A note on the privatization neutrality result with colluding private firms

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**Abstract**

In a mixed quantity-setting oligopoly, we investigate the welfare effects of privatization in the presence of an optimal output subsidy. We find that the privatization neutrality result is not satisfied whenever there is at least some cooperation between the private firms. Our result suggests that the degree of cooperation between private firms reduces the government incentives to privatize the public firm. In addition, if a consumer surplus bias of the public firm is considered, the privatization neutrality result does not hold either, and the incentives to privatize the public firm are further reduced.
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In a mixed quantity-setting oligopoly, we investigate the welfare effects of privatization in the presence of an optimal output subsidy. We find that the privatization neutrality result is not satisfied whenever there is at least some cooperation between the private firms. Our result suggests that the degree of cooperation between private firms reduces the government incentives to privatize the public firm. In addition, if a consumer surplus bias of the public firm is considered, the privatization neutrality result does not hold either, and the incentives to privatize the public firm are further reduced.
1 Introduction

The literature on mixed oligopolies where a public or semi-public firm competes against one or more private profit-maximizing firms is extensive. There is also an abundant literature considering privatization in a mixed oligopoly. One line of research has shown that there are no consequences from privatization of a public firm whenever a production subsidy ensures the first-best allocation. This has often been referred to as the privatization neutrality theorem. In a very influential paper, White (1996) shows that in a Cournot setting when a production subsidy is used only before full privatization, it always lowers welfare regardless of the number of firms in the industry. On the contrary, there are no welfare consequences when a subsidy is implemented before and after privatization. In this line, Matsumura and Okumura (2013) find that the neutrality holds if an output floor is introduced. Other examples of this line of research are Poyago-Theotoky (2001) or Myles (2002). Another line of research, however, has derived a non-neutral result of privatization. For instance, Fjell and Heywood (2004) have shown that privatization is not necessarily welfare neutral in mixed oligopolies under a production subsidy if firms move sequentially. In their paper, the model is extended to assume that after privatization the leader’s role changes. When the public firm remains a leader after privatization welfare is reduced. More recently, Matsumura and Tomaru (2012, 2013) have respectively shown that the privatization neutrality does not hold if there are foreign competitors or when an excess burden of taxation is introduced. Additionally, in a very recent paper, Lin and Matsumura (2018) have shown that the neutrality result does not hold unless public and private firms have the same cost function. However, to the best of our knowledge, the literature on mixed oligopolies has mostly ignored the possibility that private firms achieve a collusive agreement. Among the few exceptions are the recent papers by Correia-da-Silva and Pinho (2018), Escrihuela-Villar and Gutiérrez-Hita (2018), and Colombo (2016) where it is generally found that the presence of a public firm makes collusion among private firms harder to sustain. Another example is Matsumura and Okamura (2015) where the authors introduce an interdependent payoff structure into a mixed oligopoly assuming that firms consider their own and other firms’ profits. It is basically obtained that the optimal degree of privatization is higher when there is less market competition. These papers, though, do not consider the effect of private firms’ collusion on the privatization
neutral result.

As a matter of fact, mixed public-private firms are increasingly used in several European countries. Regulatory authorities make use of mixed firms following cost considerations, financial constraints or also aiming to reduce or eliminate the abuse of dominant position in markets where some degree of collusion has been observed. In this sense, some empirical studies have also analyzed how weak competition makes it difficult for local governments to obtain benefits from contracting out. For instance, in the Netherlands, Dijkgraaf and Gradus (2007) investigate whether collusion exists and its impact on tariffs for waste collection. Their results indicate the existence of collusion between private firms and that the presence of competing public firms might be essential to ensure more and fair competition. In the market for water and waste production, and based on the calculation of the Herfindahl-Hirschman Index (HHI) for some Spanish municipalities, Bel and Fageda (2011) obtain that a tendency toward concentration may diminish the advantages of partial privatization.

