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Heterogeneous expectations, collateral constraints and unconventional monetary policy

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Abstract

We consider the effects of heterogeneous beliefs in a general equilibrium model with endogenous collateral constraints and unconventional monetary policy. The heterogeneous expectations modify the way in which agents are restricted in the collateral. We numerically show that the relative optimism of the borrower makes him more leveraged and that this increases the welfare gains of unconventional monetary policy.

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1 Introduction

The last major economic crisis of 2008-2010 boosted new efforts in the economic literature towards understanding its nature and its relationship with other aspects of the economy, such as the role of optimistic/pessimistic agents in the crisis cycle and the effects of financial crisis policies in the presence of optimistic/pessimistic agents.

This paper is closely related to Araujo et al. (2015), in which a general equilibrium model with collateral and money, based on Geanakoplos and Zame (2014), is used to analyse the effects of unconventional monetary policy. However, their model does not consider the case of agents with heterogeneous beliefs. Using a basic general equilibrium model with collateral, Geanankoplos (2009) analysed heterogeneous beliefs in a context of the crisis cycle, highlighting the large losses of more optimistic and leveraged buyers.

Our main conclusions can be briefly summarized as follows. In an economy with two types of agents, buyer and lender, if the buyer is relatively more optimistic than the lender, then he will be more leveraged. If the buyer is relatively more pessimistic, he will then be short. The way in which the agents are restricted is of fundamental importance to understanding the welfare effects of unconventional monetary policy.

The remainder of this work is organized as follows: section 2 presents the model, section 3 presents the numerical results, and section 4 concludes.

2 Unconventional Monetary Policy with Heterogeneous Expectations

We consider a pure exchange economy with two periods $t = 0, 1$ and uncertainty about the states of nature in period 1, denoted by $s \in \mathcal{S} = \{1, \ldots, S\}$. There are $h \in \mathcal{H} = \{1, \ldots, H\}$ agents and 3 goods. Good 1 is perishable, good 2 is the service of the durable and good 3 is durable. The durable is used either as collateral to the privately issued financial contracts or for utilization of its service. Each household has a preference ordering defined over consumption plans $\mathbf{x}^h = (x^h_l, x^h_{1l}, \ldots, x^h_{Sl}) \in \mathbb{R}^{2(S+1)}_+$ specifying the household's consumption of each of goods $l = 1, 2$ in each of the states. We shall assume that households have identical preferences, and each seek to maximize expected utility $u^h = u(x^h_1, x^h_2) + \sum_{s=1}^{S} \pi^h_s u(x^h_{s1}, x^h_{s2})$, where $\pi^h_s > 0$ is the subjective probability agent h assigns to the occurrence of state s.

Definition 1. *If $\pi^{\tilde{h}}_s \neq \pi^{\hat{h}}_s$ for some $\tilde{h}$ and $\hat{h}$, then we say that the economy has "heterogeneous expectations" or "heterogeneous beliefs".*

There are $j \in \mathcal{J} = \{1, \ldots, J\}$ privately issued financial claims in the economy. Each asset j promises delivery of one unit of money in period 1, regardless of the states s. The collateral requirement of asset j is $C_j \in \mathbb{R}_+$; any issuer must hold $C_j \geq 0$ units of the durable in period 0 per unit sold of asset j. Given the possibility of default, the actual payoff of asset j in state s is $min(1, p_{s3}C_j)$ in units of money, where p_{s3} is price of the durable in state s of period 1. As in Araujo et al. (2012) we assume that only S assets are traded, $j = 1, \ldots, S$, such that: $C_j = 1/p_{j3}$ for each j is the endogenous determination of the collateral requirements.

Each agent has an initial endowment $e^h_1 \geq 0$ of the nondurable, $e^h_3 \geq 0$ of the durable in $t = 0$, an initial endowment $e^h_{s1} \geq 0$ of the nondurable in state s of $t = 1$ and they also receive a money transfer $d^h \geq 0$ in the first period. The Central Bank (CB) defines the interest rate

$i \geq 0$ and collects the money in the second period through a lump sum $\theta^h \in [0,1]$ taxation, where $\sum^h \theta^h = 1$.

