Correlated shocks in estimated DSGE models

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Abstract
In simulating and estimating DSGE models, we typically assume that exogenous shocks exist and that they capture aggregate uncertainty. Further, they are either interpreted as structural or as measurement error. Therefore, we almost always assume orthogonality of those shocks restricting the off-diagonal elements of the variance-covariance matrix. In this paper, we ask the question whether correlated shocks matter when we estimate typical DSGE models and what we can learn from including them. We argue that using correlated shocks is useful as a robustness test: observing correlated shocks implies that the underlying DSGE model is misspecified and we can understand the weaknesses of the underlying model. We find sizable and relevant differences for the three DSGE models estimated when we include correlated shocks. This holds for the estimation of structural parameters, driving forces of fluctuations, and the size and sign of the estimated shocks.
1 Introduction

In this paper, we ask the question whether correlated shocks matter when we estimate typical Dynamic Stochastic General Equilibrium models (DSGE, for short) and what we can learn from including them. Put differently, does the arbitrary restriction on non-correlated shocks has implications for the results of the Bayesian estimation of some typical DSGE models? Observing correlated shocks implies that the underlying DSGE model is misspecified (cf. Del Negro and Schorfheide, 2009). Therefore, allowing for correlated shocks can be a useful inference or robustness test. Further, the obtained correlations can be used to understand the weaknesses of the underlying theoretical foundation and can help to improve it. We argue that estimating DSGE models with correlated shocks should become a standard routine to test for potential misspecification. For example, according to this approach, the workhorse DSGE model by Smets and Wouters (2007) does not suffer from misspecification, while other models in the literature do, leading to different results.

Over the last three decades, DSGE models have been the workhorse models in macroeconomic analysis employed by policy makers and academics. They assume, amongst many other things, that exogenous shocks exist and that they capture aggregate uncertainty. Those shocks are either interpreted as "structural" (cf. Smets and Wouters, 2007) or as measurement error (cf. Ireland, 2004 and Schorfheide, 2013). In simulating model dynamics or in estimating those models, we - almost always - assume orthogonality of those shocks. This creates a restriction on the variance-covariance matrix where all off-diagonal elements are, therefore, zero.

While this is a strict, convenient assumption, it rules out implications from widely accepted theories. For example, in a model with multiple, different assets - say bonds and houses - and risk averse households, portfolio theory could result in an optimum where households invest in both assets, such that correlated shocks become important (cf. De Santis and Gerard, 1997). A second example comes from the findings by Diebold and Yilmaz (2014). The authors use network theory to track daily changes in the connectedness of US financial institutions over the Global Financial Crisis. They find sizable time-varying levels of connectedness in the US financial system. A shock hitting one institution can, as in the case of Lehman Brothers, spread through the network and affect other institutions. This epidemic character can lead to correlated shocks.

Empirically, using a vectorautoregression model, Chari et al. (2007) find important cross-correlations of business cycle shocks. Along this line, Schmitt-Grohé and Uribe (2011) show that a common shock to productivity and investment-specific productivity drives a large share of business cycle dynamics. In the open-economy, Justiniano and Preston (2010) and Rabanal et al. (2011) find that cross-country correlated shocks are an important factor, for example, in driving exchange rates. Fernández-Villaverde et al. (2011) investigate the correlation between level shocks and volatility shocks for the real interest rate in a set of latin american countries. They find a high correlation, between 0.69 and 0.89, between level and correlation shocks.

Our paper is closely related to Cúrdia and Reis (2011) who, to the best of our knowledge, started the discussion about correlated shocks in DSGE models. They show that allowing
for correlated shocks does change the results from estimating DSGE models. For example, they attribute significant parts of business cycle fluctuations to productivity and fiscal policy shocks rather than variations in mark-ups. Further, Andrle (2014) demonstrates misspecification of a DSGE model by showing that the model fails to (endogenously) explain the co-movement of investment and consumption without strongly correlated shocks. Ferroni et al. (2015) focus on the additional assumption that priors on the standard deviation of structural shocks exclude zero. They show that this creates a downward bias in the internal persistence of the model and, most interestingly, cluster shocks in fundamental and non-fundamental.

We find that for the three DSGE models estimated, sizable and relevant differences emerge when we include correlated shocks. This holds for the estimation of structural parameters, e.g. changes in Taylor-rule parameters, the driving forces of fluctuations, and the size and sign of the estimated shocks. Therefore, results derived from the estimation, for example standard deviations, would be different in the model with correlated shocks vs. the model without correlated shocks. Most importantly, the differences are not systematic, in the sense that there is no clear upward or downward bias. This set of results supports using correlated shocks as an additional test. We also estimate the Smets and Wouters (2007) model where the model without correlated shocks performs better (higher log-likelihood) compared to the model with correlated shocks. For this model, the robustness check using correlated shocks would have been successful.