Accordingly, the main goal of the present note is to analyze if, in a mixed oligopoly, the privatization neutrality result is also robust to the inclusion of cooperation between private firms. To that extent, we consider a mixed oligopoly market where two private firms simultaneously compete in quantities with a partially welfare-maximizing public firm and where the government establishes optimal (welfare-maximizing) uniform production subsidies. Additionally, we assume that private firms are able to imperfectly collude\(^1\) by maximizing the sum of their own profits and a fraction of the other private firm’s profits. As a consequence, this fraction may be considered as the degree of coordination which implies that firms can agree on a distribution of the output quotas different from that arising from a perfect joint profit maximization agreement. This way of modeling the intensity of competition, which has received growing attention of scholars (see for instance Symeonidis, 2008), is closely related to the coefficient of cooperation defined by Cyert and DeGroot (1973). Following the reasoning of Symeonidis (2008), this payoff function based on a relative performance might be used to parameterize the intensity of competition.\(^2\)

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\(^1\)Imperfect collusion is often used to describe situations where firms collude without reaching the Pareto frontier of profits whereas partial collusion refers to a market where only a subset of firms colludes.

\(^2\)Additionally, Escrihuela-Villar (2015) shows that this formulation and the conjectural variations approach lead to equivalent closed-form solutions.
Our main result is that in a symmetric mixed oligopoly, the privatization neutrality result is not robust to the existence of cooperation between the private firms. Consequently, when there is at least some collusion in a mixed oligopoly, a privatization policy is also important despite the existence of a welfare maximizing production subsidy. In fact, the present note shows that privatization might decrease welfare since the welfare-maximizing behavior of the public firm mitigates collusion of private firms. In addition, we also show that if the public firm has a bias toward consumer surplus in her objective function, the privatization neutrality result does not hold regardless of the competitiveness between the private firms and that the aforementioned bias further reduces the incentives of the regulator to privatize. Our analysis proves useful to identify plausible conditions under which an optimal privatization scheme is important despite the presence of optimal production subsidies. In addition, our results also suggest that under certain conditions full nationalization is optimal which is in contrast to Matsumura (1998) who finds that neither full privatization nor full nationalization is optimal in a symmetric cost mixed duopoly.

The rest of the note is structured as follows. Section 2 describes the imperfectly collusive market in the presence of a public firm and presents the main results. Section 3 presents an extension of the basic model where the public firm might have a consumer bias. Proofs are relegated to the Appendix.

2 Imperfect competition in a mixed oligopoly with production subsidies

We consider an industry with three firms simultaneously producing a homogeneous product with a quadratic cost function given by $c(q_i) = \frac{1}{2}q_i^2$. Firms indexed by $i = 2, 3$ are profit-maximizing private firms whereas the firm indexed by 1 is assumed to be a welfare-maximizing public firm.\textsuperscript{3} Welfare ($W$) accounts for cumulative firm’s profits $\sum_{i=1}^{3} \Pi_i$ and consumer surplus ($CS$), where $\Pi_i$ denotes profit of firm $i$. Industry inverse demand is

\textsuperscript{3}As stated in De Fraja and Delbono (1990), if each firm’s marginal cost is constant the public firm will impose the rule of pricing at marginal cost. Then, if there were any fixed costs, the public firm would be unable to cover the losses which would then need to be funded by the taxpayer. We abstract from this issue by considering increasing marginal costs.
piecewise linear \( p(Q) = \max(0, a - Q) \), where \( Q = \sum_{i=1}^{3} q_i \) is the industry output, \( p \) is the output price, and \( a > 0 \). We assume that entry and exit in the market are not possible.\(^4\)

In many cases, though, a partial privatization structure can be observed and, therefore, the government still holds a positive proportion of shares in privatized firms. Therefore, these semi-privatized firms cannot be pure welfare maximizers. In the present framework, we assume that the unique semi-public firm maximizes the weighted sum of own profit and welfare: \( \beta(\sum_{i=1}^{3} \Pi_i + CS) + (1 - \beta)\Pi_1 \) where \( \beta \in [0, 1] \). Consequently, the present formulation encompasses the model with one pure public firm and two private firms if \( \beta = 1 \) and the quantity competition among three private firms if \( \beta = 0 \). In other words, if the shares owned by the government increase then also \( \beta \) increases.

