the US during the period from 1951 to 2000. Tsong and Lee (2012) point out that the lower power of conventional unit root tests would result to inaccurate empirical conclusions.

The second but primary way is cointegration test emphasizing on surveying the long-run relation between nominal interest rate and inflation rate. Mishkin and Simon (1995) believe there are only long-run Fisher Effects in Australia by employing Engle-Granger (EG) cointegration test. Wallace and Warner (1993) find Fisher Effects through cointegration test with considering term structure of interest rate. Koustas and Serletis (1999) reject the Fisher Effects in Belgium, Canada, Denmark, France, Germany, Greece, Ireland, Japan, the Netherlands, the United Kingdom and the United States. Bierens (2000) applies nonparametric nonlinear co-trending analysis to describe the nonlinear properties of the Fisher Effects. Million (2004) finds significant nonlinear mean-reversion properties for the US Treasury Bill market by a Threshold AutoRegressive (TAR) test. Tsong and Lee (2012) reveal the asymmetric performance between nominal interest rate and inflation rate, and Fisher Effects also vary with quantiles of the variables.

This paper sheds new light on this classical economic hypothesis for the US. To the best of our knowledge, this is the first paper aiming to analyzing the Fisher Effects by two newly proposed methods, such as testing the stationarity of real interest rate by Quantile Unit Root test (hereafter QUR) and Quantile Cointegration test (hereafter QC) between nominal interest rate and inflation rate. The empirical results from the two methods would, without doubt, provide more insightful evidence for this topic through different angles. Besides, in terms of the models employed, QUR could provide the effects of a given shock on the mean-reverting property of the real interest rate. Moreover, the test would indicate quantile-varying properties of Fisher Effects. Furthermore, unlike the Engle-Granger (hereafter EG) cointegration test (Engle, et al., 1987) making the coefficient fixed, the QC would make the coefficients to be varying at different quantiles. Besides, these two new tests based on quantile regression are more powerful than the conventional methods.

The rest of the paper is organized as follows. Section 2 introduces the theoretical model of Fisher equation. Section 3 presents the datasets and descriptive statistics. Section 4 introduces empirical method including QUR and QC. Section 5 presents the empirical results. The last section concludes the paper.

2. Theoretical Analysis

In line with Fisher hypothesis, we make NIR_t to represent nominal interest rate. Then, the Fisher equation could be expressed as,

$$RIR_t = NIR_t - \pi_t^e \qquad (1)$$

here, RIR_t is real interest rate, π_t^e is the expected inflation rate. Due to accessibility of the expected inflation rate, we use real inflation for substitution. Under the condition of rational expectations, the expected inflation rate could be expressed as follows,

$$\pi_t^e = \pi_t + e_t \qquad (2)$$

where, π_t is actual inflation rate which is easily accessible, e_t is a stationary series with zero mean. Thus, the equation (1) could be transformed as follows,

$$RIR_t = NIR_t - \pi_t - e_t \qquad (3)$$

Under the assumption of rational expectations, the Fisher Effects exist when the nominal interest rate and inflation rate are cointegrated with cointegration vector $(1,-1)'$. However, we usually practically test the following regression,

$$RIR_t = \alpha + \beta\pi_t + \varepsilon_t \qquad (4)$$

here, the full Fisher Effects hold when RIR_t and π_t are cointegrated with $\beta = 1$. If $\beta < 1$, Fisher (1930) indicates that it may be caused by monetary illusion. Besides, Mundell (1963) indicates that negative correlation between nominal interest rate and inflation rate will also make the coefficient less than one. Furthermore, another approach testing the Fisher Effects is to survey the stationarity of real interest rate in equation (3) by unit root tests.