2 Results

We estimate three influential DSGE model among the myriad of models in the literature.1 We select the paper by Bernanke et al. (1999), as it introduces the now commonly used financial accelerator, the Iacoviello (2005) paper, as it considers the housing market, and the Justiniano et al. (2011) paper, as an example of a now standard medium-size DSGE model with a large number of shocks.

These three papers are all published in top journals, have a large number of citations, and have also been selected because they have been published six years apart from each other, representing advances in modelling and estimation techniques. Further, they do consider increasing numbers of shocks (three, four, and six). Overall, they should be a good representation of the entire population of closed-economy DSGE models.

It should also be stressed that we do not compare the results across models. We are only interested in whether, for a given model, considering correlated shocks significantly changes the findings. Therefore, we estimate a different set of parameters for each model, consider a different set of observed time series (which is also necessary given the different number of shocks across models), but use the same time period for the estimation. We use Bayesian methods to estimate all model versions (see Herbst and Schorfheide, 2015 for details of the method). We check for convergence and use five MCMC chains with 100,000 draws each.

1We also estimate the Smets and Wouters (2007) model but could not find significant differences between the model with and without correlated shocks.
Iacoviello (2005) and Justiniano et al. (2011) models. Interestingly, we do not find a correlation between monetary and fiscal policy shocks. The Iacoviello (2005) model features the largest number of shocks and we find various cost-push shocks and a positive correlation between cost-push and technology shocks. The marginal positive and negative correlations across shocks. For example, investment-specific technology shocks are positively correlated with aggregate technology shock. In contrast, the marginal efficiency of investment shock to the marginalefficiency of capital. In the Bernanke et al. (1999) model, we find a negative correlation between monetary policy and the aggregate technology shock and the shock to the marginalefficiency of investment. In Table 1 presents our results. First, the results show that most of the correlations are significant. We begin by discussing the estimated correlations between the shocks for each model. Table 1 shows the estimated correlations between the shocks for each model. For example, in the Bernanke et al. (1999) model, we find a negative correlation between the aggregate technology shock and the shock to the marginalefficiency of investment. In the Iacoviello (2005) model, we find a negative correlation between monetary policy and cost-push shocks and a positive correlation between cost-push and technology shocks. The Justiniano et al. (2011) model features the largest number of shocks and we find various positive and negative correlations across shocks. For example, investment-specific technology shocks are positively correlated with aggregate technology shock. In contrast, the marginal efficiency of capital shock is negatively correlated with the investment-specific technology shock and the aggregate technology shock, in line with the results from the Bernanke et al. (1999) model. Interestingly, we do not find a correlation between monetary and fiscal policy shocks.

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<tr>
<td>$\sigma_{MP,Z}$</td>
<td>$-0.73$</td>
<td>$-0.8$</td>
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<td>$\sigma_{MP,ME}$</td>
<td>$0.89$</td>
<td>$(0.84,0.93)$</td>
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<tr>
<td>$\sigma_{Z,ME}$</td>
<td>$-0.56$</td>
<td>$(-0.71,-0.42)$</td>
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<tr>
<td>$\sigma_{MP,CP}$</td>
<td>$-0.38$</td>
<td>$(-0.52,-0.22)$</td>
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<td>$\sigma_{MP,HP}$</td>
<td>$-0.02$</td>
<td>$(0.17,0.12)$</td>
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<td>$\sigma_{CP,HP}$</td>
<td>$0.84$</td>
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<td>$\sigma_{CP,Z}$</td>
<td>$-0.13$</td>
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<td>$\sigma_{IS,Z}$</td>
<td>$0.36$</td>
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<td>$\sigma_{IS,PF}$</td>
<td>$0.32$</td>
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<td>$\sigma_{IS,MP}$</td>
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<td>$\sigma_{Z,FP}$</td>
<td>$-0.27$</td>
<td>$(-0.39,-0.15)$</td>
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Table 1: Estimated correlation between shocks for all three models considered. Shock names are: MP: Monetary policy, Z: aggregate technology, ME: marginal efficiency of capital, CP: Cost-push, HP: housing preference, IS: investment-specific technology, PF: preference, FP: fiscal policy.
Next, we discuss the differences in the estimated parameters shown in figure 1. In this figure, we plot the difference between the estimated parameter obtained from the model without correlated shocks minus the one obtained from the model with correlated shocks. First, we find that there is no clear pattern of over- or underestimating parameters when considering correlated shocks. In the Bernanke et al. (1999) model, the largest difference is obtained for the survival probability of entrepreneurs, the inverse of the Frisch elasticity, and the autocorrelation coefficient of the shock to the marginal efficiency of investment. While the survival probability is 0.89 in the model with correlated shocks, it is only half as large without correlated shocks. This has important implications, for example, for the dynamics of net worth. Further, the inverse of the Frisch elasticity is smaller (by 0.15) in the model with correlated shocks compared to the model without correlated shocks. This has implications for the link between wages and labor supply: a lower value implies a lower elasticity of the labor supply curve with respect to changes in wages. In the Iacoviello (2005) model, the largest difference is obtained for capital adjustment costs, the Taylor-rule coefficient on inflation, and the autocorrelation coefficient on the technology shock. Capital adjustment costs are 1.98 in the model with correlated shocks, 2.18 in the model without correlated shocks. Hence, capital will accumulate more in the model with correlated shocks. Finally, in the Justiniano et al. (2011) model we find the largest differences. The largest ones are obtained for the inverse Frisch elasticity, habit persistence in consumption, wage stickiness, the Taylor-rule coefficient on inflation, the autocorrelation parameter for the marginal efficiency of capital shock. For example, habit persistence is 0.96 and 0.81 in the model with correlated and uncorrelated shocks, respectively. This implies a much more persistent adjustment path of consumption in the model with correlated shocks. The inverse of the Frisch elasticity is slightly larger in the model with correlated shocks (2.86 vs. 2.38, with smaller differences for the Frisch elasticity: 0.35 vs. 0.42). Wage stickiness, directly affecting the volatility of wages and, hence, the hiring decision of firms, is 0.77 in the model with correlated shocks but is 0.65 in the model without correlated shocks.