As mentioned in the introduction, we characterize the degree of cooperation between private firms considering a particular model where the output is determined in such a way that both private firms maximize the sum of their own profits and a fraction of the other private firm’s profits, namely \( \Pi_i + \alpha \Pi_j \) where \( i, j = 2, 3 \ (i \neq j) \), and \( \alpha \in [0, 1] \) is assumed to be symmetric and constant. This parameter \( \alpha \) can be interpreted as representing the cooperation of the market, with a smaller \( \alpha \) indicating a more competitive market. This interdependent payoff approach enables us to treat the degree of market cooperation as a continuous variable. In this model, the equilibrium outcome converges to the standard Cournot mixed oligopoly market when \( \alpha \) approaches 0 and the private firms’ joint profit-maximizing allocation when \( \alpha = 1 \).\(^5\) Therefore, a direct link between a positive \( \alpha \) and the degree of collusion can be established. We assume, therefore, coordination only between private firms whereas the (semi) public firm is concerned with social welfare and individual

\(^{4}\)The results of the present paper also carry over to an oligopoly model with \( n \) private firms. We present the duopoly model for ease of exposition. Additionally, the comparative statics with respect to \( n \) would just confirm the intuitions provided in the comprehensive literature on mixed oligopolies (see for instance De Fraja and Delbono, 1990) where, with a large number of private firms, privatization might increase welfare since the public firm must produce a very high level of output, driving private profits to a very low level.

\(^{5}\)We note that the equilibrium outcome converges to a competitive outcome in which private firms are price takers when \( \alpha \) approaches \(-1\). However, since we focus on the case of cooperation only between private firms, a negative \( \alpha \) could be interpreted as private firms acting more competitively than the public firm. Our results though also carry over to the case where \( \alpha \in [-1, 0] \).
It is a straightforward exercise to obtain the equilibrium outputs

\[ q_1(\alpha, \beta) = \frac{(2 + \alpha)}{(2 - \beta)} q_i(\alpha, \beta), \quad q_i(\alpha, \beta) = \frac{\alpha(2 - \beta)}{10 + 3\alpha - \beta(4 + \alpha)} \]  

(1)

where \( q_1(\alpha, \beta) \geq q_i(\alpha, \beta) \), and profits

\[ \Pi_1(\alpha, \beta) = [(3 - 2\beta)/2] \cdot [q_1(\alpha, \beta)]^2; \quad \Pi_i(\alpha, \beta) = [(3 + 2\alpha)/2] \cdot [q_i(\alpha, \beta)]^2. \]  

(2)

It can be easily checked that public firm’s output increases with \( \alpha \) and \( \beta \) while the reverse is true for private firms. Since our focus is on the effects of an optimal subsidy on welfare, we consider the degree of privatization \( \beta \) as exogenously given. The implementation of an optimal welfare-maximizing uniform production subsidy (that we denote by \( s > 0 \)) affects all firms. Accordingly, welfare must also incorporate the cost of the subsidy; then,

\[ W(\alpha, \beta) = \sum_{i=1}^{3} \Pi_i(\alpha, \beta) + CS(\alpha, \beta) - s(\sum_{i=1}^{3} q_i(\alpha, \beta)). \]  

(3)

The subsidy is included in the total welfare as part of the public and private firms’ profits but also as an equivalent expenditure. Using the standard formulation in the literature we assume that, in the first stage, the government chooses \( s \) to maximize \( W(\alpha, \beta) \). In the second stage, firms simultaneously choose their outputs. We use the subgame perfect Nash equilibrium as the equilibrium concept. The equilibrium quantities and profits with an optimal uniform production subsidy are given by

\[ \hat{q}_1(\alpha, \beta) = \frac{\alpha(\beta^2(2+\alpha)(4+\alpha)+(2+\alpha)(6+\alpha) - 2\beta(8+\alpha(7+\alpha)))}{2(24-2\beta(4+\alpha)^2+\beta^2(4+\alpha)^2+\alpha(8+\alpha))}, \]

\[ \hat{q}_i(\alpha, \beta) = \frac{\alpha(6+\alpha+(\beta-2)\beta(4+\alpha))}{24-2\beta(4+\alpha)^2+\beta^2(4+\alpha)^2+\alpha(8+\alpha)}, \]  