The unconventional monetary policy is denoted by $\omega \in [0,1]$, which represents the fraction of aggregate durable of the economy that is purchased by the CB.

Given the prices of the goods $p \in \mathbb{R}^{3(S+1)}_{++}$ and the asset prices $q \in \mathbb{R}^J_+$, each household h chooses a consumption plan $\mathbf{x}^h \in \mathbb{R}^{2(S+1)}_+$, a portfolio of $\psi^h \in \mathbb{R}^J_+$ asset purchases, $\varphi^h \in \mathbb{R}^J_+$ asset issuances, $\mu^h \geq 0$ money, and a quantity $x^h_3 \geq 0$ of the durable good, in order to solve the following optimization problem:

$$
\begin{aligned}
&\max_{\mathbf{x}^h \geq 0,\, \psi^h \geq 0,\, \varphi^h \geq 0,\, \mu^h \geq 0,\, x^h_3 \geq 0} u^h(\mathbf{x}^h) \\
&\text{s.t.} \\
&p_1(x^h_1 - e^h_1) + p_2(x^h_2 - x^h_3) + p_3(x^h_3 - e^h_3) + q \cdot (\psi^h - \varphi^h) + (1+i)^{-1}(\mu^h - d^h) \ \leq \ 0, \\
&p_{s1}(x^h_{s1} - e^h_{s1}) + p_{s2}(x^h_{s2} - x^h_3) - \sum_{j=1}^{S} (\psi^h_j - \varphi^h_j) \min\{1, p_{s3} C_j\} \\
&+ \theta^h \left[(1+i)(p_3 - p_2)\omega \sum_h e^h_3 + \sum_h d^h - p_{s3}\omega \sum_h e^h_3 \right] - \mu^h \leq 0, \quad \forall\, s \in \mathcal{S} \\
&x^h_3 \ \geq \ \sum_{j=1}^{S} \varphi^h_j C_j
\end{aligned} \tag{1}
$$

An equilibrium for this economy is as follows:

Definition 2. *Let $(u^h(\cdot), e^h, d^h)$ be an economy with monetary specification $(i, \omega, \{p_{s1}\}_{s \in S})$. An equilibrium for this economy is a vector $((\overline{\mathbf{x}}, \overline{x}_3, \overline{\psi}, \overline{\varphi}, \overline{\mu}); (\overline{p}, \overline{q}); \overline{C})$ consistent with the monetary policy specification such that:*

(i) *for each $h \in \mathcal{H}$, $(\overline{\mathbf{x}}^h, \overline{x}^h_3, \overline{\psi}^h, \overline{\varphi}^h, \overline{\mu}^h)$ solves problem (1), given prices $(\overline{p}, \overline{q})$, the interest rate i and collateral requirements $\overline{C}$;*

(ii) $\sum_{h=1}^H \overline{x}^h_1 = \sum_{h=1}^H e^h_1$;

(iii) $\sum_{h=1}^H \overline{x}^h_2 = \sum_{h=1}^H e^h_3$;

(iv) $\sum_{h=1}^H \overline{x}^h_3 = (1-\omega) \sum_{h=1}^H e^h_3$;

(v) $\sum_{h=1}^H \overline{x}^h_{s1} = \sum_{h=1}^H e^h_{s1}$ *for each $s \in \mathcal{S}$;*

(vi) $\sum_{h=1}^H \overline{x}^h_{s2} = \sum_{h=1}^H e^h_3$ *for each $s \in \mathcal{S}$;*

(vii) $\sum_{h=1}^H (\overline{\psi}^h - \overline{\varphi}^h) = 0$;

(viii) $\sum_{h=1}^H \overline{\mu}^h = (1+i)(\overline{p}_3 - \overline{p}_2)\omega \sum_h e^h_3 + \sum_h d^h$;

