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Environmental efficiency of price and quantity policies

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Abstract

Under the set-up of Weitzman (1974), we examine the relative environmental efficiency of price (i.e., emission tax) and quantity (i.e., emission cap) policies given the expected net benefits of emissions no less than some fixed value. We discover that quantity policies are more environmentally efficient than price policies regardless of relative steepness of emissions' marginal benefit and marginal cost functions. However, if the constraint of minimum expected net benefits of emissions is not considered, both price and quantity policies are equally efficient.

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1. Introduction

How to select sustainable and effective policies in uncertain environments to reduce pollutant emissions has been a major concern of all countries. In his pioneer work, Weitzman (1974) shows that price policies (i.e., emission tax) are preferred to quantity policies (i.e., emission cap) when emission's marginal cost function is steeper than its marginal benefit function, while quantity policies are preferred in the converse situation. Nevertheless, an emission tax may be politically unappealing such as in the US (see Pizer; 1999, 2002), and an emission cap can cause large compliance costs due to uncertainties even it is feasible. Thus, other policies emerge, such as the indexed quantity policy addressed by Ellerman and Wing (2003), the general indexed quantity policy investigated by Newell and Pizer (2008), and the hybrid policy explored by Pizer (2002), Jacoby and Ellerman (2004), McKibbin and Wilcoxen (2002), and Webster et al. (2010). These studies contribute to the literature by ranking environmental policies based on the maximized expected net benefits of emissions.

With global warming becoming more and more serious, irregular and extreme climates inflict large-scale floods and droughts all over the world. How to effectively lower pollutant emissions within a limited time period becomes a very emergent and unavoidable challenge for mankind. Paris Agreement requesting countries to reduce emissions through the mechanisms of nationally determined contributions (NDCs) is only a beginning. We should find the most environmentally efficient policies to achieve the lowest emission level as soon as possible. Thus, this paper compares relative environmental efficiency of two simple and often-used price and quantity policies given the constraints of minimum expected net benefits of emissions under the set-up of Weitzman (1974). Unlike Weitzman (1974), we discover that quantity policies are more environmentally efficient than price policies regardless of relative steepness of emission's marginal benefit and marginal cost functions. That is because the shadow price of an additional expected net benefit of emissions under price policies is higher than that under quantity policies. Thus, to achieve the same expected net benefit of emissions, countries adopting price policies will discharge more than those using quantity policies. However, if the constraint of minimum expected net benefits of emissions is ignored, both policies are equally efficient.

The rest of this paper is structured as follows. Section 2 presents our model and results and Section 3 gives conclusions. All proofs are in the Appendix.

2. The Model and Results

Let e represent a firm's pollutant emission level. Following Weitzman (1974), we adopt quadratic cost and benefit functions of emissions around the optimal emission level $\hat{e}$,[1]

$$C(e,\ \theta) = a(\theta) + (C' + \alpha(\theta))(e - \hat{e}) + \frac{C''}{2}(e - \hat{e})^2 \text{ and} \tag{1}$$

$$B(e,\ \eta) = b(\eta) + (B' + \beta(\eta))(e - \hat{e}) + \frac{B''}{2}(e - \hat{e})^2 \tag{2}$$

respectively, where θ and η are random shocks from the cost and benefit sides of emissions, and $a(\theta)$, $\alpha(\theta)$, $b(\eta)$, and $\beta(\eta)$ are stochastic functions of θ and η. By contrast, C', C'', B', and B'' are constant coefficients. As in Weitzman (1974), we assume $E(\alpha(\theta)) = E(\beta(\eta)) = E[\alpha(\theta),\ \beta(\eta)] = 0$ and $B'' < 0 < C''$. Accordingly, we have the respective marginal cost and marginal benefit of emissions

$$C_1(e,\ \theta) = [C' + \alpha(\theta)] + C''(e - \hat{e}) \text{ and} \tag{3}$$

$$B_1(e,\ \eta) = [B' + \beta(\eta))] + B''(e - \hat{e}) \tag{4}$$

with $E[C_1(\hat{e},\ \theta)] = C'$ and $E[B_1(\hat{e},\ \eta)] = B'$. We presume $B' > 0$ because larger emissions will bring higher benefits for the society, and $C_1(e,\ \theta) < 0$ for all θ because the firm will spend lower (abatement) costs as its emissions increase. Moreover, large

[1] In Weitzman (1974), the choice variable of the social planner is clean air, while our choice variable is the emission level.

B' or $|C'|$ is needed to guarantee the existence of the solutions, which will be addressed in the Appendix. The assumptions are summarized below.