3. Econometric Methodology

3.1 Quantile Unit Root Test (QUR)

We first consider a time-series y_t following AR(q) process with a constant term, and ε_t denotes the serially uncorrelated error term,

$$y_t = c + \sum_{i=1}^{p} \gamma_i y_{t-i} + \varepsilon_t \qquad (5)$$

Then, we rewrite the model as ADF type unit root test, which can be expressed as,

$$y_t = c + \alpha y_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta y_{t-j} + \varepsilon_t \qquad (6)$$

This paper focused on testing the stationarity of RIR which is calculated by $NIR_t - \pi_t$. To be noted, α represents the autoregressive coefficient and describes the persistence of y_t. Obviously, the series y_t is stationary if the coefficient α<1. Following Koenker and Xiao (2004), the ADF form could be rewritten at quantile τth as,

$$Q_{y_t}(\tau | \Omega_{t-1}) = c(\tau) + \alpha(\tau) y_{t-1} + \sum_{j=1}^{p-1} \phi_j(\tau) \Delta y_{t-j} \quad (7)$$

here, $Q_{y_t}(\tau | \Omega_{t-1})$ is τth quantile of ε_t conditional on the past information set Ω_{t-1}. $c(\tau)$ measures the series shock of y_t at different quantiles. Besides, $\alpha(\tau)$ captures the reversion speed of y_t given each quantile. In addition, we calculate half-lives of a shock as $Half-lives = \ln(0.5)/\ln(\alpha(\tau))$. Here, AIC information criteria is employed to select the optimal lag. To obtain the coefficient $c(\tau), \alpha(\tau)$ and $\sum_{j=1}^{p-1} \phi_j(\tau)$, we minimize the following equation,

$$\min \sum_{t=1}^{n} (\tau - I_t(y_t < c(\tau) + \alpha(\tau) y_{t-1} + \sum_{j=1}^{p-1} \phi_j(\tau) \Delta y_{t-j})) | y_t - c(\tau) - \alpha(\tau) y_{t-1} - \sum_{j=1}^{p-1} \phi_j(\tau) \Delta y_{t-j} | \quad (8)$$

Here, $I_t(\cdot) = 1$ if $y_t < Q_\tau(y_t | \Omega_{t-1})$, otherwise $I_t(\cdot) = 0$. Koenker and Xiao (2004) further propose t-statistic to test the null hypothesis $\alpha(\tau) = 1$, which could be expressed as,

$$t_n(\tau_i) = \frac{\hat{f}(F^{-1}(\tau_i))}{\sqrt{\tau_i(1-\tau_i)}} (y'_{-1} P_{(1, \Delta y_{t-1}, \ldots, \Delta y_{t-j})} y_{-1})^{1/2} (\hat{\alpha}(\tau) - 1) \quad (9)$$

where, $f(\cdot)$ is probability functions of y_t, and $F(\cdot)$ is cumulative density function of series y_t. y_{-1} is the vector of lagged series y_t and P_X is the projection matrix onto the space orthogonal to $(1, \Delta y_{t-1}, \cdots, \Delta y_{t-j})$. $\hat{f}(F^{-1}(\tau_i))$ is estimated by consistency theory indicated in Koenker and Xiao (2004), as follows,

$$\hat{f}(F^{-1}(\tau_i)) = \frac{(\tau_i - \tau_{i-1})}{G'(\Phi(\tau_i) - \Phi(\tau_{i-1}))} \quad (10)$$

To be noted, $G(\cdot) = (1, \Delta y_{t-1}, \cdots, \Delta y_{t-j})$, $\Phi(\tau_i) = c(\tau_i), \alpha(\tau_i), \phi_1(\tau_i), \cdots, \phi_{p-1}(\tau_i)$ and $\tau \in [\underline{\lambda}, \overline{\lambda}]$. We set $\underline{\lambda} = 0.1$ and $\overline{\lambda} = 0.9$. Obviously, we could test the unit root hypothesis at different quantiles in comparison with traditional ADF test only emphasizing on the conditional central tendency.

To assess the performance of QUR test, Koenker and Xiao (2004) suggested a Kolmogorov-Smirnov (QKS) test which could be presented as follows,[1]

$$QKS = \sup_{\tau_i \in [\underline{\lambda}, \overline{\lambda}]} | t_n(\tau) | \quad (11)$$

In this paper, we select the maximum of $t_n(\tau)$ to build the QKS statistics over the quantiles $\tau_i \in (0.1, 0.2, \cdots, 0.9)'$. Although the limiting distributions of both $t_n(\tau)$ and QKS tests are non-standard, Koenker and Xiao (2004) propose re-sampling procedures to derive critical values.[2] In this paper, we make the bootstrap iterations to 5000 times to generate critical values.

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[1] See Koenker and Xiao (2004) for detailed information about QKS statistic.

