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### Efficiency or speculation? A dynamic analysis of the Bitcoin market

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#### Abstract

Bitcoin has recently been labelled as a “dangerous speculative bubble” by Nobel Prize-winning economists Joseph Stiglitz and Robert Shiller, as the Bitcoin's market value now exceeds the GDP of over 130 countries. In this study, the multifractality and efficiency of the Bitcoin price index are tested, using a nonlinear data analysis technique called the multifractal detrended fluctuation analysis (MF-DFA). In addition, we assess the time-variations in the market efficiency level through using a rolling-window framework. Our evidence shows that the efficiency of the Bitcoin market changes over time and this market seems to be more efficient during downward than upward periods. We also find that Bitcoin is marked by a persistent long memory phenomenon in its short- term components, which could be interpreted as a possible speculation by investors.

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*Abstract*

Bitcoin has recently been labelled as a “dangerous speculative bubble” by Nobel Prize-winning economists Joseph Stiglitz and Robert Shiller, as the Bitcoin’s market value now exceeds the GDP of over 130 countries. In this study, the multifractality and efficiency of the Bitcoin price index are tested, using a nonlinear data analysis technique called the multifractal detrended fluctuation analysis (MF-DFA). In addition, we test the time-variations in the market efficiency level through using a rolling-window framework. Our evidence shows that the efficiency of the Bitcoin market changes over time and this market seems to be more efficient during downward than upward periods. We also find that Bitcoin is marked by a persistent long memory phenomenon in its short- term components, which could be interpreted as a possible speculation by investors.

## 1. Introduction

The rapid development of global capital markets is changing the relevance and empirical validity of the Efficient Market Hypothesis (EMH). Over the last fifty years, the EMH has been a topic of intense debate and discussion. The EMH explains the movements in asset prices and the ability of investors to make abnormal profits. Since the EMH emerged in the 1960s (Fama, 1970), it has been a subject of investigation by a huge number of academic researchers seeking to ascertain its validity. Under the assumption of rational investors, this hypothesis postulates that asset prices completely reflect information and expectations, and that stock prices instantly reflect all available information. Fama (1970) differentiates among three forms of market efficiency but the most frequently assessed is the weak form. A market is proclaimed to be weak form efficient, if investors cannot utilize past information to predict future returns. Fama (1970) focused on the analysis of stock market efficiency, while Roll (1972) and Danthine (1977) were among the first authors to investigate the efficiency of commodity markets but provided controversial evidence. The efficiency of foreign exchange markets were also tested (see, for example, Cornell and Dietrich, 1978). Koutsoyiannis (1983) concentrated on the efficiency of the gold market and concluded that the market efficiency in this market cannot be refuted. The weak form EMH has been largely investigated in the literature for many traditional financial assets (He and Wang, 2017; Tiwari et al., 2017; Shahzad et al., 2017) and for several commodities (Wang et al., 2011; Kristoufek and Vovsrda, 2016).

The discussion about the efficiency of Bitcoin market is relatively scarce despite its increasing relevance among investors. Although this cryptocurrency has commonly been studied, the research community has remained focused on the legal, macroeconomic and financial aspects, the hedge and safe haven capabilities and the potential factors explaining its price. This paper does not seek to argue what the “true” or “fundamental” value of Bitcoin is, or to identify further the determinants of its price, but rather to test (i) whether the speculative behavior plays an important role in the Bitcoin market, and (ii) if the Bitcoin market follows the efficient market hypothesis. The majority of studies on the Bitcoin issue claim that this cryptocurrency is a speculative bubble rather than a long-term investment (Bouoiyour and Selmi, 2015; Ciaian et al., 2016). Bartos (2015) find that this market responds immediately to the arrival of new information, and thus can be proclaimed as an efficient market. Urquhart (2016) documents that the Bitcoin market is not weakly efficient over the selected full sample period.

The contribution of the paper is twofold: First, while the speculative nature of Bitcoin has been often proxied by the general interest in this cryptocurrency, this study seeks to explain the multifractal behavior of Bitcoin and the long-memory phenomenon observed in the short-term by the speculating attitudes of investors. Second, we test whether the efficiency of Bitcoin changes over time. Researchers have successfully applied the Multifractal Detrended Fluctuation Analysis (MF-DFA) technique to prove the multifractal behavior and the efficiency of different financial time series (Kristoufek and Vovsrda, 2016; Mensi et al., 2017; Shahzad et al., 2017; Tiwari et al., 2017). However, this study is the first to implement the MF-DFA method to investigate the multifractal properties of Bitcoin prices. In general, the price fluctuations in financial markets are governed by a very complex law. This complexity is due to the nonlinear interactions among heterogeneous agents and by events happening in an external environment. The MF-DFA is a dynamic approach that accounts for irregularities that may be embedded in the Bitcoin market’s behavior including nonlinearities, asymmetries, fat-tails and volatility clustering.