Assumption 1: $E(\alpha(\theta)) = E(\beta(\eta)) = E[\alpha(\theta),\ \beta(\eta)] = 0,\ B' > 0,\ C_1(e,\ \theta) < 0$ for all θ with large B' or $|C'|$, and $B'' < 0 < C''$.

Unlike Weitzman's (1974), Newell and Pizer's (2008), and Webster et al.'s (2010) maximizing emissions' expected net benefits, we let the social planner pursue environmental efficiency of emissions given the expected net benefits of emissions achieving some minimum level. That is, the social planner will choose $\hat{e}$ to solve the problem of

$$\min_{e \geq 0} e \quad \text{s.t. } EB(e,\ \eta) - EC(e,\ \theta) \geq \underline{W}, \tag{5}$$

where $\underline{W} > 0$ is the required minimum expected net benefit of emissions. The constraint imposed in (5) asks countries to keep a minimum level of net benefits when seeking the lowest emissions. This constraint is plausible, often-adopted in economics, and necessary for the existence of interior solutions (i.e., positive optimal emissions).[2] We will discuss how this constraint present or not will affect our results later.

Denote $L = e - \lambda[EB(e,\ \eta) - EC(e,\ \theta) - \underline{W}]$ the Lagrange function of the problem in (5) with the Lagrange multiplier λ. The associated solutions are as follows.

Lemma 1: *Given (1)-(2), (5), and Assumption 1, the optimal emission* $\hat{e} = \hat{e}(\underline{W}) > 0$ *and the optimal Lagrange multiplier* $\hat{\lambda} > 0$ *satisfy conditions*

$$\begin{aligned} EB_1(\hat{e},\ \eta) &= EC_1(\hat{e},\ \theta) + \frac{1}{\hat{\lambda}} \text{ and} \\ EB(\hat{e},\ \eta) &= EC(\hat{e},\ \theta) + \underline{W}. \end{aligned} \tag{6}$$

[2] The set-ups of Weitzman's (1974) and ours are like a firm's profit-maximization and cost-minimization problems under some minimum-output requirements, respectively. The minimum output constraint is not needed in the former problem, while it is necessary in the latter problem. Without this constraint, the cost-minimizing firm will produce zero output. Similarly, if there is no constraint in (5), it is optimal for the firm to discharge zero emission.

On the other hand, as in Weitzman (1974), we derive the optimal price policy using a two-stage game. In the first stage, the social planner announces the optimal price (i.e., emission tax) $\tilde{p}$ to minimize the expected emission level given the expected net benefits of emissions no less than $\underline{W}$. Then, given $\tilde{p}$ and a realized θ, the firm chooses its optimal emission $\tilde{e}$ to minimize the sum of emission taxes and emission costs. Afterward, $(\tilde{p},\ \tilde{e})$ is solved by backward induction as follows. First, given price p and realized cost shock θ, the firm will choose $e = h(p,\ \theta)$ to solve the problem of

$$\min_{e \geq 0}\ p \cdot e + C(e,\ \theta).$$

Then $e = h(p,\ \theta)$ will satisfy condition

$$p = -C_1(h(p,\ \theta),\ \theta) \tag{7}$$

with $\frac{\partial h}{\partial p} = \frac{-1}{C''} < 0$ and $\frac{\partial h}{\partial \theta} = 1 > 0$.

Next, given $e = h(p,\ \theta)$, the social planner chooses $\tilde{p}$ to solve the problem of

$$\min_{p \geq 0}\ E[h(p,\ \theta)] \quad \text{s.t. } E[B(h(p,\ \theta),\ \eta)] - E[C(h(p,\ \theta),\ \theta)] \geq \underline{W}. \tag{8}$$

Denote $L = E[h(p,\ \theta)] - \lambda\{E[B(h(p,\ \theta),\ \eta)] - E[C(h(p,\ \theta),\ \theta)] - \underline{W}\}$ this problem's Lagrange function with the Lagrange multiplier λ. Solving the problem in (8) yields the following.[3]

Lemma 2: *Given (1)-(2), (8), and Assumption 1, we have* $\tilde{p} = \tilde{p}(\underline{W}) > 0$ and $\tilde{\lambda} > 0$ *satisfy conditions*

$$\tilde{p} = \frac{1}{\tilde{\lambda}} - E[B_1(h(\tilde{p},\ \theta),\ \eta)] \text{ and} \tag{9}$$

$$\underline{W} = E[B(h(\tilde{p},\ \theta),\ \eta)] - E[C(h(\tilde{p},\ \theta),\ \theta)], \tag{10}$$

where $\tilde{\lambda}$ *is the optimal Lagrange multiplier. Then, denote* $\tilde{e} = \tilde{e}(\underline{W},\ \theta) \equiv h(\tilde{p}(\underline{W}),\ \theta)$ *the firm's optimal emissions under the price policy.*

[3] Similarly, if the constraint in (8) is taken away, the firm's optimal emission is zero.

