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On a trade-off in the evolution of ownership

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Abstract

This paper shows that the emergence of ownership faces a trade-off between interacting group size in a population and the fighting cost for resources in a Hawk-Dove-Bourgeois game played by entities in the population. More precisely, for each fighting cost, large size of the group induces neutral stability of the Bourgeois whereas small size ensures evolutionary stability of the Hawk. Each player's adaption process is supposed as follows : At each period, a sub-group of the population is randomly drawn, and the Hawk-Dove-Bourgeois game is played in the group. Each player in a group observes the payoff distribution of the members in the group and switches own strategy to one with higher payoff.

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1. Introduction

Since Smith(1982) introduced the Hawk-Dove-Bourgeois game, there has been a lot of research regarding the evolution of property of rights among animal species and human beings. It is well known that this game has a stable state where in all players choose Bourgeois. This paper provides a new perspective for this result. It shows that under the evolutionary dynamics with a local matching structure (Hible, 2011), there is a trade-off between interacting group size and the cost of establishing property of rights in a society.

2. The model

2.1 Adaptation dynamics

We consider a population consisting of infinitely many players. Time is continuous $t \in \mathbf{R}_+$. At each point in time t, each player plays a game with the following rules. Each player randomly becomes a member of a sub-group that has a population of $n > 2$ players, and each of the n players in this group is matched randomly with another player in the same group to play the game.

We denote the set of players of the sub-group by $N = \{1, 2, ..., n\}$. We suppose that each player has a common set of strategies $S = \{s_1, ..., s_k\}$. When a player chooses $s_i \in S$ and the opponent player is randomly matched with the player choosing $s_j \in S$, the former player gets a payoff of $a_{ij} \in \mathbf{R}_+$. Let $A = (a_{ij})_{i,j=1,...,k}$ be the payoff matrix such that the (i, j) element of A is a_{ij}. Let $\Delta = \{(x_{s_1}, ..., x_{s_k}) | \forall i = 1, ..., k, x_i \geq 0, \sum_{i=1}^{k} x_i = 1\}$ denote the set of mixed strategies over S, and each $x \in \Delta$ indicates the state of the population with its distribution of strategies.

Noting that each sub-group is randomly constructed, the expected payoff of each player choosing $s_i \in S$ at a state $x \in \Delta$ is given by $u_i(e_i, x) = e_i \cdot Ax$, where e_i is a unit vector of which the i th component is 1, and $u(x, x) = x \cdot Ax$ is the average payoff of x.

After playing the game, the number of players s_i increases if and only if the payoff of choosing s_i is higher than the average payoff. Under this adaptation process, the dynamics of the whole population is given by the flowing equation, and it is called **local replicator dynamics** (Hible, 2011).

Definition 1. *The local replicator dynamics is given by*

$$\dot{x}_i = e_i \cdot \widetilde{A}x - x \cdot \widetilde{A}x, \tag{1}$$

where the dot represents the time derivative dx_i/dt, and $\widetilde{A} = A - (A + A^t)/n$.

As $N \to \infty$, the local replicator dynamics approaches the corresponding replicator dynamics : $\dot{x}_i = e_i \cdot Ax - x \cdot Ax$ (Taylor and Jonker, 1979)

2.2 The Hawk-Dove-Bourgeois game

The standard Hawk Dove game is given by a payoff matrix

$$\begin{array}{c c} & \begin{array}{cc} H & D \end{array} \\ \begin{array}{c} H \\ D \end{array} & \begin{pmatrix} \frac{v-c}{2} & v \\ 0 & \frac{v}{2} \end{pmatrix} \end{array},$$

where $v > 0$ is the value of a resource, and $c > v$ indicates the cost for fighting over the resource. Moreover, we suppose that there are two cases for this game. In the first case, the player might get the value of the resource before the other player is able to find the resource. In second case, the other player might occupy the value of the resource before the player is able to find the resource. Since the population is infinite, we suppose that each case occurs with an equal probability of 1/2. The following strategy is called **Bourgeois**. If a player adopts the Bourgeois strategy, then the player chooses H in the first case and D in the second. The payoff matrix of the Hawk Dove game with the Bourgeois strategy can be written as