The remainder of the paper is organized as follows. Section 2 presents the methodology and the data. In Section 3 reports and discusses the empirical results. Section 4 concludes the paper.

## 2. Methodology and data

Following Kantelhardt et al. (2002), the MF-DFA method is a generalization of the Detrended Fluctuation Analysis, which consists of five steps. Let assume that  $\{x_t, t = 1, \dots, N\}$  be a time series of length  $N$ .

*Step 1:* we determine the “profile”  $y_k$  of the time series  $x(k)$  for  $k = 1, \dots, N$ , as:

$$y_k = \sum_{t=1}^k [x_t - \bar{x}], k = 1, \dots, N \quad (1)$$

where  $\bar{x}$  denotes the average over the whole time series.

*Step 2:* we divide the “profile”  $y_k$  into  $N_s \equiv N/s$  non-overlapping segments of equal length where  $s$  is the scale.

*Step 3:* we estimate a local trend by fitting a polynomial to the data. Thereafter, we calculate the variances by the two following formulas, depending on the segment  $v$ :

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^s \{Y[(v-1)s + i] - y_v(i)\}^2 \quad (2)$$

for  $v = 1, 2, \dots, N_s$ , and

$$F^2(s, v) = \frac{1}{s} \sum_{i=1}^s \{Y[N - (v - N_s)s + i] - y_v(i)\}^2 \quad (3)$$

for  $v = N_s + 1, \dots, 2N_s$ .

*Step 4:* By averaging the variances over all segments, we obtain the  $q^{\text{th}}$  order fluctuation function:

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F^2(s, v)]^{q/2} \right\}^{1/q} \quad (4)$$

where the index variable  $q$  can take any real values except zero. For  $q = 2$ , the standard DFA procedure is retrieved.

*Step 5:* we investigate the multiscaling behavior of the fluctuation functions  $F_q(s)$  by determining the slope of log-log plots of  $F_q(s)$  vs.  $s$  for various values of  $q$ .

$$F_q(s) \sim s^{h(q)} \quad (5)$$

The time series is multifractal if  $h(q)$  depends on  $q$ .

It is well documented that the generalized Hurst exponent  $h(q)$  defined by the MF-DFA is linked to the multifractal scaling exponent  $\tau(q)$  known as the Rényi exponent:

$$\tau(q) = qh(q) - 1 \quad (6)$$

Ultimately, we test the efficiency of the Bitcoin market by using the inefficiency index based on the multifractal dimension ( $IDM$ ), given by:

$$IDM = \frac{1}{2} (|h(-5) - 0.5| + |h(5) - 0.5|) = \frac{1}{2} \Delta h \quad (7)$$

The Bitcoin market is efficient if the value of  $IDM$  is close to zero, while if the values of  $IDM$  are strong they will indicate a less efficient market.

### 3. Empirical results

#### 3.1 Testing for the multifractal behavior of Bitcoin

In this paper, we implement the MF-DFA method over daily price data for the Coin Desk Bitcoin Price Index, covering the time period from July 1, 2010 to July 31, 2017. We begin our analysis by estimating the Hurst exponent for (i) the overall span of the data (from July 1, 2010 to July 31, 2017, and using (ii) the rolling window approach. The idea is expected to appropriately monitor how the Hurst exponent evolves over time by

constructing three alternative windows of distinct lengths. These windows have a size of 40%, 50% and 60% of the total numbers of observations (i.e., 1010, 1260 and 1520 observations, respectively), following Plakandaras et al. (2017) who suggested that there are no specific guidelines to select the window size but it should not be too large or too small because the findings might be subject to data snooping across distinct window sizes. Hence, the motivation here is only to show the robustness of our results with different window lengths where each window is selected based on the guidelines outlined by Plakandaras et al. (2017).