(ix) $C_j = 1/\overline{p}_{j3}$ *for each $j \in J$*

The wealth that agent h chooses to transfer from state 0 to state s in the second period is denoted by the vector $y^h = (y_1^h, \ldots, y_S^h)$. When there are two states ($S = 2$), which is the case in our numerical analysis, the transference of wealth is defined by $y_s^h = \left(\frac{1+i}{p_{s1}}\right)\left[\mu^h + \frac{1}{1+i}(\psi_2^h - \varphi_2^h)\right] + \left(\frac{p_{s3}}{p_{s1}}\right)\left[x_3^h + (\psi_1^h - \varphi_1^h)C_1\right]$, $s = 1, 2$. This shows that the agent's optimization problem can be completely rewritten in terms of the vector of transferences. Each agent h chooses y^h that maximizes an indirect utility function subject to $p_{21}y_2^h \leq p_{11}y_1^h$ and $y_2^h \geq 0$. The first inequality is called the *short-sale constraint*, and the second one is the *leverage constraint*.[1]

Definition 3. *For $\mathcal{S} = 2$ and suppose that $s = 1$ is the good state and $s = 2$ is the bad state. If $\pi_{s=1}^{\tilde{h}} > \pi_{s=1}^{\hat{h}}$ then it is said that $\tilde{h}$ is relatively more optimistic than $\hat{h}$.*

3 Numerical Results

In this section we describe three numerical examples that illustrate how heterogeneous beliefs modify the collateral constraints and the welfare consequences of unconventional monetary policy. [2]

Following Araujo et al. (2015), the aggregate endowment of the economy at $t = 0$ is 7 units of good 1 ($\sum_{h=1}^2 e_1^h = 7$) and 7 units of good 3 ($\sum_{h=1}^2 e_3^h = 7$). At $t = 1$ we have 15 units of good 1 in $s = 1$ (good state), i.e. $\sum_{h=1}^2 e_{11}^h = 15$, and 6 units of good 1 in $s = 2$ (bad state), i.e. $\sum_{h=1}^2 e_{21}^h = 6$. There is no endowment of good 2. The amount of durable endowed by an agent at $t = 0$ will determine his role in the financial markets, a buyer or a lender. In the numerical analysis, the durable of agent $h = 1$ will be set to 0, so he will be called a *buyer*. Agent 2 will be called a *lender*. In this specific case of aggregate endowment, the endogenous collateral of the assets will be $C_1 = \frac{7}{15}$ and $C_2 = \frac{7}{6}$.[3] The CB sets $p_{s1} = 1$ for $s = 1, 2$ as the inflation target, the interest rate i to 0.1 and the taxation $\theta^1 = 0.9$ and $\theta^2 = 0.1$. The endowment of money is $d^1 = 0.0009$ and $d^2 = 0.0001$. All these parameters follow Araujo et al. (2015) in order to make our analysis comparable. The utility function is the standard Cobb-Douglas, defined by $u(x) = \log(x)$. In examples 1 and 2 we set $\omega = 0$ to study the buyer's constraints and, for the utility analysis, we saw the utility changes due to a marginal change in the central bank purchase to $\omega = 0.001$. In contrast, in example 3, we assessed the utility of the agents as function of ω, allowing it to take values in the interval $[0, 1]$.

3.1 Example 1: fixed probabilities and several endowment distributions

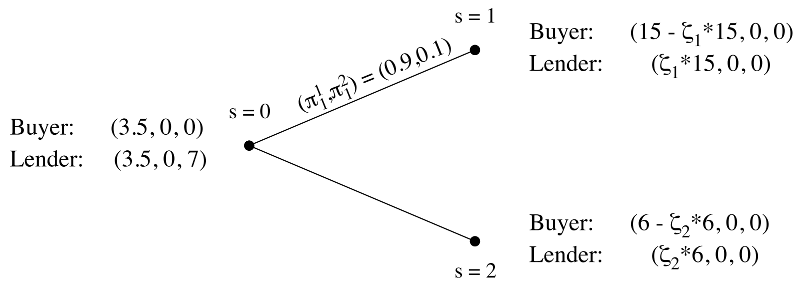
The endowment distribution and the probability are as follows:

[1] For more details on these properties see Araujo et al. (2015) .