[2] See Koenker and Xiao (2004) for detailed information about Bootstrap procedure.

3.2 Quantile Cointegration Test

We consider a traditional cointegration model,

$$y_t = \alpha + \beta_t x_t + e_t \qquad (12)$$

where, both variables y_t and x_t are integrated as I(1) process, and e_t is a stationary time series. Then following Xiao (2009), we decompose the lead and lag terms $\sum_{j=-m}^{m} \gamma_{jt} \Delta x_{t-j}$ of the regressor and a pure innovation term ε_t to solve the endogeneity problem, which can be expressed as,

$$y_t = \alpha + \beta_t x_t + \sum_{j=-m}^{m} \gamma_{jt} \Delta x_{t-j} + \varepsilon_t, t = 1, 2, ..., n, \qquad (13)$$

here, $y_t = NIR_t - \overline{NIR}$, $x_t = \pi_t - \overline{\pi}$, $\overline{NIR_t}$ and $\overline{\pi}$ represent sample mean of nominal interest rate and inflation rate, respectively. Here, β_t and $\gamma_{jt}(j = -m \sim m)$ depends on t. Next, the τ_{th} quantile of y_t based on information set Ω_t up to t, which can be written as,

$$Q_{y_t}(\tau \mid \Omega_{t-1}) = \alpha(\tau) + \beta(\tau) x_t + \sum_{j=-m}^{m} \gamma_j(\tau) \Delta x_{t-i} \qquad (14)$$

where, $\alpha(\tau)$ is the nominal interest rate shock at quantile τ, and $\beta(\tau)$ represents the cointegration coefficient which is used to test the long-run relation between nominal interest rate and inflation rate. Besides, $\beta(\tau)$ measures the mean-reverted properties in response to the shock.

Then, we could estimate the parameters in equation (14) by minimizing the following equation,

$$\min \sum_{t=1}^{n} (\tau - I(y_t < \alpha(\tau) + \beta(\tau) x_t + \sum_{j=-m}^{m} \gamma_j(\tau) \Delta x_{t-i})) \mid y_t - \alpha(\tau) - \beta(\tau) x_t - \sum_{j=-m}^{m} \gamma_j(\tau) \Delta x_{t-i} \mid \qquad (15)$$

here, $I(\cdot)$ is an indicator function, when $y_t < \alpha(\tau) + \beta(\tau) x_t + \sum_{j=-m}^{m} \gamma_j(\tau) \Delta x_{t-i}$, $I(\cdot) = 1$, otherwise, $I(\cdot) = 0$. Then, we could test whether Fisher Effects hold by testing the null hypothesis: $H_0 : \beta(\tau) = 1$. Besides, the QC could provide more time-varying results of Fisher Effects over the quantiles. For instance, the null hypothesis $H_0 : \beta(\tau) = \hat{\beta}$ over the quantiles ($\tau = 0.1, 0.2, ..., 0.9$) can be tested based on the statistic of $\sup \mid \hat{V}_n(\tau) \mid$, which could be expressed as,[3]

$$\sup \mid \hat{V}_n(\tau) \mid \Rightarrow \sup \mid \frac{1}{f_\varepsilon(F_\varepsilon^{-1}(\tau)} [\int_0^1 \underline{B}_{\Delta x} \underline{B}_\psi]^{-1} \int_0^1 \underline{B}_{\Delta x} d(\underline{B}_\psi^* - f_\varepsilon(F_\varepsilon^{-1}(\tau))) B_\varepsilon^* \mid \qquad (16)$$

where $\hat{\beta}$ is estimated by OLS method in (13), and $\hat{V}_n(\tau) = n(\hat{\beta}(\tau) - \hat{\beta})$; $\Rightarrow$

[3] See Xiao (2009) for detailed information about $\sup \mid \hat{V}_n(\tau) \mid$ statistic.

indicates weak convergence; $f(\cdot)$ and $F(\cdot)$ are respectively the p.d.f and c.d.f of the error term e_t in equation (12); $\psi_\tau(u)=\tau-I(e<0)$; $\underline{B}$ is the demeaned Brownian motion; $\underline{B}_{\Delta x}$ and $\underline{B}_\psi$ are both Brownian motion, but independent. We make use of the maximum of $\hat{V}_n(\tau)$ over the quantiles ($\tau=0.1,0.2,...,0.9$) to construct the statistic. Xiao (2009) proposes a bootstrap method to accurate the critical values due to the test's asymptotic distribution and several nuisance parameters.[4] In this study, we run the bootstrap loop with 5000 iterations.