Based on Lemmas 1 and 2, we can explore the environmental efficiency of both price and quantity policies. Equation (7) implies

$$p = -C_1(h(p,\ \theta),\ \theta) = -[C' + \alpha(\theta) + C''(h(p,\ \theta) - \hat{e})] \tag{11}$$

by (3). Rearranging (11) and evaluating (11) at $p = \tilde{p}(\underline{W})$ yield

$$\tilde{e}(\underline{W},\ \theta) = h(\tilde{p}(\underline{W}),\ \theta) = \hat{e}(\underline{W}) - \frac{\tilde{p}(\underline{W}) + C' + \alpha(\theta)}{C''}. \tag{12}$$

Evaluating (4) at $e = h(\tilde{p}(\underline{W}),\ \theta)$ and taking its expectation with respect to θ and η yield

$$E[B_1(h(\tilde{p}(\underline{W}),\ \theta),\ \eta)] = E[B' + \beta(\eta) + B''(h(\tilde{p}(\underline{W}),\ \theta) - \hat{e})] = B' - \frac{B''[\tilde{p}(\underline{W}) + C']}{C''} \tag{13}$$

by (12). Then substituting (13) into (9) and rearranging (9) give

$$\tilde{p}(\underline{W}) = \frac{C''}{(C'' - B'')}[\frac{1}{\tilde{\lambda}} - B' + \frac{B''C'}{C''}]. \tag{14}$$

Accordingly, substituting (14) into (12) yields

$$\begin{aligned}
& \tilde{e}(\underline{W},\ \theta) - \hat{e}(\underline{W}) \\
= \; & \frac{-[\tilde{p}(\underline{W}) + C' + \alpha(\theta)]}{C''} \\
= \; & \frac{-1}{(C'' - B'')}[\frac{1}{\tilde{\lambda}} - B' + \frac{B''}{C''}C'] - \frac{1}{C''}[C' + \alpha(\theta)] \\
= \; & \frac{-1}{(C'' - B'')}[\frac{1}{\tilde{\lambda}} - C' - \frac{1}{\hat{\lambda}} + \frac{B''}{C''}C'] - \frac{1}{C''}[C' + \alpha(\theta)] \\
= \; & \frac{-1}{(C'' - B'')}[\frac{1}{\tilde{\lambda}} - \frac{1}{\hat{\lambda}}] - \frac{\alpha(\theta)}{C''},
\end{aligned} \tag{15}$$

where the second equality is by (14), and the third equality is due to $EC_1(\hat{e},\ \theta) = C'$, $EB_1(\hat{e},\ \eta) = B'$, and $B' = C' + \frac{1}{\hat{\lambda}}$ by (6). Taking the expectation of (15) with respect to θ yields

$$E[\tilde{e}(\underline{W},\ \theta) - \hat{e}(\underline{W})] = \frac{-1}{(C'' - B'')}[\frac{1}{\tilde{\lambda}} - \frac{1}{\hat{\lambda}}]. \tag{16}$$

Since $C'' > 0 > B''$, the sign of $E[\tilde{e}(\underline{W},\ \theta) - \hat{e}(\underline{W})]$ is completely determined by relative sizes of $\frac{1}{\hat{\lambda}}$ and $\frac{1}{\tilde{\lambda}}$. For optimal Lagrange multipliers $\hat{\lambda}$ and $\tilde{\lambda}$ in problems (5) and

(8), the Envelope Theorem implies $\frac{d\hat{e}(\underline{W})}{d\underline{W}} = \hat{\lambda} > 0$ and $\frac{dE(h(\tilde{p}(\underline{W}),\, \theta)}{d\underline{W}} = \tilde{\lambda} > 0$. Because $E[\tilde{e}(\underline{W},\, \theta)] = \hat{e}(\underline{W}) - \frac{\tilde{p}(\underline{W})+C'}{C''}$ by (12), we have

$$\frac{dE[\tilde{e}(\underline{W},\, \theta)]}{d\underline{W}} = \frac{d\hat{e}(\underline{W})}{d\underline{W}} - \frac{\frac{d\tilde{p}(\underline{W})}{d\underline{W}}}{C''}. \tag{17}$$