In short, this exercise serves to ascertain that intertemporal Hurst exponent fluctuations are well documented. We observe that the Hurst exponents exhibit large fluctuations and track each other but without any clear trend from April 2013 to April 2017 (see Figure 1). The 1010 observation DFA window diverges some in the second half of 2016.<sup>1</sup> This change can be related to some occasional events where there are beliefs that certain events happened in 2016 are mainly behind the Bitcoin price buoyancy. Some recent studies indicate that the global uncertainty is one of the potential driving forces of the Bitcoin price (see for example, Bouri et al., 2017). In addition, using a Bayesian quantile regression, Bouoiyour and Selmi (2017) show that the uncertainty surrounding China's deepening slowdown, Brexit and India's demonetization are the major contributors of the rises in the Bitcoin price when the market is improving. The anxiety over the results of the 2016 US presidential election was shown to be a positive determinant pushing up the price of Bitcoin when the market is functioning around the normal regime. However, the Venezuelan currency demonetization in December 2016 was found to be a fundamental factor affecting the Bitcoin price when the market is heading into decline.

**Figure 1. Rolling Hurst exponents for the three windows estimated with the DFA methodology**

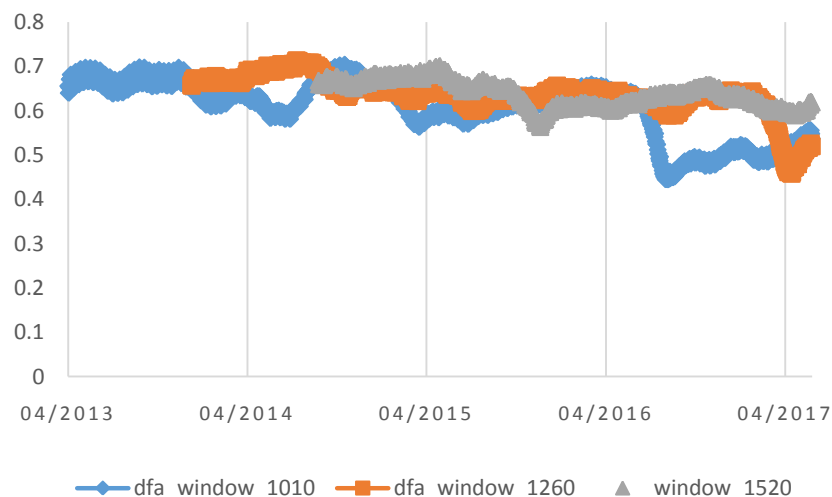


Figure 2 describes the multiscaling behavior of the fluctuations  $Fq(s)$  versus the time scales  $s$ . One crossover point can be observed which is attributed to a change in the properties of the time series at dissimilar scales of time.

<sup>1</sup>We also test the occurrence of nonlinearities in the daily Bitcoin series within a rolling window framework, using the MacLeod–Li test (McLeod and Li, 1983), the cobivariate Hinich test (Hinich and Patterson, 1985), the Tsay (1986) test and the BDS test (Brock et al., 1996). All of the tests reject the null hypothesis of linearity at the 1% level of significance in rolling windows. More detailed findings are available upon request from the authors.