[2] The computation of the equilibria follows Schommer (2013) and a Macbook Pro with an Intel Core i7, 2.5GHz and 16 GB 1600 MHz DDR3 was used for the computations. The tolerance in all cases was 10^{-08}.

[3] Assuming that the preferences are homothetic the relative prices can be determined from the economy's endowment pattern alone, see lemma 2 in Araujo et al. (2015).

Figure 1: Endowment and Probability Distribution



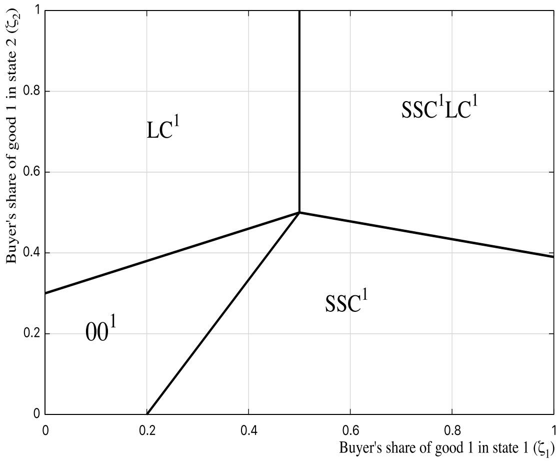
The variable ζ_1 will represent the *proportion* of perishable owned by the buyer in state 1 (i.e., $\zeta_1 = e^1_{11}/\sum_h e^h_{11} \in [0,1]$) and ζ_2 will represent the proportion of perishable owned by the buyer in state 2 (i.e., $\zeta_2 = e^1_{21}/\sum_h e^h_{21} \in [0,1]$). For this example we chose probabilities $(\pi^1_1, \pi^2_1) = (0.9, 0.1)$, which is the case where the buyer is optimistic (assigns greater probability to the good state) and the lender is pessimistic. We also compared this example with the homogeneous case $(\pi^1_1, \pi^2_1) = (0.5, 0.5)$, which is the one analysed by Araujo et al. (2015).

In the figure 2, we consider how the equilibrium changes as ζ_1 (on the horizontal axis) and ζ_2 (on the vertical axis). We use the following shorthand to report the way in which the collateral constraints bind in a given equilibrium. The symbol LC^h means that agent h has a binding leverage constraint in the economies of this region. Analogously, SSC^h means that he has a binding short sale constraint in these economies. Additionally, 00^h means that neither of the constraints are binding for agent h. For the same range of variation in the period-1 endowment patters, we show the signs of the marginal utilities when ω is increased by small enough amount: thus "++" means that the welfare of both agents increases; "$-+$" means that the welfare of the buyer (agent 1) decreases and the welfare of the lender (agent 2) increases; "00" to indicate that both derivatives are zero.

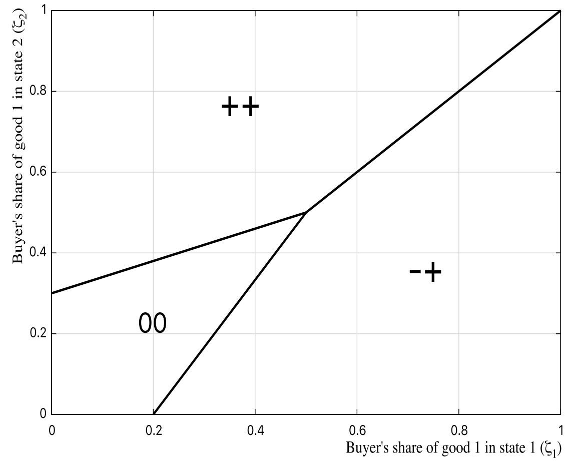
The difference between the shapes of the regions in panels (a) and (c) is due to the presence of heterogeneous expectations in the economy. We see that the higher the heterogeneity in the economy, the greater is the LC^1 region. Considering this movement in the LC^1 region, it is no surprise that the region of Pareto improvement increases as shown in panel (d). Indeed, Araujo et al. (2015) proved that unconventional monetary policy provides a welfare improvement (Pareto improvement) only in leverage constrained economies. This numerical result then shows how the heterogeneous expectations of the economy can potentialize the effects of the policy.