4. Data Selection and Descriptive Statistics

Monthly data is retrieved from Fred Economic Data of US Federal Reserve Bank of St. Louis covering the period from 1960:01 to 2017:02.[5] We select the 3-month treasury bill as a proxy of nominal interest rate (hereafter NIR for simplification purpose). Besides, the percentage change of CPI from year ago is used to represent inflation rate (hereafter π). The real interest rate (hereafter RIR) calculated by the equation $RIR=NIR-\pi$. The datasets used are plotted in Figure 1.

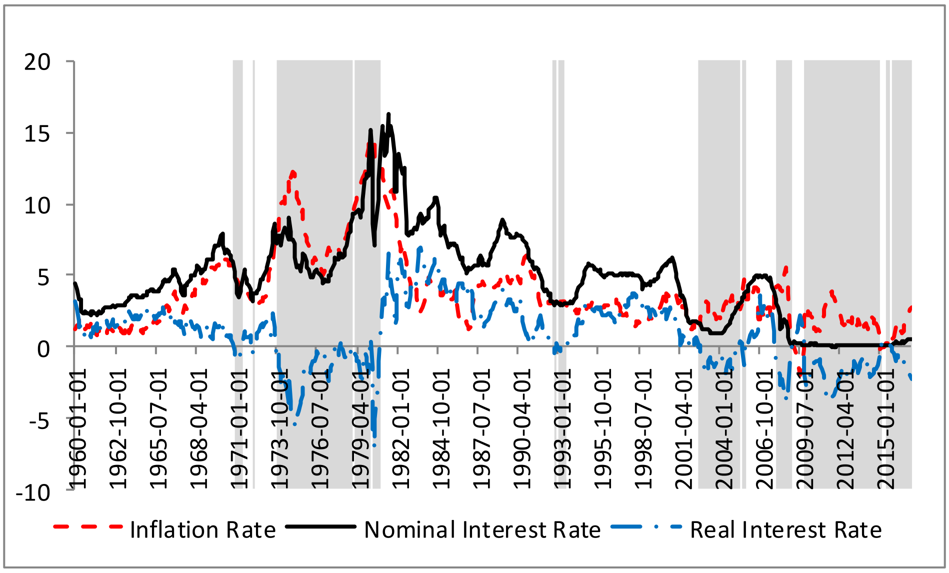

Figure 1 Data Figure

Shaded area denotes the period when the real interest rate is negative. The first long period with negative RIR starts from 1973:10 to 1980:10, which lasts for 85 months. The second period starts from 2009:11 to 2017:02. After the 2007 subprime crisis, the US Fed has implemented multi-rounded Quantitative Easing (QE) monetary policy to stabilize the macroeconomic fluctuations. The NIR continuously goes down, and the RIR keeps negative during this period.

Table 1. Description Statistics

Series	NIR	π	RIR
Mean	3.8033	4.6752	0.8718

[4] See Xiao (2009) for detailed information about Bootstrap method.

[5] The website is http://research.stlouisUS Fed.org/fred2/.

Median	3.0669	4.7650	1.0301
Maximum	14.5923	16.3000	6.8835
Minimum	-1.9588	0.0100	-7.1959
Std. Dev.	2.8746	3.1656	2.2157
Skewness	1.5236	0.6731	-0.1869
Kurtosis	5.3472	3.8565	3.1485
Jarque-Bera	422.8872**	72.7670**	4.6264
Probability	0.0000	0.0000	0.0989
Observation	686	686	686

Note: ** and * indicate 5% and 10% significant level.

Table 1 reports descriptive statistics of NIR and π. Obviously, π shows more fluctuations in comparison with NIR and RIR. The standard deviation of NIR is 2.8746. However, the RIR performs more stable with standard deviation 2.2157. Besides, the skewness of RIR is -0.1869 and the Kurtosis is over 3.1485. For Jarque-Bera statistics, NIR and π reject the null hypothesis at 5% significant level and real interest rate rejects the null at 10% significant level. The non-normality of the variables supports the viewpoint that using the quantile method is reasonable (Xiao, 2009).