**Figure 2. The Plotting of  $\log Fq(s)$  vs. logs of the Bitcoin returns**

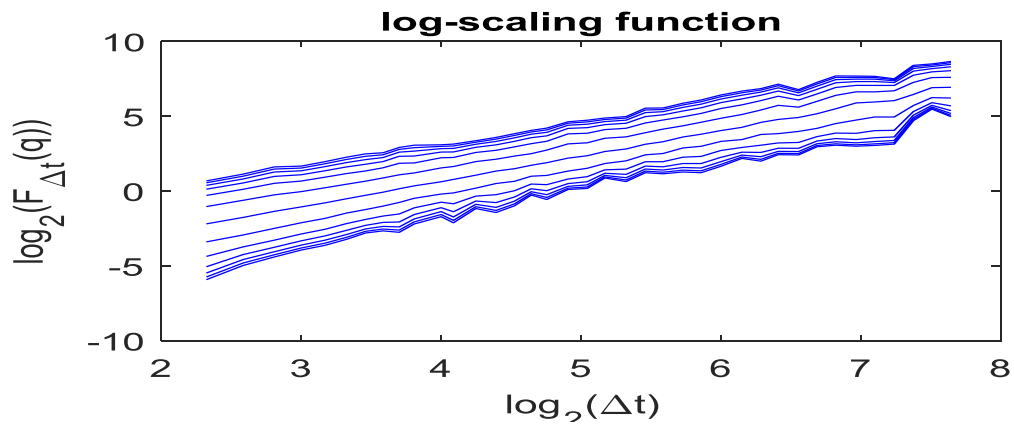
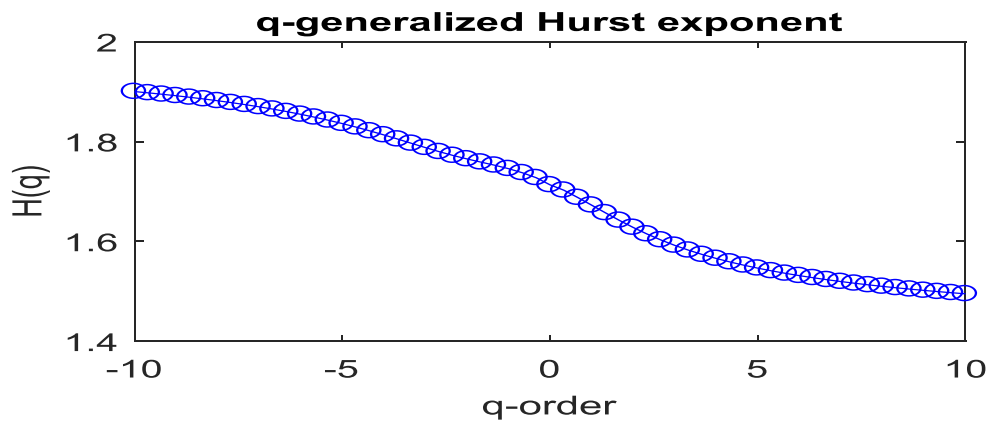


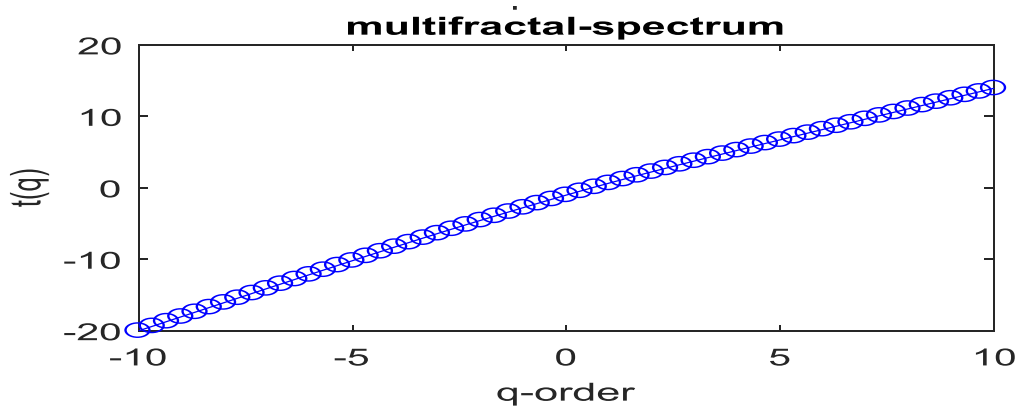
Figure 3 indicates that the function  $h(q)$  presents a nonlinear decreasing form for increasing values of  $q$ , which underscores the multifractal nature of Bitcoin.

**Figure 3. The Generalized Hurst Exponent  $h(q)$  vs.  $q$  for the Bitcoin returns**



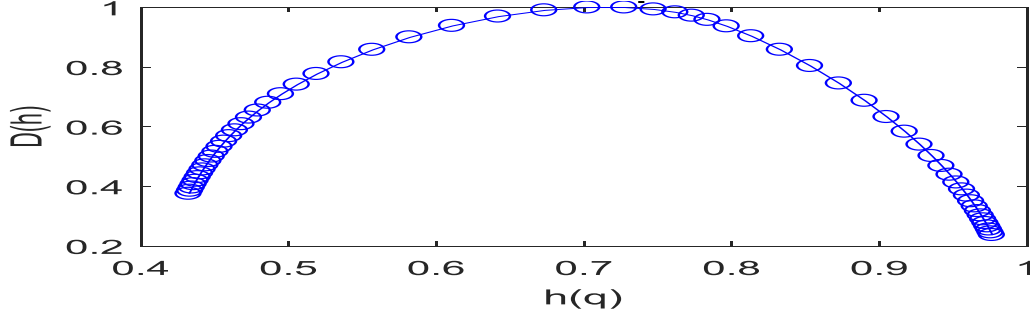
Another way to capture the multifractal behavior of Bitcoin is to measure the Rényi exponent  $\tau(q)$ . We observe from Figure 4 a nonlinear shape of the curve, highlighting the multifractality of Bitcoin.