Figure 2: Pareto improvement in leverage constrained economies

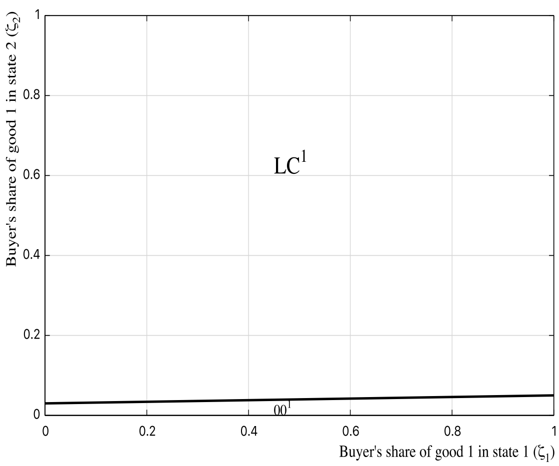
(a) Buyer's constraints:
homogeneous case $(\pi_1^1, \pi_1^2) = (0.5, 0.5)$



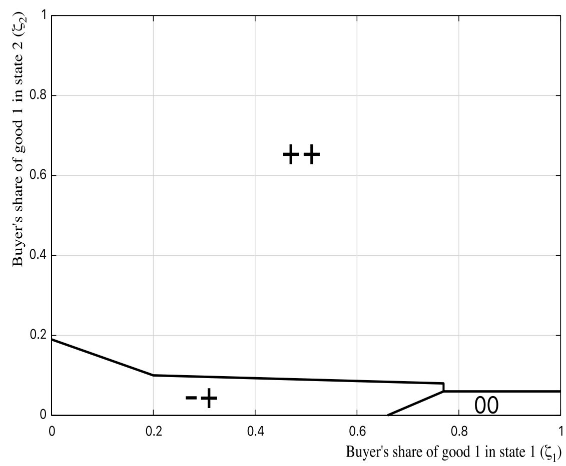
(b) Agent's welfare:
homogeneous case $(\pi_1^1, \pi_1^2) = (0.5, 0.5)$



(c) Buyer's constraints:
heterogeneous case $(\pi_1^1, \pi_1^2) = (0.9, 0.1)$



(d) Agent's welfare:
heterogeneous case $(\pi_1^1, \pi_1^2) = (0.9, 0.1)$

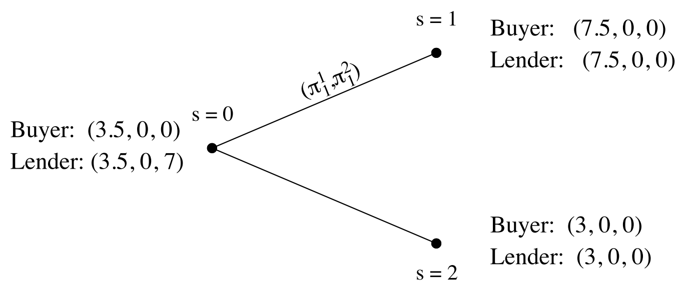


Concerning the lender's constraints, in the homogeneous case he has no binding constraint in any of the economies but in the heterogeneous case he is short sale constrained in the economies on the left side of the box. This explains the existence of some leverage constrained economies in (d) without Pareto improvement.