(4)

\[ \hat{\Pi}_1(\alpha, \beta) = \left[ \frac{3}{2} + \frac{4\alpha\beta}{\beta^2(2+\alpha)(4+\alpha)+(2+\alpha)(6+\alpha) - 2\beta(8+\alpha(7+\alpha))} \right] \cdot [\hat{q}_1(\alpha, \beta)]^2 \]

\[ \hat{\Pi}_i(\alpha, \beta) = \left[ \frac{3}{2} + \alpha \right] \cdot [\hat{q}_i(\alpha, \beta)]^2. \]

Consequently, in a linear demand and convex cost function setup, our model represents an extension of the one presented in Lin and Matsumura (2018) by including the degree of

\footnote{Admittedly, welfare-improving collusion between public and private firms might also be formed assuming, for instance, that firms are also concerned with corporate social responsibility (see, for instance, Haraguchi and Matsumura, 2018).}
competition between private firms. The next proposition shows that in a mixed oligopoly where firms have the same cost functions, the neutrality result fails unless private firms do not coordinate their outputs.

**Proposition 1.** In our mixed oligopoly with imperfect competition, total welfare does not depend on $\alpha$ if and only if $\alpha = 0$. On the other hand, if $\alpha \in (0, 1]$, total welfare always increases with $\beta$. In this case, furthermore, $\partial^2 W(\alpha, \beta)/\partial \beta \partial \alpha > 0$.

Proposition 1 shows that the privatization neutrality result is not robust to the assumption of collusion between private firms. In other words, if the degree of competition between private firms is also considered, privatization affects welfare despite the inclusion of a welfare-maximizing production subsidy. More precisely, if private firms restrict competition (through a $\alpha > 0$), a larger $\beta$ increases welfare due to the output expansion of the public firm. In addition, an increase in the degree of cooperation between private firms further decreases the regulator’s incentives to privatize the public firm.

Summarizing, our analysis proves useful to identify that the main result in Lin and Matsumura (2018) crucially relies on the type of competition assumed. Consequently, the optimal privatization strategy should carefully consider the competitiveness of the market also in the presence of uniform optimal production subsidies.

### 3 Extension: Consumer surplus vs total welfare

Following White (2002), we consider in this section that the public firm might have an objective function, arised from a certain political orientation, different from that observed under the standard welfare-maximizing one. The objective function of the public firm thus might take a wide variety of forms ranging from pro-consumer to pro-business.\(^7\)

\(^7\)We note that in the present model even with $\beta = 0$ the former public (now private) firm does not collude with the other private firms. Admittedly, assuming that firm 1 also colludes with the other firms would lead us back to the symmetric framework of Lin and Matsumura (2018) and the privatization neutrality result would be restored. We believe though that it is a more natural assumption to consider that a semi-public or a recently privatized firm would not engage in collusive activities. We thank a referee for bringing this issue to our attention.

\(^8\)In addition, one could also argue that the use of a “total welfare” standard may be interpreted as treating transfers between agents as welfare-neutral. However, it is also plausible that transfers associated
Accordingly, we consider here the effect of a possible bias toward consumer surplus in the public firm maximization problem. To that extent, we assume here that this bias toward consumer surplus is captured by the parameter \( \theta \in [0, 1] \) as long as we assume that the public firm maximizes the following weighted sum: \( \beta(\theta (\Pi_1 + CS) + (1 - \theta)W) + (1 - \beta)\Pi_1 \) when competing against the private firms. Basically, a positive \( \theta \) can also be interpreted as a public firm more aggressively competing with the private firms where, in the limit case of \( \theta = 1 \), private firms’ profits do not appear in the objective function of the public firm. The present formulation thus encompasses the case presented in the previous section if \( \theta = 0 \). Most likely, this interpretation cannot be readily carried over to the whole public sector and, therefore, we assume that in the first stage, the government still chooses \( s \) to maximize \( W \).