5. Empirical Results and Economic Implications

5.1 Empirical Results from QUR test

In line with Mishkin and Simon (1995), testing the unit root for RIR calculated by $NIR_t - \pi_t$ is equivalently to investigate the full Fisher Effects in the long-run. In other words, if there is cointegration between NIR and π, the RIR is stationary. For comparative reasons, we first implement traditional unit root test, including ADF test (Dickey and Fuller, 1981), PP test (Phillips and Perron, 1988) and KPSS test (Kwiatkowski et al., 1992).

Table 2 Conventional Unit Root Test for Real Interest Rate

ADF Unit Root Test Results

series	Level		First Differences	
	Constant	Constant with Trend	Constant	Constant with Trend
RIR	-2.5942(13)	-2.7946(13)	-8.7704(12)**	-8.7716(12)**

PP Unit Root Test Results

series	Level		First Differences	
	Constant	Constant with Trend	Constant	Constant with Trend
RIR	-3.4467(8)**	-3.6544(7)	-19.6656(17)**	-19.6474(17)**

KPSS Unit Root Test Results

series	Level		First Differences	
	Constant	Constant with Trend	Constant	Constant with Trend
RIR	0.4570(21)	0.2778(21)**	0.0294(12)	0.0269(12)

Note: ** denotes significance at5% level. The number in parenthesis indicates the lag order selected based on the recursive t-statistic, as suggested by Perron (1989). The number in the brackets indicates the truncation for the Bartlett Kernel, as suggested by the Newey-West test.

Obviously, ADF test for RIR says non-stationary conclusion with and without considering trend. PP test indicates the RIR is stable at 5% significant level with constant. Besides, the KPSS test presents that the series is not stable at 5% significant level with constant, but significantly stationary including both constant and trend term. Lastly, after taking first difference, the tests hold stationary conclusion. Although the series are at least I(1) process, Ng and Perron (2001) point out that these tests suffer from severe size distortions which may result to the rejection of the unit root hypothesis in favor of stationarity.

Then, we implement QUR test to re-examine the stationarity of RIR to directly reveal the Fisher Effects, which could provide more accurate and insightful economic implications.

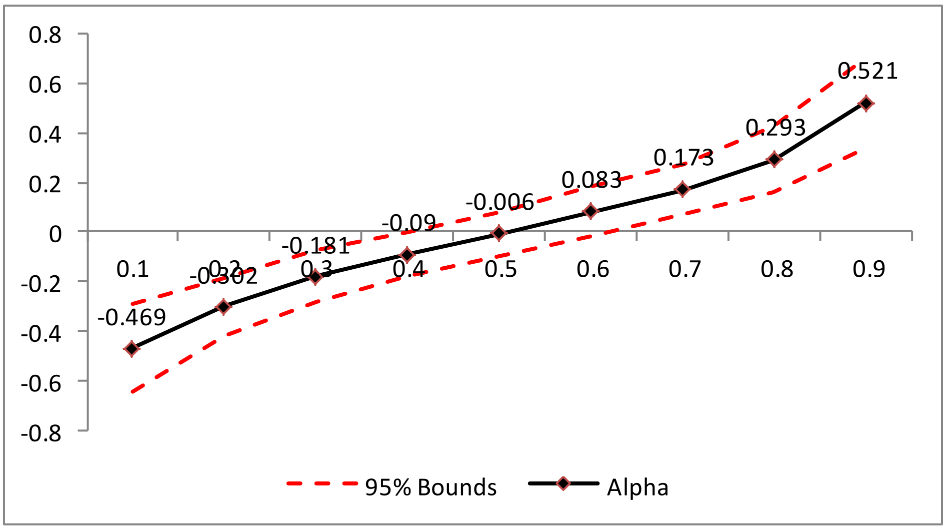

Figure 2 Move Path of $\alpha(\tau)$

Table 3 Quantile Unit Root Test for real interest rate.