**Figure 4. The Rényi Exponent  $\tau(q)$  vs.  $q$**



It is also shown from Figure 5 that the singularity spectrum curve of Bitcoin has an inverted parabola shape, confirming its multifractal nature.

**Figure 5. The Singularity Spectrum of the Bitcoin return**



The generalized Hurst exponents for various small and large time scales can reflect the autocorrelated behavior of the Bitcoin market in the short- and long-term horizons. For our case, we examine the different behaviors for the scales of both less than and more than 30 trading days. Table 1 displays the generalized Hurst exponents for  $s < 30$  and  $s > 30$  with  $q$  varying from  $-5$  to  $5$ . We note that all of the generalized Hurst exponents are larger than  $0.5$  for  $s < 30$ , highlighting that all kinds of the Bitcoin variations seem to be persistent in the short-term. However, the generalized Hurst exponents for  $s > 30$  diminish as  $q$  increases. Overall, the short-term behavior of the Bitcoin market is persistent but the long-term behavior is anti-persistent.

**Table 1. The generalized Hurst exponents of Bitcoin returns with  $q$  varying from  $-5$  to  $5$**

| $q$ | Original series: Bitcoin returns |          |
|-----|----------------------------------|----------|
|     | $s < 30$                         | $s > 30$ |
| -5  | 0.6453                           | 0.5987   |
| -4  | 0.7124                           | 0.6324   |
| -3  | 0.6815                           | 0.6192   |
| -2  | 0.5596                           | 0.5982   |
| -1  | 0.5318                           | 0.5641   |
| 0   | 0.5064                           | 0.5389   |
| 1   | 0.5976                           | 0.4126   |
| 2   | 0.5159                           | 0.3851   |
| 3   | 0.6342                           | 0.3214   |
| 4   | 0.6651                           | 0.2658   |
| 5   | 0.7038                           | 0.1943   |

Table 2 provides the mean values of  $IDM$  in Eq. (7) during periods of upward and downward linear trends of the Bitcoin prices. To test the significance of the difference of  $IDM$ , we utilize the following equation:

$$IDM_i = \alpha + \beta * D_i + \varepsilon_i \quad (8)$$

Where  $IDM_i$  describes the value of  $IDM$  defined in Eq. (7) for the Bitcoin return series in the  $i^{th}$  rolling window.  $D_i$  is a binary variable where  $D_i$  equals 1 if the Bitcoin price in the  $i^{th}$  time window shows an upward trend, and  $D_i$  equals 0 otherwise. Finally,  $\varepsilon_i$  is the stochastic noise.

The results are reported in Table 2. For different window lengths, we note that the *IDM* mean value during the upward period is weaker than during the downward period. Thus, the Bitcoin market seems to be more efficient over downward periods.

**Table 2. IDM mean values during downward and upward periods**

| Window lengths | Downward trends | Upward trends | t-statistics |
|----------------|-----------------|---------------|--------------|
| 1010           | 0.4867          | 0.3491        | -6.3722***   |
| 1260           | 0.4105          | 0.2652        | -8.1063***   |
| 1520           | 0.3248          | 0.1817        | -11.4261***  |

### 3.2. Sources of the multifractality

The characteristic of the data is examined in order to determine whether the source of multifractality stems from the fat-tail probability density function or from the long memory characteristics in the small and large fluctuations. To address these issues, we use the shuffled and the surrogated time series. Since the correlations do not exist in the shuffled data, the second type of multifractality can easily be determined, while the first type cannot be removed by the shuffling procedure because it arises from the frequency of observations. We can deduce that the main source of multifractality is due to the presence of fat-tail in the probability density function when the situation of  $h(q)$  in the original series does not change. However, if there is a multifractality originated from both types (i.e., the fat-tail probability density function and the different long memory features in the small and large fluctuations), then the shuffled series will unveil less a pronounced multifractality than the original series. To prove our argument, we have provided results for original data and compared these results with the shuffled series and the surrogated series. Table 3 summarizes the results of the generalized Hurst exponent  $h(q)$  versus  $q$  order for both the original and shuffled series. We find that the shuffled Bitcoin return series still have multifractality features, even though it seems lower than the original data. This highlights that the multifractality of Bitcoin returns mainly arises from the fat tails.