$$\Pi = \begin{array}{c c} & \begin{array}{ccc} H & D & B \end{array} \\ \begin{array}{c} H \\ D \\ B \end{array} & \begin{pmatrix} \frac{v-c}{2} & v & \frac{3v-c}{4} \\ 0 & \frac{v}{2} & \frac{v}{4} \\ \frac{v-c}{4} & \frac{3v}{4} & \frac{v}{2} \end{pmatrix} \end{array}.$$

The game with payoff matrix Π is called the **Hawk-Dove-Bourgeois game**. Replacing A in Definition1 with Π, the payoff matrix $\widetilde{A}$ is

$$\begin{pmatrix} \frac{v-c}{2}(1-\frac{2}{n}) & v(1-\frac{1}{n}) & \frac{3v-c}{4}-\frac{1}{n}(v-\frac{c}{2}) \\ -\frac{v}{n} & \frac{v}{2}(1-\frac{2}{n}) & \frac{v}{4}(1-\frac{4}{n}) \\ \frac{v-c}{4}-\frac{1}{n}(v-\frac{c}{2}) & \frac{3v}{4}(1-\frac{4}{3n}) & \frac{v}{2}(1-\frac{2}{n}) \end{pmatrix}.$$

Figure1 depicts the trajectory of the local replicator dynamics over Δ.[1] In this figure, the black nodes "●" indicate stable rest points and the white nodes "○" indicate unstable ones. From this figure, we guess that there would be a *Threshold*. If the number of subgroups is less than n^*, strategy H would be stable. Later in this paper we look at the case of $n^* = 10$.

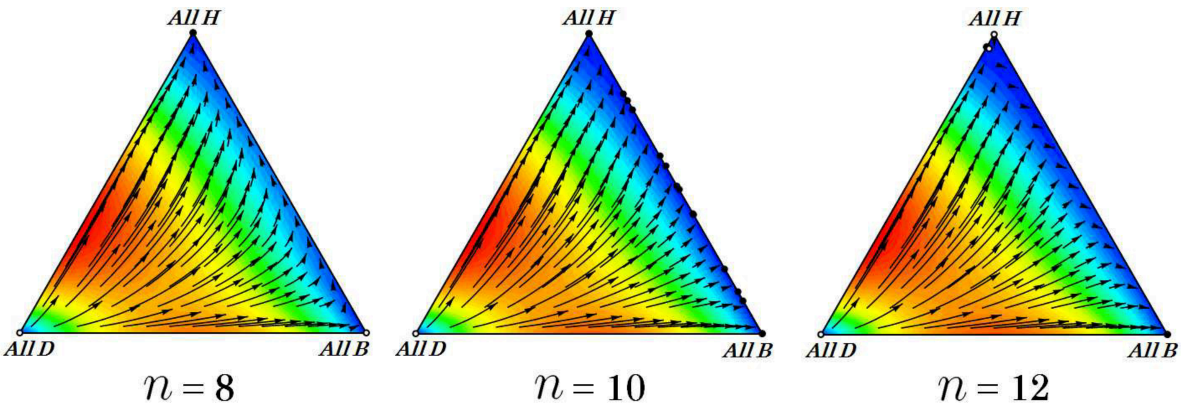

Figure 1: Phase diagram of the $\widetilde{A}$ game for parameters $c = 5$ and $v = 4$.

To check the stability of the population state, we use the concept of **Neutral stable strategy**(NSS ; Bomze and Weibull,1995)

Definition 2. *A strategy $x^* \in \Delta$ is NSS if*
(i) $u(x^*, x^*) \geq u(x, x^*), \forall x \in \Delta, x \neq x^*$,
(ii) $\exists y \in \Delta s.t. u(x^*, x^*) = u(y, x^*) \Rightarrow u(x^*, y) \geq u(y, y)$.