3.2 Example 2: fixed endowments and several probability distributions

The endowment distribution chosen for this example is symmetric at $t = 1$. Thus, the numerical results are more likely to be caused by differences in the subjective probabilities of the agents. The box that will be shown has π_1^1 on the x-axis, which is the probability the buyer assigns to the good state $s = 1$, and π_1^2 on the y-axis, which is the probability the lender assigns to the good state. (See figures 3 and 4)

Figure 3: Endowment Distribution

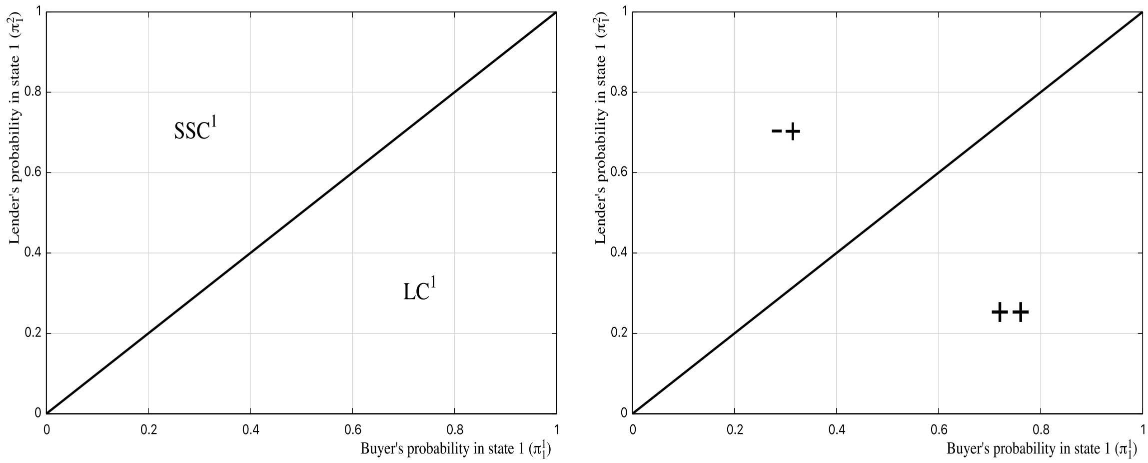


The main result in this case is that the heterogeneous beliefs is important in determining which constraint of the buyer will be binding. When he is more optimistic than the lender, he will be leverage constrained (LC^1) and this result is in tune with Geanankoplos (2009). When he is relatively more pessimistic, he will be short sale constrained (SSC^1) (see figure 4 panel (a)).

Indeed, the more optimistic agent thinks that the good state $s = 1$ is more likely to happen. Therefore, he wants to bring the maximum wealth possible from the bad state $s = 2$ to the good state $s = 1$. The need to accomplish this kind of transference makes him prone to be leverage constrained, making $y_2^h = 0$. Analogously, the more pessimistic agent chooses to decrease the wealth transferred to the good state, y_1^h, and increase the transference y_2^h to the bad state; thus he is more likely to be short sale constrained, making $p_{21}y_2^h = p_{11}y_1^h$.

Figure 4: Buyer's constraints

(a) Buyer's constraints (b) Agent's welfare



The constraints of the lender in this case are not binding in almost all economies. However, in some economies within the bottom-right region, when the relative optimism of the buyer is maximized, he is constrained in the short sale.

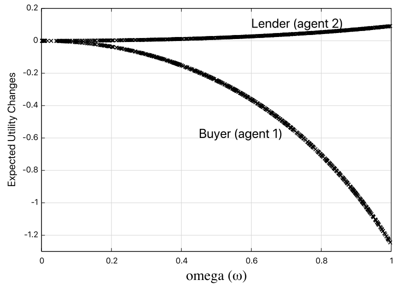
3.3 Example 3: fixed endowment and probability distribution with several levels of CB purchases (ω)

For this analysis we used the same endowment distribution of figure 3 with four probability distributions, including the homogeneous case for comparison. We are interested in analyzing the utility of the agents as a function of $\omega \in [0, 1]$.