**Table 3. The generalized Hurst exponent  $h(q)$  values for different orders: Original series vs. shuffled series**

|                       | Original series |          |          | Shuffled series |          |          |
|-----------------------|-----------------|----------|----------|-----------------|----------|----------|
|                       | $h(q=-5)$       | $h(q=0)$ | $h(q=5)$ | $h(q=-5)$       | $h(q=0)$ | $h(q=5)$ |
| Bitcoin return series | 0.5786          | 0.5021   | 0.6931   | 0.5543          | 0.4689   | 0.6157   |

To be more effective in our analysis, we assess the multifractal spectrum width for the original and shuffled series. The results displayed in Table 4 clearly show that the multifractal spectrum widths of the Bitcoin return series are narrower in the shuffled data than in the original ones. This implies that while we have removed the correlations in the original series, multifractality remains in the shuffled, arising from the fat tails.

**Table 4. Multifractal Spectrum Width values**

|                       | Original series | Shuffled series |
|-----------------------|-----------------|-----------------|
| Bitcoin return series | 0.3691          | 0.2958          |

To effectively address whether the existence of multifractality is related to the fat-tails in the probability density function, we compute the surrogated data through the STAP method and display the generalized Hurst exponent findings of them together with the original data outcomes. The results reported in Table 5 reveal that the surrogated Bitcoin return series' statistics are not quite dissimilar from the original data. Given this



consideration, the gap between the various  $q$  orders for Bitcoin returns in the surrogated data is very modest. In addition, the  $h(q)$  values for the  $h=-5$  and  $h=0$  are almost the same. This means that there is a fat tails effect in the multifractality of the Bitcoin return data in addition to the long memory features. In short, these results underscore that investors or traders that make transactions on Bitcoins may be exposed to great risks. From a computational viewpoint, it is well known that among the most potential factors which determine these types of fat tails in the return distributions are the instant and extreme events in financial markets.

**Table 5. The generalized Hurst exponent  $h(q)$  values for different orders: Original series vs. Surrogated series**

|                       | Original series |          |          | Surrogated series |          |          |
|-----------------------|-----------------|----------|----------|-------------------|----------|----------|
|                       | $h(q=-5)$       | $h(q=0)$ | $h(q=5)$ | $h(q=-5)$         | $h(q=0)$ | $h(q=5)$ |
| Bitcoin return series | 0.5786          | 0.5021   | 0.6931   | 0.5621            | 0.4913   | 0.6483   |

#### 4. Conclusion

This paper seeks to assess from a new perspective the Bitcoin property related to the evolution of its market efficiency, by using the multifractal detrended fluctuation analysis. We find that generally observed irregularities including nonlinearities, asymmetries, fat-tails and volatility clustering are embedded in the Bitcoin market's behavior, which makes it a more risky but more profitable market for investors.

Further results show that Bitcoin market has more pronounced predictability power during downward periods than upward periods, indicating that investors could use the forecasting predictability to evaluate the risk based on the market condition and make better portfolio choices in downward and upward periods. Hence, investors have to use a tactical approach since holding a position towards Bitcoin over short-term horizons or during upward periods may lead to great investment losses.

We find also that all kinds of Bitcoin fluctuations are persistent in the short-term which could be interpreted by the great speculative and volatile behavior of Bitcoin.

Last but not least, we find that Bitcoin is still far from being closer to efficiency, and this may be due to many reasons including its infancy, speculative and volatile behavior, inelastic money supply and lack of legal security. Bitcoin will still very volatile as its future development remains unclear. The regulatory decisions will have also a significant effect on investors' attitudes. Currently, regulatory agencies remain weighing on a legal frame for cryptocurrencies and in particular Bitcoin. Moreover, in the face of recurring cyber attacks including MtGox, Instawallet, or Bithump, the Bitcoin ecosystem will have to strengthen its security standards to become largely accepted by traditional investors (Klein et al. 2018). Such evidence points to a pricing inefficiency and deeply encourage practitioners to introduce better instruments such as the Exchange Traded Funds as alternatives to traders and investors interested in having exposure to Bitcoin. In fact, the launch of Bitcoin futures would help tilt the scale a bit in the direction of Bitcoin. It would mitigate the risks associated to the lack of regulatory framework for Bitcoin. The Commodity Futures Trading Commission introduced specific rules for all speculators and investors in the futures contracts. Certainly, this would attract professional traders and then improve the trading volume in the market (Selmi et al. 2018).

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