Overall, we find relatively large differences in some estimated parameters that, importantly, affect model dynamics. Further, there is no clear pattern whether correlated shocks lead to an upward or a downward bias in estimating the structural parameters of the three DSGE models considered. We also stress that not all correlations are significant: in the Iacoviello (2005) model, housing preference shocks are not statistically significantly correlated with the other shocks. Relevant for policymakers, we also find differences in the Taylor-rule parameters describing monetary policy.

### 2.2 Variance Decomposition

In the previous section, we have shown that various shocks are significantly correlated and that this generates relevant differences in the estimated parameters. This section extends the analysis and considers differences in the driving forces of variations in the key macroeconomic variables (output and inflation) for the three DSGE models considered. Figure 2 presents our results.

In the Bernanke et al. (1999) model, we find that not considering correlated shocks leads to an overestimation of the effect of monetary policy in driving output (by approx. 25
Figure 1: Difference in estimated parameters across the models with and without correlated shocks. It is defined as the difference between the estimated parameter obtained from the model without correlated shocks minus the one obtained from the model with correlated shocks.
percentage points) and an underestimation of the effect of the aggregate technology shock (roughly 35 percentage points). A similar result is obtained for the inflation rate. For the Iacoviello (2005) model, we find that for output the cost push shock (shock to inflation) is overestimated, while the housing preference shock is underestimated. For inflation, we find the opposite result. Finally, in the Justiniano et al. (2011) model, we find that the impact of the monetary policy shock on output is overestimated while the investment-specific technology shock is underestimated. For inflation, we find that the monetary policy shock is underestimated, giving an overestimate for the government spending and aggregate technology shock.

Those findings are important as policymakers need to have a good understanding of the driving forces of business cycle variation in order to tailor a policy response. For example, supply-side shocks will, in general, have different effects compared to demand-side shocks.

### 2.3 Estimated Shocks

Finally, in this section we want to document differences in the time series of the estimated shocks. As an example, we consider the aggregate technology shock that is present in all three DSGE models. Figure 3 presents the estimated shocks for the baseline and the correlated shock model. Again, for policymakers it is important to know the source of the business cycle variation and the size of the shock. Our results show that considering correlated shocks does lead to different shock sizes and, even more importantly, different signs of the shocks. For the Bernanke et al. (1999) model, we find relatively small differences except for the early 1980’s, characterized by the double-dip recession. In the Iacoviello (2005) and the Justiniano et al. (2011) model, the differences are relatively small, and we do not observe large spikes as in the Bernanke et al. (1999) model. Correlated shocks also alter the volatility of the estimated shocks with heterogeneous effects. In the Bernanke et al. (1999) model, the volatility of the
simulated time series increases in the correlated shock specification compared to the baseline case, while there is a decrease in the Iacoviello (2005) and Justiniano et al. (2011) model. These are important insights for policy-makers.