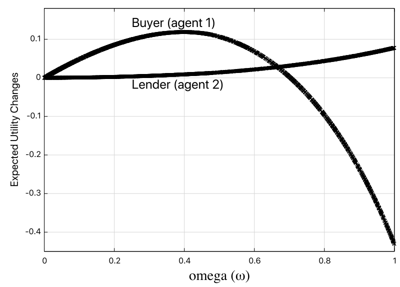
Each chart of figure 5 shows the relative change of the utility of the agents in comparison with the initial $\omega = 0$ case. In the homogeneous case (see panel (a) of figure 5) we do not have Pareto improvement, since the welfare of buyer is monotonically decreasing in ω over the entire feasible range of values. Generally speaking, when considering the heterogeneous cases, the higher the relative optimism, the greatest the range of Pareto improvement in ω. In panel (d), for example, the Pareto improvement happens until $\omega = 0.85$, approximately. And the maximum gain for the buyer‘s utility happens when he is optimistic and the lender is neutral, with almost 40%, as can be seen in panel (c).

Figure 5: Fixed endowment and fixed probabilities

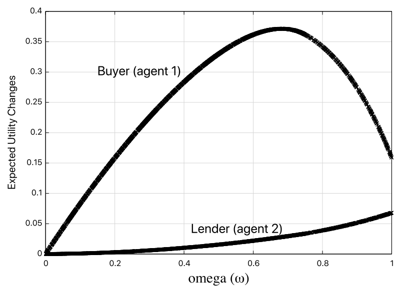
(a) case $(\pi_1^1, \pi_1^2) = (0.5, 0.5)$



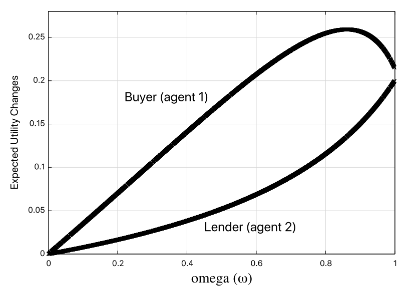
(b) case $(\pi_1^1, \pi_1^2) = (0.75, 0.5)$



(c) case $(\pi_1^1, \pi_1^2) = (0.9, 0.5)$



(d) case $(\pi_1^1, \pi_1^2) = (0.9, 0.1)$



4 Conclusions

We consider the consequences of heterogeneous beliefs in a general equilibrium model with endogenous collateral constraints and unconventional monetary policy. In our model, agents assign different probabilities to the states of nature. There are two types of agents in the economy, the buyer and the lender, and the unconventional monetary policy is defined by the purchase of risky assets by the Central Bank.

We have numerically shown that relative beliefs have a huge influence in determining the collateral constraints of agents. The relatively more optimistic one tends to be leverage constrained, and the relatively more pessimistic one tends to be short sale constrained.

It was found that when the buyer is relatively optimistic, the unconventional monetary policy was *more* effective in the sense that more economies benefited with a Pareto improvement. Thus, optimism *potentiates* the effect of this policy. In contrast, pessimism *softens* the effect of the policy since fewer economies benefit with a Pareto improvement (in this case). The theoretical reason behind this is that unconventional monetary policy can only produce a Pareto improvement in economies with leverage constrained agents, and optimism tends to make them leverage constrained.

Finally, optimism/pessimism modifies the amount of risky assets that the Central Bank must buy to reach the highest level of utility of the agents. The maximum gains in utility occur in economies where the buyer is more optimistic and the lender is neutral.

References

Araujo, A., Kubler, F., and Schommer, S. (2012) "Regulating Collateral-requirement when Markets are Incomplete" *Journal of Economic Theory* **147**, 450-476.

Araujo, A., Schommer, S., and Woodford, M. (2015) "Conventional and Unconventional Monetary Policy with Endogenous Collateral Constraints" *American Economic Journal: Macroeconomics* **7 (1)**, 1-43.

Geanakoplos, J. and Zame, W. (2014) "Collateral Equilibrium, I: a basic framework" *Economic Theory* **56**, 443-492.

Geanankoplos, J. (2009) "The Leverage Cycle" *NBER Macroeconomics Annual* **24 (1)**, 1-66.

Schommer, S. (2013) "Computing General Equilibrium with Incomplete Markets Collateral and Default Penalties" *Annals of Operations Research* **206**, 367-383.