τ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\alpha(\tau)$	-0.4690**	-0.3020**	-0.1810**	-0.0900**	-0.0060	0.0830**	0.1730**	0.2930**	0.5210**
$\beta(\tau)$	0.9850	0.9850	0.9800**	0.9820**	0.9770**	0.9720**	0.9740**	0.9660**	0.9520**
$t_n(\tau)$	-0.6880	-1.3210	-2.0500	-2.1990	-2.7560**	-3.1940**	-2.5930**	-2.5200**	-2.6390**
Half-lives	45.8624	45.8624	34.3096	38.1606	29.7889	24.4070	26.3114	20.0381	14.0912
QKS statistic	3.194**	CV1:3.660	CV5:3.146	CV10:2.925					

Notes: **denotes significance at 5% level. CV10, CV5, and CV1 are the critical values of statistical significance at 10%, 5%, and 1%, respectively. Numbers in parenthesis denote bootstrap p-values with the bootstrap replications set to be 5000. For $\alpha(\tau)$, the unit-root null is examined with the $t_n(\tau)$ statistic. The lag length q is selected based on robust Schwarz information criterion as suggested by Galvao (2009) with a maximum lag set to be 12.

Obviously, the QKS statistic indicates the RIR series is stationary, which is 3.1940 significant at 5% level. Thus, we conclude full Fisher Effects between NIR and π. For a further step, the QUR test could provide more quantile-varying views on mean-reverted properties and lasting period of a shock.

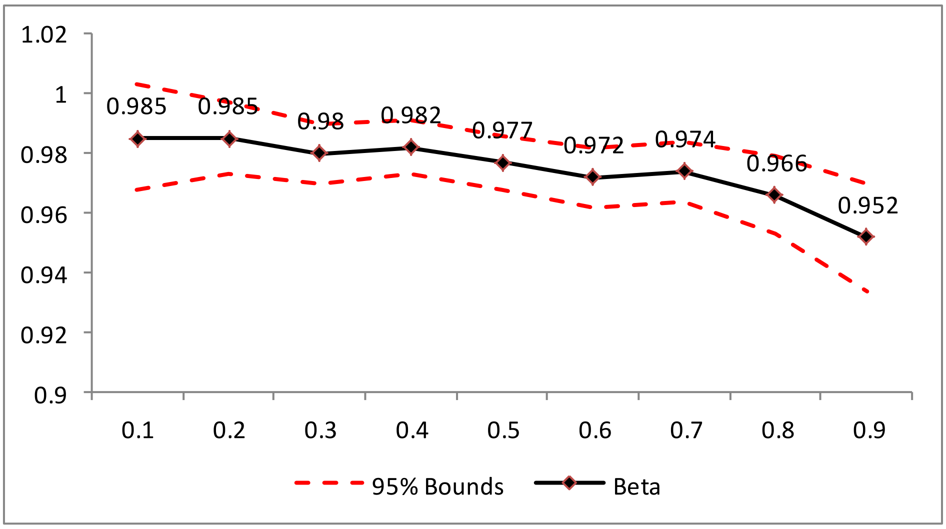

Figure 3 Move Path of $\beta(\tau)$

Figure 2 and 3 present the path of $\alpha(\tau)$ and $\beta(\tau)$. Some interesting findings could be summarized. First, the shocks over the quantiles from 0.1 to 0.5 are negative, but turn to be positive at upper quantiles. The path of $\beta(\tau)$ is downward with the peak value 0.9850 at lower quantiles 0.1 and 0.2. However, $\beta(\tau)$ is more likely to be significant at upper quantiles. Besides, the mean-reverting speeds represented by $\beta(\tau)$ are also varied over the quantiles ($\tau = 0.1, 0.2, \ldots, 0.9$). The finding indicates that the asymmetric effects exist in the full Fisher Effects in the short-run. With fully considering the path of $\alpha(\tau)$ and $\beta(\tau)$, we conclude that the negative shocks make permanent impacts on the real interest rate. Secondly, the half-lives are over 20 months over the quantiles from 0.1 to 0.8. Thus, we could find asymmetries in the full Fisher Effects for the US.

5.2 Empirical Results from Quantile Cointegration test

Table 4 reports the stationarity of NIR and π, respectively. According to results of ADF, PP and KPSS test, the NIR and π perform unit-root behavior since the ADF and PP test fail to reject the null hypothesis of non-stationarity, but KPSS test rejects the stationary null hypothesis. In other words, the two series employed are all I(1) process, which satisfies the requirements of cointegration test.