On the other hand, due to $\tilde{p}$ satisfying equation (10), we can get

$$\frac{d\tilde{p}(\underline{W})}{d\underline{W}} = \frac{1}{E[B_1(h(\tilde{p},\, \theta),\, \eta) \cdot \frac{\partial h}{\partial p}] - E[C_1(h(\tilde{p},\, \theta),\, \theta) \cdot \frac{\partial h}{\partial p}]} < 0 \tag{18}$$

by the implicit function theorem, and $E[B_1(h(\tilde{p},\, \theta),\, \eta)\frac{\partial h}{\partial p}] - E[C_1(h(\tilde{p},\, \theta),\, \theta)\frac{\partial h}{\partial p}] < 0$ by equation (25), $\frac{\partial h(\tilde{p},\, \theta)}{\partial p} < 0$, and $\tilde{\lambda} > 0$. Accordingly, equation (17) becomes

$$\tilde{\lambda} - \hat{\lambda} = -\frac{\frac{d\tilde{p}(\underline{W})}{d\underline{W}}}{C''} > 0 \tag{19}$$

due to $C'' > 0$ and (18). It in turn suggests $\frac{1}{\tilde{\lambda}} < \frac{1}{\hat{\lambda}}$, and thus equation (16) becomes

$$E[\tilde{e}(\underline{W},\, \theta) - \hat{e}(\underline{W})] = \frac{-1}{(C'' - B'')}[\frac{1}{\tilde{\lambda}} - \frac{1}{\hat{\lambda}}] > 0 \tag{20}$$

by $C'' > 0 > B''$ and (19). Then (20) suggests $E[\tilde{e}(\underline{W},\, \theta)] > \hat{e}(\underline{W})$, which means that quantity policies are more environmentally efficient than price policies. These results are summarized below.

Proposition 1. *Given (1)-(2) and Assumption 1, we find that quantity policies are more environmentally efficient than price policies regardless of the relative steepness of emission's marginal benefit and marginal cost functions. Moreover, at equilibria, both kinds of policies achieve the same expected net benefits of emissions.*

The intuition of Proposition 1 is as follows. Equation (19) suggests that an additional expected net benefit of emissions will lead to firm's larger optimal emission increments under price policies relative to under quantity policies. That is because the rising benefit will not only cause firm's higher optimal emissions, but also lower its optimal price under price policies, which in turn will raise its optimal emissions further by $\frac{\partial h(p,\, \theta)}{\partial p} < 0$ due to the announced prices in the first stage affecting firm's optimal emissions decided in the second stage. However, this latter effect will not occur under quantity policies having one stage only. These imply that the shadow price of the rising expected net benefits of emissions under price policies is higher than that under quantity policies, as suggested by (19). Accordingly, to achieve the same expected net benefits of emissions, countries adopting price policies will discharge more than those using quantity policies. In other words, quantity policies are more environmentally efficient than price policies.

Finally, it is worthy to mention the results without considering the constraint given in (5). Under the circumstance, we will have $\hat{e} = 0$ or $\hat{\lambda} = 0$ because the firm's optimal emission level is zero. Similarly, if the constraint in (8) is ignored, the firm's optimal emission level is zero as well-that is, $\tilde{e} = 0$ or $\tilde{\lambda} = 0$. Accordingly, $\hat{\lambda} = \tilde{\lambda}$ in (19) suggests that both price and quantity policies are equally efficient as stated below.

Corollary 1. *Suppose that the regulator imposes no constraint of minimum expected net benefits of emissions when deciding the optimal policies. Then both price and quantity policies are equally environmentally efficient and will induce zero emission of the firm.*

3. Conclusions

This paper explores environmental efficiency of price and quantity policies under the set-up of Weitzman (1974). We show that quantity policies are at least as environmentally efficient as price policies. This finding holds whether emissions' marginal benefit functions are steeper or flatter than their marginal cost functions. We think that similar analyses should be conducted on other environmental policies such as hybrid and (general) indexed quantity policies in the future.