Besides the size of the shock, we also find differences in the sign of the shock. This probably is even more important than the difference in the size of the shock as the policy response would be different for a positive vs. a negative shock. For all three models, we find that the sign of the difference does change frequently over time, implying different signs of the technology shock across the two versions of the model. In the previous section we have shown that the relative importance of the driving forces of business cycle variations varies across the specifications. In this section, we add that for each shock the importance in the time dimension is affected. For example, in the Justiniano et al. (2011) model we find that the technology shock in the baseline model is much more volatile and has larger deviations compared to the correlated shock model. This has implications for policy makers as it implies that the economy will be hit more frequently with (on average) larger shocks. For researchers it could affect the interpretation of which shocks drive recessions. For example, in the Justiniano et al. (2011) model the recession in the early 2000’s appears to be driven by a negative productivity shock when we look at the model without correlated shocks. When we look at the model with correlated shocks, the contribution of the technology shock appears to be much smaller.

These issues have implications for policy makers. First, it implies that the economy will be hit more frequently with (on average) larger shocks. For researchers it could affect the interpretation of which shocks drive recessions. Second, the policy response is likely different for demand- vs. supply-side shocks. For researchers it indicates that the model needs large variations along the supply-side to match the data. This could help to identify shortcomings in the model design.

Finally, we find interesting results for the distribution of the difference between the estimated technology shock with and without correlated shocks. While for the Justiniano et al. (2011) model, the distribution is close to a normal distribution (skewness: 0.08), the distributions for the Bernanke et al. (1999) and the Iacoviello (2005) model are not close to a normal one. For Bernanke et al. (1999), the distribution is right-skewed (skewness: 1.35) while for the Iacoviello (2005) model we find a left-skewed distribution (skewness: -0.76). This implies, on average, more positive differences between the two variants for the Bernanke et al. (1999) model and more negative once for the Iacoviello (2005) model. The implication of this finding is that in the Bernanke et al. (1999) model we observe more, larger shocks in the model without correlated shocks. This is important because it implies that technology shocks will hit the economy more often and their impact will be larger. This is relevant for policy makers in pinning down optimal policy and for researchers as this could indicate that the model is dominated by supply-side shocks. It could imply that changes to the demand-side of the model are needed to improve the model. The opposite holds for the Iacoviello (2005) model.
Figure 3: Estimated shocks for the model without correlated shocks (black, solid line) and the model with correlated shocks (red, dash-dotted line).

3 Conclusion

We investigate whether correlated shocks matter when we estimate workhorse DSGE models. Observing correlated shocks implies that the underlying DSGE model is misspecified. We argue that including correlated shocks in the estimation should become a standard routine to test the model. Further, the results can be used to understand the weaknesses of the underlying theoretical foundation of the model and can help to improve it.

We find that for the three state-of-the-art DSGE models, representing the majority of employed DSGE models in a closed-economy setting, results with correlated shocks do vary from the results without them. In particular, we document that differences in the estimation of structural parameters, e.g. changes in Taylor-rule parameters, the driving forces of fluctuations, and the size and sign of the estimated shocks. Importantly, there is no clear pattern of an upward or downward bias introduced by correlated shocks. Our results support our claim to use correlated shocks as an additional test. This holds true for researchers and for policymakers: including a correlated shock specification, at least as a robustness check, is important to avoid mismeasurement of policy responses.

Including correlated shocks implies that the moments of the estimated shocks and their frequency might be altered. In our analysis, we find that the economy will be hit more often with larger shocks. This holds true for the Bernanke et al. (1999) model and the opposite holds true for the Iacoviello (2005) model. These issues might helpfull to identify
shortcomings in the model design. For researchers, results might imply changing the view on shocks. For instance, the Justiniano et al. (2011) model implies the policy response might be different for demand vs. supply shocks and pinning down the model requires large variations along the supply-side to match the data. The results on distributive data from Bernanke et al. (1999) and Iacoviello (2005) show that there is no uniform pattern from including correlated shocks and some models might require fine-tuning the supply or demand-side of the underlying model.
References


4 Appendix

4.1 Data and Priors

We estimate the model using the data set from Smets and Wouters (2007). We use the following time series: GDP, investment, consumption, inflation, employment, and the Federal funds rate. Data is on a quarterly frequency for the United States and covers the period from 1947Q3 to 2004Q4 (230 observations). Non-stationary series are first-differenced.

Given the different number of shocks for each model, we use a different number of time series. For estimating the Bernanke et al. (1999) model, we use output, investment, and the interest rate. For the Iacoviello (2005) paper, we use output, investment, interest rate, and inflation. Finally, for the Justiniano et al. (2011) paper, we use output, investment, interest rate, inflation, consumption, and employment.

All shocks follow an Inverse Gamma distribution with mean 0.5 and standard error 2. All correlations between shocks are assumed to be normally distributed within the interval (-0.9,0.99) with mean 0.7 and standard deviation 0.5. Priors for the estimated parameters are chosen in line with the literature and are kept for the estimation with correlated shocks. Details on the prior choice for the other parameters is available upon request. Note that the choice of priors would only matter if we would believe that the prior choice affects the outcome if we introduce correlated shocks.