Table 4 Conventional Unit Root Test for Nominal Interest Rate, Inflation Rate

ADF unit root test results		
series	Level	First Differences

	Constant	Constant with Trend	Constant	Constant with Trend
NIR	-2.1149(13)	-2.8762(13)	-6.4620(12)**	-6.5147(12)**
π	-2.6057(1)	-2.8642(1)	-17.8013(0)**	-17.7937(0)**

PP unit root test results

series	Level		First Differences	
	Constant	Constant with Trend	Constant	Constant with Trend
NIR	-1.8602(13)	-2.4113(14)	-17.6574(24)**	-17.5988(25)**
π	-2.6766(11)	-2.9422(11)	-17.6676 (4)**	-17.6594(4)**

KPSS unit root test results

series	Level		First Differences	
	Constant	Constant with Trend	Constant	Constant with Trend
NIR	1.2046(21)**	0.5289(21)**	0.0775(15)	0.0298(15)
π	0.7476(21)**	0.3353(21)**	0.0561(11)	0.0353(11)

Note: ** denotes significance at 5% level. The number in parenthesis indicates the lag order selected based on the recursive t-statistic, as suggested by Perron (1989). The number in the brackets indicates the truncation for the Bartlett Kernel, as suggested by the Newey-West test (1987).

Due to the non-stationary conclusion obtained from unit root test, we make use of EG cointegration test to reveal the long-run relation between NIR and π, the empirical results are presented in Table 5.

Table 5 Engle-Granger two-step cointegration test results

	ADF	Coefficient (p-value)
The U.S.	-2.3144	0.8088 (0.0000)

Notes: The 5% critical values for the ADF test are -3.37, retrieved from Table B.9 in Hamilton (1994). ** indicates significance at the 5% level.

The nominal interest rate and the inflation rate are not cointegrated. The coefficient is less than one with significance, we believe the Fisher Effect puzzle in the US. The reason of the Fisher Effect puzzle is mainly caused by lower testing efficiency of EG cointegration test which does not allow cointegrating vector changing over time. To obtain more accurate empirical results, we employ a quantile regression based cointegration test to describe the long-run relation in different business cycles, the long-run relation between the nominal interest rate and the inflation rate may vary in different quantiles. The varied macroeconomic conditions could be indicated by nominal interest rate and inflation rate in different quantiles.

Table 6 presents the empirical results from QC test with considering the magnitudes of shocks and long-term cointegrated coefficients over quantiles ($\tau = 0.1, 0.2, \ldots, 0.9$). Then, the value of $Sup|V_n|$ and p-value are used to investigate the long-run cointegration between NIR and π. In contrast to empirical results from EG cointegration test, the $Sup|V_n|$ statistic in QC test rejects the null hypothesis of quantile cointegration at 1% significant level, which strongly supports long-run cointegration between NIR and π. The empirical results indicate the Fisher Effects hold in the US.

The shock $\alpha(\tau)$ related to NIR shows quantile-varying path over the quantiles ($\tau = 0.1, 0.2, \ldots, 0.9$). In brief, shocks are negative in the lower quantiles $\tau = 0.1, \ldots, 0.4$, turn to positive in the upper quantiles $\tau = 0.5, \ldots, 0.9$. The magnitude of the shocks $\alpha(\tau)$ ranges from -2.8169 to 2.5178. Besides, the cointegrating coefficients are also quantile-varying.

Figure 4 presents the path of cointgrating coefficients. Obviously, the coefficients are not fixed, which contrasts to the results from EG method. We find the coefficients of quantile cointegration test wave around the coefficient obtained from the EG cointegration test over the quantiles $\tau = 0.1, \ldots, 0.6$. In general, the coefficient turns bigger in the upper quantile $\tau = 0.7$, and at 90% quantile it can be up to 1.1014 significant at 5% level. This finding holds Fisher Effects exist in the US economy in the upper quantile. But over the lower quantiles, the US economy reveals Fisher puzzle.