The next proposition considers the effect of a positive \( \theta \) on the result stated in Proposition 1.

**Proposition 2** In our mixed oligopoly with imperfect competition, total welfare does not depend on \( \beta \) if and only if \( \alpha = 0 \) and \( \theta = 0 \). On the other hand, if either \( \alpha \) and/or \( \theta \) are positive, total welfare always increases with \( \beta \). In this case, furthermore, \( \partial^2 W/\partial \beta \partial \theta > 0 \).

Proposition 2 states that the privatization neutrality result relies not only on the Cournot competition between private firms but also on the unbiased welfare maximization of the public firm. In fact, the inclusion of a consumer bias in the public firm reinforces the optimality of keeping the shares of the public firm owned by the government. Finally, we note that following Matsumura and Okumura (2013) an optimal output floor regulation could be set to obtain the privatization neutrality result also with imperfect competition among private firms. Consequently, Proposition 2 is useful to emphasize the importance of a possible bias in the public firm objective function, since a positive \( \theta \) implies that privatization is not welfare neutral regardless of the behavior of private firms or the potential output regulation.

to several policy measures might greatly exceed in magnitude any deadweight loss, so that a decision to ignore transfers may be quite important. Consequently, several papers (see, for instance, Pittman, 2007) call for antitrust agencies to move in the direction of a consumer surplus standard rather than a total welfare standard.
Appendix

Proof of Proposition 1. From (4) we can easily obtain the expression for the welfare,
\[ W(\alpha, \beta) = \frac{a^2((6+\alpha)^2-2\beta(24+\alpha(12+\alpha))+\beta^2(24+\alpha(12+\alpha)))}{4(24-2\beta(4+\alpha)^2+\beta^2(4+\alpha)^2+\alpha(8+\alpha))}. \]
Then, \[ \frac{\partial W(\alpha, \beta)}{\partial \beta} = \frac{2\alpha^2(1-\beta^2)\alpha^2}{(24-2\beta(4+\alpha)^2+\beta^2(4+\alpha)^2+\alpha(8+\alpha))^2} = 0 \] when \( \alpha = 0 \). In addition, \[ \frac{\partial^2 W(\alpha, \beta)}{\partial \beta^2} = 0 \] is only equal to 0 whenever \( \alpha = 0 \) and \( \theta = 0 \). Unfortunately, the expression for \[ \frac{\partial^2 W(\alpha, \beta)}{\partial \beta \partial \theta} \] cannot be further simplified to be included in the paper. However, we used the program Wolfram Mathematica 7.0 to check that \[ \frac{\partial^2 W(\alpha, \beta)}{\partial \beta \partial \theta} > 0. \] Further details are available from the authors upon request.

Proof of Proposition 2. When the public firm maximizes the weighted sum described in Section 3, total welfare can be easily calculated and
\[ W(\alpha, \beta) = \frac{a^2(4(9+6(\beta-2)\beta-12\beta+7(\beta-2)\beta^2+4(\beta-1)^2\beta^2)-4(\beta-1)^2(29-3)\alpha+(\beta-1)^2\alpha^2)}{4(24-4\beta(7+\alpha)+\alpha(8+\alpha)-2\beta(4-2\theta+\alpha)^2+\beta^2(4-2\theta+\alpha)^2)}. \]
Then, \[ \frac{\partial W(\alpha, \beta)}{\partial \beta} = \frac{2\alpha^2(1-\beta)(4\beta^2+a-\theta^2)}{(24-4\beta(7+\alpha)+\alpha(8+\alpha)-2\beta(4-2\theta+\alpha)^2+\beta^2(4-2\theta+\alpha)^2)^2} \] is only equal to 0 whenever \( \alpha = 0 \) and \( \theta = 0 \). Further details are available from the authors upon request.

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