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Appendix

Proof of Lemma 1: Given $L = e - \lambda[EB(e,\ \eta) - EC(e,\ \theta) - \underline{W}]$, the Kuhn-Tucker conditions are

$$\begin{aligned} \frac{\partial L}{\partial e} &= 1 - \lambda[EB_1(e,\ \eta) - EC_1(e,\ \theta)] \geq 0,\ e \cdot \frac{\partial L}{\partial e} = 0, & (21) \\ \frac{\partial L}{\partial \lambda} &= \underline{W} - EB(e,\ \eta) + EC(e,\ \theta) \leq 0,\ \lambda \cdot \frac{\partial L}{\partial \lambda} = 0. & (22) \end{aligned}$$

If $\lambda = 0$, then $\frac{\partial L}{\partial e} = 1 > 0$, and $\hat{e} = 0$ by (21), which is not an interesting solution. Thus, we focus on the case of $\lambda > 0$. Under the circumstance, the optimal emission $\hat{e} = \hat{e}(\underline{W}) > 0$ will satisfy condition $EB(\hat{e},\ \eta) - EC(\hat{e},\ \theta) = \underline{W}$ by (22). Assumption 1 implies that function $[EB(e,\ \eta) - EC(e,\ \theta)]$ is strictly concave in e with maximum value $\frac{(B'-C')^2}{2(C''-B'')} > 0$ occurred at $e^* = \hat{e} + \frac{(B'-C')}{(C''-B'')} > \hat{e}$. Large enough B' or $|C'|$ can guarantee the existence of $\hat{e} = \hat{e}(\underline{W})$ for a given value of $\underline{W}$. Moreover, the optimal Lagrange multiplier $\hat{\lambda} > 0$ will satisfy condition $EB_1(\hat{e},\ \eta) = EC_1(\hat{e},\ \theta) + \frac{1}{\hat{\lambda}}$ by (21). □

Proof of Lemma 2: Given $L = E[h(p,\ \theta)] - \lambda\{E[B(h(p,\ \theta),\ \eta)] - E[C(h(p,\ \theta),\ \theta)] - \underline{W}\}$, the Kuhn-Tucker conditions are

$$\begin{aligned} \frac{\partial L}{\partial p} &= E[\frac{\partial h(p,\ \theta)}{\partial p}] - \lambda\{E[B_1(h(p,\ \theta),\ \eta) \cdot \frac{\partial h(p,\ \theta)}{\partial p}] - E[C_1(h(p,\ \theta),\ \theta) \cdot \frac{\partial h(p,\ \theta)}{\partial p}]\} \geq 0, \\ & \quad p \cdot \frac{\partial L}{\partial p} = 0, & (23) \\ \frac{\partial L}{\partial \lambda} &= \underline{W} - E[B(h(p,\ \theta),\ \eta)] + E[C(h(p,\ \theta),\ \theta)] \leq 0,\ \lambda \cdot \frac{\partial L}{\partial \lambda} = 0. & (24) \end{aligned}$$

If $\lambda = 0$, then $\frac{\partial L}{\partial p} = E[\frac{\partial h(p,\,\theta)}{\partial p}] < 0$ by $\frac{\partial h(p,\,\theta)}{\partial p} < 0$, which contradicts $\frac{\partial L}{\partial p} \geq 0$ required by (23). Accordingly, we must have $\tilde{\lambda} > 0$, which in turn implies $\frac{\partial L}{\partial \lambda} = 0$ by (24). Thus, $\tilde{p} = \tilde{p}(\underline{W})$ satisfies condition $E[B(h(\tilde{p},\ \theta),\ \eta)] - E[C(h(\tilde{p},\ \theta),\ \theta)] = \underline{W}$. As in Lemma 1, the existence of $\tilde{p} > 0$ is guaranteed by large B' or $|C'|$. Thus, we have

$$E[\frac{\partial h(\tilde{p},\ \theta)}{\partial p}] = \lambda\{E[B_1(h(\tilde{p},\ \theta),\ \eta) \cdot \frac{\partial h(\tilde{p},\ \theta)}{\partial p}] - E[C_1(h(\tilde{p},\ \theta),\ \theta) \cdot \frac{\partial h(\tilde{p},\ \theta)}{\partial p}]\} \quad (25)$$

by (23). On the other hand, under condition (7) evaluated at $p = \tilde{p}$, equation (25) becomes $E[\frac{\partial h(\tilde{p},\,\theta)}{\partial p}] = \lambda E[B_1(h(\tilde{p},\ \theta),\ \eta) \cdot \frac{\partial h(\tilde{p},\,\theta)}{\partial p}] + \lambda \tilde{p} E[\frac{\partial h(\tilde{p},\,\theta)}{\partial p}]$. Thus, $\tilde{\lambda} > 0$ will satisfy condition $\tilde{p} = \frac{1}{\tilde{\lambda}} - \frac{EB_1 \cdot \frac{\partial h}{\partial p}}{E\frac{\partial h}{\partial p}} = \frac{1}{\tilde{\lambda}} - E[B_1(h(\tilde{p},\ \theta),\ \eta)]$ by $E(\alpha(\theta),\ \beta(\eta)) = 0$. $\square$

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