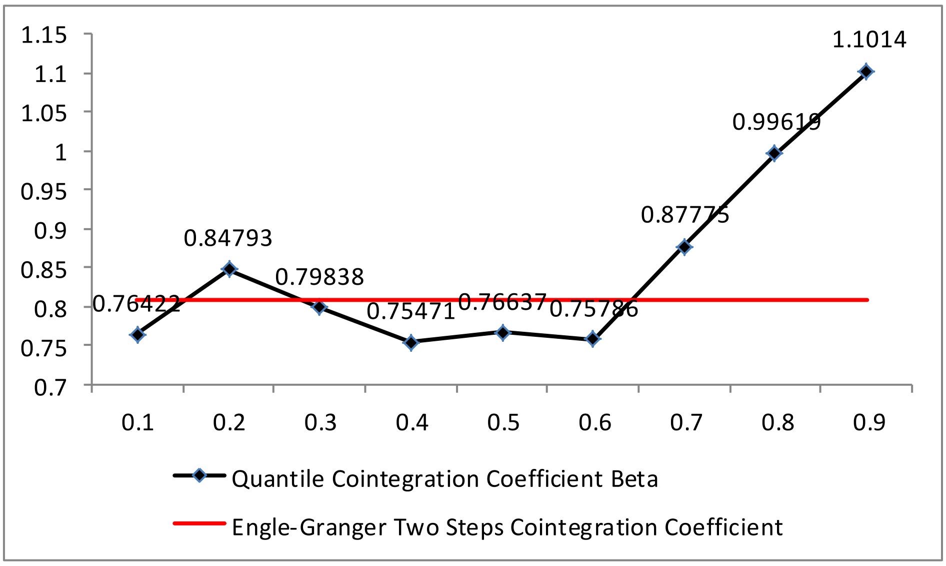

Figure 4 Move Path of Cointegration Coefficient

Table 6 Quantile cointegration results for nominal interest rates and inflation rate.

τ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\alpha(\tau)$	-2.8169**	-2.0249 **	-1.2459**	-0.4444**	0.1496	0.6376**	1.1948**	1.7770**	2.5178**
$\beta(\tau)$	0.7642 **	0.8479 **	0.7984 **	0.7547 **	0.7664**	0.7579**	0.8778**	0.9962**	1.1014**
Half-lives	2.5777	4.2020	3.0783	2.4630	2.6049	2.5000	5.3158	N.A.	N.A.
$\sup\lvert V_n\rvert = 174.7440$ **	CV1: 31.9353	CV5:21.2534	CV10:16.7685						

Notes: ** denotes significance at 5% level. CV10, CV5, and CV1 are the critical values of statistical significance at 10%, 5%, and 1%, respectively. The bootstrap replications set to be 5000. Notes: $Sup\lvert V_n\rvert$ is used to test the quantile cointegration between nominal interest rate and inflation rate.

These asymmetric empirical results from QC test are interesting and have not been mentioned in the EG method. These findings have important economic implications. The US Fed has implemented various monetary policies to control the macroeconomy with the most powerful tool, the interest rate. The monetary authority makes different monetary strategies with considering the performance of the inflation which is one of the prime goals for the US Fed. When the inflation rate is in the extremely high quantiles meaning hyperinflation, the central bank of the US always implements the aggressive tight monetary policy, increasing the interest rate, to bring down the inflation. However, when the inflation is mild, the US Fed may slightly change the interest rate in case of negative impacts of the monetary policy on the economic growth. To sum up, we could find that the Fisher Effects holds on quantile varyingly in different quantiles.

6. Conclusion

This paper revisits the classical economic theory, Fisher hypothesis, by two different methods, including Quantile Unit Root test and Quantile Cointegration test. Although these two methods test on different variables, they obtain similar empirical results. Unlike the lower power of conventional methods, such as ADF test, PP test, KPSS test and Engle-Granger Cointegration test performed in the existing literatures, we find asymmetric effects existing in Fisher hypothesis for the US. From the perspective of the results from Quantile Unit Root test, we believe the real interest rate is stationary, which is in line with full Fisher Effects. Besides, the mean-reverting process is always asymmetric, and the negative shock associated to real interested rate permanently affects the move path. Although the conventional EG method indicates no Fisher Effects, the more powerful Quantile Cointegration test applauds the full Fisher Effects in the upper quantile, but reveals Fisher puzzle in lower quantiles. These findings have significant economic implications for the US monetary authorities.

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