If condition (ii) satisfies the requirements of strict inequality, then x^* is the **Evolutionarily stable strategy** (ESS).

[1]This software package is available at https://www.ssc.wisc.edu/ whs/dynamo/.

Proposition 1. *If the sub-group size n is beyond the threshold, H is replaced by strategy B as a stable strategy in the population.*
(i) *If $n < 2c/(c-v)$, then H is ESS*
(ii) *If $n = 2c/(c-v)$, then H, B, and each strategy of $\omega \in X = \{(x_H, 0, 1-x_H) | x_H \in (0,1)\}$ are NSS.*
(iii) *If $n > 2c/(c-v)$, then B is NSS.*

Proof. (i) In this case, H dominates all the other strategies in $\widetilde{A}$. (ii) Since for any $\omega \in X$, e_H, and e_B, $u(\omega, e_H) = u(e_H, e_H)$ and $u(\omega, e_B) = u(e_B, e_B)$, $\omega \in X$ is NSS. We can easily check the neutral stability of H and B in the same way as above. (iii) From $\widetilde{A}$, Nash strategies satisfying (i) of Definition2 are $\omega' = [\frac{vn}{c(n-2)}, \frac{n(c-v)-2c}{c(n-2)}, 0]$ and B. Our remaining task is to check the stability condition of Definition2 for these strategies. It is easy to verify that $u(e_B, \omega') = u(\omega', \omega')$, and so B is NSS. However, ω' is unstable, because $u(\omega', e_B) \geq u(e_B, e_B)$ is

$$\frac{v}{4c(n-2)}[4(1-v) - 4c(1-\frac{2}{n}) - n(1-3v)] \geq \frac{v}{2}(1-\frac{2}{n}).$$

This inequity rearranges to

$$2 < n \leq -(1-\frac{v}{c}) + \sqrt{(1-\frac{v}{c})^2 + 2} \iff \frac{3}{2} < \frac{v}{c},$$

which contradicts $\frac{v}{c} < 1$. □

In case (iii) of Proposition1, B is only NSS but not ESS because Bourgeois can not distinguish the difference between mixed strategies over H and D and itself.

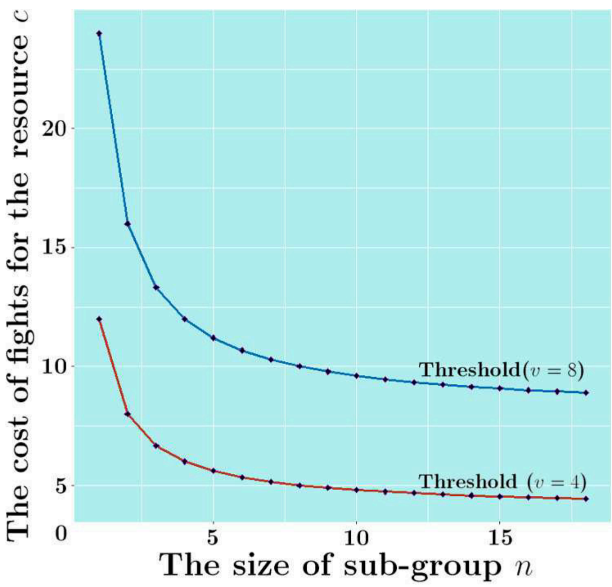

Figure 2: Bourgeois strategy becomes more successful as the cost for fighting over the resource increases.

3. Discussion

We have confirmed that, if size n of subgroup is beyond a threshold, then the Bourgeois strategy becomes neutral stable, whereas the Hawk is an ESS when n is below the threshold. Figure2 displays the area of (n, c), where B is NSS for each value of resource

$v = 4, 8$. In Figure2 we connect the threshold points (n, c) for each v and call this the **threshold curve**. B is always NSS at the north east area from each point (n, c) on each threshold curve.

In summary, we find there is a trade-off between the interacting group size of n and the cost of fighting for getting the resource to establish a property of rights in a society. Moreover, since each threshold curve has a down ward slope, we see that strategy B becomes more successful and H more disadvantageous as c increases.

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